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Cooperative Spectrum Sensing Via Sequential Detection: A Method to Reduce the Sensing Time

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Abstract

Spectrum sensing is one of the most important functions in cognitive radio systems. In this paper, we focus on reducing the sensing time in a cooperative spectrum sensing paradigm. In the proposed scheme, a sequential detection technique is employed to provide a robust and quick detection system. Each of the secondary users measures the log-likelihood probability of the received signals and then sequentially reports to the base station. Here, the maximum ratio combining (MRC) technique is employed to reduce the average sample number (ASN) in order to reduce the sensing time. This proposed scheme is analyzed and simulated to illustrate the performance in comparison with the other given methods. Analysis and simulation are provided to validate the proposed method.

Key words: Cognitive Radio, Sequential Detection, Spectrum Sensing, Cooperative Spectrum Sensing, Maximum Ratio Combining.

Ⅰ. Introduction

Stimulated by the increase in wireless services, a great increase in radio spectrum demand has resulted in a scarcity of any new radio frequency band allocations. This has prompted the need for a secondary system that is able to coexist with the existing licensed spectrum. Fortunately, this idea is practical because the scarcity of radio spectra is actually related to ineffective utilization rather than an actual shortage. A survey of the fixed granted band ranging from 30 MHz to 3 GHz has suggested the presence of numerous white spaces that would allow a secondary system to operate [1]. The Federal Communication Commission (FCC)'s interest in white spaces in TV bands [2] has prompted the investigation and development of this secondary system.

However, the secondary system must not harm the primary system transmission. The new system must also have low cost infrastructure installation and be compatible with the legacy primary system. From this aspect, cognitive radio technology (CR) is considered to be a competitive candidate when compared with approaches that use data base registry and beacon signaling [1]. At present, the IEEE 802.22 has developed a standard for the TV-band cognitive radio-based devices [3]. During this development, the problem of how to gain reliable detection in an environment of fading in the TV band has been the key challenge. Many spectrum sensing schemes were investigated for IEEE 802.22 standard and these can be categorized into two classes: single sensing and cooperative sensing schemes. Among spectrum sensing approaches, cooperative spectrum sensing is viewed as an effective method for overcoming the uncertainty of the noise channel. In this paper, we focus on how to improve the performance of a cooperative sensing method using a quick and robust detection method: sequential detection.

Traditionally, a spectrum sensing method employs the Neyman-Pearson criterion within a fixed sensing time and a predefined threshold. Unlike this traditional approach, the method of so-called sequential detection, first proposed by Wald, has an unfixed sensing time [4]. Recent research related to this method can be found in [5]. In addition, other recent papers exploit the technique to investigate spectrum sensing in cognitive radio networks, as shown in [6], [7]. In [6], Chaudhari et al. proposed a method that sequentially combines log-likelihood ratios tested from different cognitive radios, whereas the authors in [7] applied the technique to cyclostationary detection. Recently, Qiyue Zou, et al. [8] em-

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ployed sequential detection to cope with the effects of noise uncertainty. All of these techniques provide ways to reduce the sensing time and the number of signal samples.

Relying on previous work, in this paper, we consider a model in which the base station (BS) uses the MRC technique to reduce the detection time during the sequential spectrum detection. Sequential detection is also investigated in the cooperative spectrum sensing scheme. This paper considers the optimization for selection of the optimal weights that give the minimum average sample number. The structure of the article is divided into 5 parts: (Ⅰ) Introduction, (Ⅱ) System model, (Ⅲ) Analysis, (Ⅳ) Simulation, and (Ⅴ) Conclusion.

Ⅱ. System Model

Inheriting the previous framework in [8], in this paper, we employ collaborative spectrum sensing via sequential detection. Fig. 1 illustrates the model. In this paradigm, the log-likelihood statistical test is measured by each CR and then reported to the base station (BS). The base station is responsible for sequentially cumulating this statistical test and making a decision whenever the stop condition is reached. We also employ the MRC technique to minimize the average sample number, in order to reduce the sensing time. In this paper, we assume that the received signal collected by each CR has a Gaussian distribution. Two hypotheses, H_0 and H_1, are denoted as cases of absence and presence of the primary signal, respectively. The signal acquired by the m-th CR is given as follows

$$\mathrm{H}_0 : x_m[n] = s_{0,m}[n]$$
$$\mathrm{H}_1 : x_m[n] = s_{1,m}[n],\ n = 1,2,\ldots$$

where $s_{0,m}[n]$ is the n-th sample acquired by the m-th CR under H_0 and $s_{1,m}[n]$ is similarly for the respective signal under H_1. The probability of each sample under each hypothesis is accordingly given as $p_{0,m}(x_m[n])$ for the H_0 and $p_{1,m}(x_m[n])$ for the H_1. We also denote, by P_F and P_M, respectively, the false alarm and the miss detection of BS.

We describe the algorithm termed Algorithm 1 to collect the samples for global sensing. Within the model, each CR carries out its task by collecting and measuring the log-likelihood statistical test as given by:

$$z_m[n] = \ln\left(\frac{p_{0,m}(x_m[n])}{p_{1,m}(x_m[n])}\right),\ m \le M, \tag{1}$$

where M is the number of CRs required to cooperate.

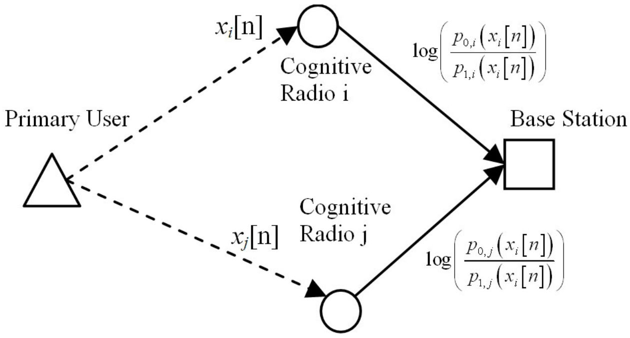

Fig. 1. System model.

The results are reported to the BS, where they are combined in order to make the global detection, as follows:

$$\tilde{z}[n] = \omega^T z[n] = \omega_1 z_1[n] + \omega_2 z_2[n] + .. + \omega_M z_M[n], \tag{2}$$

where ω is denoted as the weight vector $\omega = [\omega_1, \omega_2, .., \omega_M]^T$ and $\omega \ge 0$, and z is denoted as the log-likelihood vector for the signal reported by each CR $z[n] = [z_1[n], z_2[n], .., z_M[n]]^T$. In this model, we assume that the BS completely known regarding the distribution of measurements. The BS implements the global sensing by carrying out the cumulative sum LLR, as given by:

$$LLR(n) = \sum_{k=1}^{n} \tilde{z}[k] \tag{3}$$

BS continues to accumulate the statistical test, sample by sample, when the cumulative sum is still in the range of B and A, $B < LLR(n) < A$.Therefore, more samples are acquired until it reaches either a condition of $LLR(n) \le B$ or $LLR(n) \ge A$.

Algorithm 1

0: Base station set k = 0, LLR(0) = 0
1: **repeat**
2: $k = k + 1$
3: the m-th radio carries out the sensing and measure the likelihood statistic ratio $\ln\left(p_{0,m}(x_m[n]) / p_{1,m}(x_m[n])\right)$
4: the base station collects the log-likelihood ratio and makes a weighted cumulative sum as given, $LLR(k) = LLR(k-1) + \sum_{m=1}^{M} \omega_m \ln\left(\frac{p_{0,m}(x_m[n])}{p_{1,m}(x_m[n])}\right)$
5: **until** ($LLR(k) \le B$ *or* $LLR(k) \ge A$)
6: when $LLR(k) \le B$, H_0 is declared. Otherwise if $LLR(k) \ge A$, H_1 is declared.

Under the two hypotheses, $x_m[n]$ follows the Gaussian distribution. Under the H_0 hypothesis, we have $x_m \sim N\left(0,\sigma_{0,m}^2\right)$ and under H_1, $x_m \sim N\left(0,\sigma_{1,m}^2\right)$. We can expand (1) as follows:

$$\tilde{z}[n]=\sum_{m=1}^{M}\left(\frac{\omega_m}{2}\left(\frac{1}{\sigma_{0,m}^2}-\frac{1}{\sigma_{1,m}^2}\right)x_m^2[n]-\omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right)\right) \tag{4}$$

where g_m is defined as $g_m=\ln\left(\sigma_{0,m}/\sigma_{1,m}\right)$ and definition of f_m is that $f_m=\left(\sigma_{0,m}^{-2}-\sigma_{1,m}^{-2}\right)$. In the absence of a primary user, the expectation and variance of $\tilde{z}$ under the two hypotheses are respectively given as follows,

$$E_{H_0}\{\tilde{z}\}=\mu_0=\sum_{m=1}^{M}\omega_m\left(\sigma_{0,m}^2+g_m\right),$$
$$\mathrm{var}_{H_0}\{\tilde{z}\}=\sigma_0^2=2\omega_m^2 f_m^2\sigma_{0,m}^4,$$
$$E_{H_1}\{\tilde{z}\}=\mu_1=\sum_{m=1}^{M}\omega_m\left(\sigma_{1,m}^2+g_m\right),$$
$$\mathrm{var}_{H_1}\{\tilde{z}\}=\sigma_1^2=2\omega_m^2 f_m^2\sigma_{1,m}^4.$$

Ⅲ. Analysis

In this section, we investigate the performance of the detection and optimize the number of acquired samples subjected to the required false alarm and miss detection. In this analysis, we assume that the expectation and variance of the random variable $\tilde{z}$ are smaller than their thresholds. We denote B and A as the lower and upper thresholds, as given in the algorithm. This assumption for each case of hypothesis can be expressed as $|\mu_0|, |\sigma_0|, |\mu_1|, |\sigma_1| << |A|, |B|$. By this assumption, we can approximate the cumulative sums of the log-likelihood ratio whenever it reaches the stop condition as $LLR(n) \approx B$ for issuing the H_0 decision, and as $LLR(n)\approx A$ for the decision H_1. As confirmed by the previous work of Wald [9], we can obtain the proposition 1 statement.

3-1 Proposition 1

If the second moment of $\tilde{z}$ under the H_i is not zero, the number of samples taken by the sequential detection in Algorithm 1 is finite: $P\left(n<\infty \middle| H_i\right)=1$, for $i=\{0,1\}$.

Moreover, $\tilde{z}$ is a linear summation of chi-square random variables and its second moment always exists. In addition, it has other important properties, as described in proposition 2.

3-2 Proposition 2

Under the hypotheses of H_0 or H_1, a value of h_0, $h_1 \neq 0$ can always be found that satisfies $E_{H_0}\left\{e^{h_0\tilde{z}}\right\}=1$ and $E_{H_1}\left\{e^{h_1\tilde{z}}\right\}=1$.

Proof: See Appendix A.

The properties of fundamental identity that are confirmed by [9] and proposition 2 allow us to obtain the following equations:

$$E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=\left(E_{H_0}\left\{e^{h_0\tilde{z}}\right\}\right)^n=1,$$

where

$$E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=\left(1-P_F\right)E_{H_0}\left(e^{h_0 LLR(n)}\middle| LLR(n)\le B\right) + P_F E_{H_0}\left(e^{h_0 LLR(n)}\middle| LLR(n)\ge A\right)$$

and it is always $E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=1$. Similarly, we also have

$$E_{H_0}\left\{e^{h_1 LLR(n)}\right\}=\left(E_{H_0}\left\{e^{h_1\tilde{z}}\right\}\right)^n=1 \quad ,$$

where

$$E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=P_M E_{H_1}\left(e^{h_1 LLR(n)}\middle| LLR(n)\le B\right) +\left(1-P_M\right)E_{H_1}\left(e^{h_1 LLR(n)}\middle| LLR(n)\ge A\right)$$

and it is always $E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=1$.

We have made assumption as follows,

$$E_{H_i}\left\{LLR(n)\middle| LLR(n)\le B\right\}\approx B,$$
$$E_{H_i}\left\{LLR(n)\middle| LLR(n)\ge A\right\}\approx A.$$

We can obtain that $E_{H_i}\left(e^{h_i LLR(n)}\middle| LLR(n)\le B\right)\approx e^{h_i B}$ and $E_{H_i}\left(e^{h_i LLR(n)}\middle| LLR(n)\ge B\right)\approx e^{h_i A}$. Hence, the false alarm and miss detection probability can be derived as in the following equations:

$$E_{H_0}\left\{e^{h_0 LLR(n)}\right\}=P_F e^{h_0 A}+\left(1-P_F\right)e^{h_0 B}=1$$

Therefore,

$$P_F=\frac{1-e^{h_0 B}}{e^{h_0 A}-e^{h_0 B}}. \tag{5}$$

In addition,

$$E_{H_1}\left\{e^{h_1 LLR(n)}\right\}=\left(1-P_M\right)e^{h_1 A}+P_M e^{h_1 B}=1 \quad .$$

Thus, the miss detection probability can be expressed as,

$$P_M = \frac{e^{h_1 A} - 1}{e^{h_1 A} - e^{h_1 B}}. \tag{6}$$

Given h_0 and h_1, equations (4) and (5) enable us to derive the thresholds A and B. It is easier to identify h_0 and h_1 if we make the assumption that the large samples are taken in sequential manner (this assumption is true even when ASN is minimized using a suitable weight for each received signal). Due to the large number of samples, the weighted combining based log-likelihood signal $\tilde{z}$ can be assumed as the Gaussian random variable that follows the distribution of $\mathcal{N}(\mu_0, \sigma_0^2)$ under H_0, and $\mathcal{N}(\mu_1, \sigma_1^2)$ under H_1. From proposition 2, we already obtained that $E_{H_i}(e^{h_i \tilde{z}}) = 1, i = \{0,1\}$. Using the Laplace transform method, we can achieve the following,

$$E_{H_i}(e^{h_i \tilde{z}}) = e^{\mu_i h_i + \frac{1}{2}\sigma_i^2 h_i^2} = 1.$$
$$\Rightarrow h_i = \frac{-2\mu_i}{\sigma_i^2} \tag{7}$$

As confirmed by A. Wald in [9], we can rewrite the average sample number in each case of H_i as given by,

$$E_{H_0}\{N_{stop}\} = \frac{(1-P_F)E_{H_0}^{*} + P_F E_{H_0}^{**}}{E\{\tilde{z}\}},$$
$$E_{H_1}\{N_{stop}\} = \frac{P_M E_{H_0}^{*} + (1-P_M) E_{H_0}^{**}}{E\{\tilde{z}\}}$$

where $E_{H_i}^{*} = E_{H_i}(LLR(n) | LLR(n) \le B)$ and $E_{H_i}^{**} = E_{H_i}(LLR(n) | LLR(n) \ge A)$. We can rewrite the above equations as given by:

$$E_{H_0}\{N_{stop}\} \approx \frac{(1-P_F)B + P_F A}{E\{\tilde{z}\}} = \frac{A - B - Ae^{h_0 B} + Be^{h_0 A}}{E_{H_0}\{\tilde{z}\}} \tag{8}$$

and

$$E_{H_1}\{N_{stop}\} \approx \frac{P_M B + (1-P_M)A}{E\{\tilde{z}\}} = \frac{A - B - Ae^{h_0 B} + Be^{h_0 A}}{E_{H_1}\{\tilde{z}\}}. \tag{9}$$

We make the assumption that we can approximate the values of $e^{h_0 A}$ and $e^{h_0 B}$ as follows:

$$e^{h_0 A} = 1 + h_0 A + \frac{(h_0 A)^2}{2} + O(h_0 A) \approx 1 + h_0 A + \frac{(h_0 A)^2}{2}$$

where is denoted as the big-o notation. Similarly, we have,

$$e^{h_0 B} = 1 + h_0 B + \frac{(h_0 B)^2}{2} + O(h_0 B) \approx 1 + h_0 B + \frac{(h_0 B)^2}{2}$$

Substituting the values of h_0, h_1, μ_0, and μ_1, we obtain derivation of (8) and (9), as given in the following expressions:

$$E_{H_0}\{N_{stop}\} = \frac{-AB}{\sigma_0^2 - \mu_0(A+B)} \approx \frac{-AB}{\sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{0,m}^4 - (A+B)\sum_{m=1}^{M}\omega_m(\sigma_{0,m}^2 + g_m)} \tag{10}$$

and

$$E_{H_1}\{N_{stop}\} = \frac{-AB}{\sigma_1^2 - \mu_1(A+B)} \approx \frac{-AB}{\sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{1,m}^4 - (A+B)\sum_{m=1}^{M}\omega_m(\sigma_{1,m}^2 + g_m)} \tag{11}$$

In following this analysis, our purpose is to select the weights yielding the lowest average sample number. Equivalently, the problem can be generalized as given:

$$\text{Minimize } Max\{E_{H_0}\{N_{stop}\}, E_{H_1}\{N_{stop}\}\}$$
$$s.t. \quad \sum_{m=1}^{M}\omega_m = 1, \quad \omega_1, .., \omega_m, .., \omega_M > 0$$

The problem can be resolved by dividing it into two minimization problems as shown in Algorithm 2, as described as follows:

Algorithm 2
Step 1:

$$\underset{\omega}{\text{Maximize}} \quad \left\{\sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{0,m}^4 - (A+B)\sum_{m=1}^{M}\omega_m(\sigma_{0,m}^2 + g_m)\right\}$$

$s.t.$

$$\sum_{m=1}^{M}\omega_m = 1$$
$$0 < \sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{1,m}^4 - (A+B)\sum_{m=1}^{M}\omega_m(\sigma_{1,m}^2 + g_m) <$$
$$< \sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{0,m}^4 - (A+B)\sum_{m=1}^{M}\omega_m(\sigma_{0,m}^2 + g_m)$$
$$\omega_1, .., \omega_m, .., \omega_M > 0$$

We denote vector $\hat{\omega}$ as the selected weight of this step.

Step 2:

$$\underset{\omega}{\text{Maximize}} \quad \left\{ \sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{0,m}^4 - (A+B) \sum_{m=1}^{M} \omega_m \left(\sigma_{1,m}^2 + g_m \right) \right\}$$

s.t.

$$\sum_{m=1}^{M} \omega_m = 1$$

$$0 < \sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{0,m}^4 - (A+B) \sum_{m=1}^{M} \omega_m \left(\sigma_{0,m}^2 + g_m \right) <$$

$$< \sum_{m=1}^{M} 2\omega_m^2 f_m^2 \sigma_{1,m}^4 - (A+B) \sum_{m=1}^{M} \omega_m \left(\sigma_{1,m}^2 + g_m \right)$$

$$\omega_1, .., \omega_m, .., \omega_M > 0$$

We denote $\breve{\omega}$ as the selected weight of this step.

Step 3 :

The selected weights are identified as the given:

$$\omega_{\min} = \arg\ \min \left\{ E_{H_0} \left\{ N_{stop}, \hat{\omega} \right\}, E_{H_1} \left\{ N_{stop}, \breve{\omega} \right\} \right\}$$

Ⅳ. Simulation

In this part, we survey a case when BS selects $M = 4$ SU users in its region. Each of the SUs tolerates an amount of zeros mean AWGN with the variances, as given: $\sigma_{0,m}^2 = 1,\ 0.9,\ 0.8,\ 0.7$. In the case of PU presence, the variances of the acquired signal at each CR are as follows, $\sigma_{1,m}^2 = 1.1,\ 1,\ 0.9,\ 0.8$. The desired false alarm and miss detection probability are the same and are set at the values from 10^{-3} to 0.1. Fig. 2 is shown to compare the average sample number of the proposed method to the equally combining method with conventional detection and to the sequential detection using the equally gain combining (EGC) technique (which uses an equal weighting value). The conventional detection [8] using the fixed number of samples has the highest number of samples to achieve the required false alarm and miss detection probability. This is clear evident to show how robust is the sequential technique. In order to survey the performance of the proposed approach, we compared the proposed method and sequential detection using the EGC technique, which uses equal weighting values. A significant reduction in the average sample number is noted for the proposed method. Fig. 3 shows how well the sequential detection in those schemes follows the designed false alarm and misses detection. The simulation shows that, at some points, especially in the small range of the designed false alarms and miss detections, the conventional detection provides a mildly higher false alarm than the requirements. Both of the schemes of sequential detection also guarantee the designed requirements.

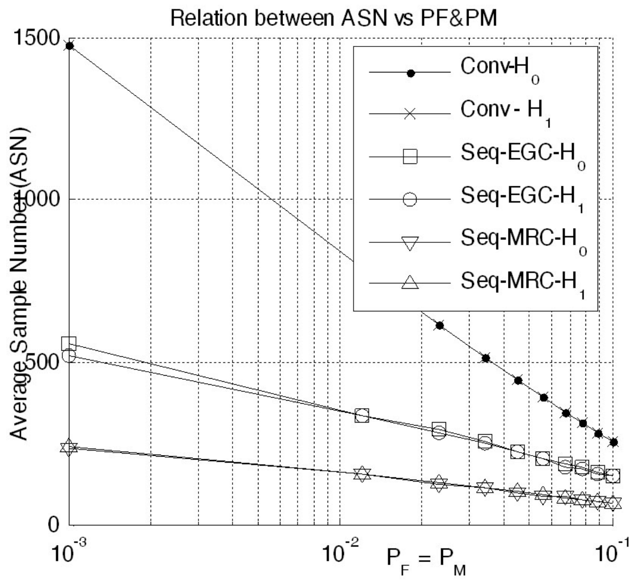

Fig. 2. The comparison of the average sample number between the traditional detection, the sequential detection using the EGC technique vs. the proposed sequential detection using the MRC technique.

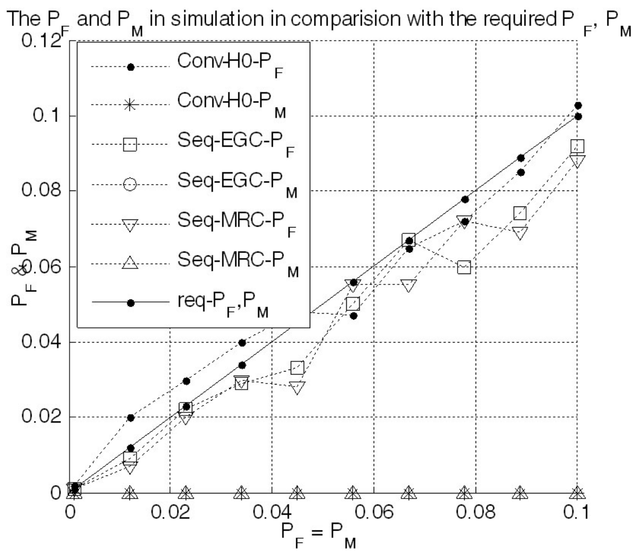

Fig. 3. The comparison of error in detection between the traditional detection, the sequential detection using the EGC technique vs. the proposed sequential detection using the MRC technique.

Ⅴ. Conclusion

In this paper, we have concentrated on reducing the average sample number in cooperative spectrum sensing via sequential detection. We have given an analysis for using the method of MRC in this model of spectrum sensing. Algorithms to obtain the model of sensing and optimization are also proposed and given during the analysis. Simulation shows that the proposed method has significantly reduced the time of sensing, which is very important in a Cognitive Radio Network.

Appendix A

In this part, we prove that under H_0 or H_1, it is always possible to find a value of h_0, $h_1 \neq 0$ that satisfies $E_{H_0}\{e^{h_0 \tilde{z}}\}=1$ and $E_{H_1}\{e^{h_1 \tilde{z}}\}=1$.

Proof:

For the H_0 case, we have

$$E_{H_0}\{e^{h_0 \tilde{z}}\} = \int_{-\infty}^{+\infty} e^{h_0 \sum_{m=1}^{M} \tilde{z}} \, dP(x_1, x_2, .., x_M). \tag{A1}$$

Because x_1, x_2 ..., x_M are independently distributed, the integral can be rewritten as:

$$E_{H_0}\{e^{h_0 \tilde{z}}\} = \prod_{m=1}^{M} \int_{-\infty}^{+\infty} \exp\left(\left(\frac{\omega_m}{2}\left(\frac{1}{\sigma_{0,m}^2} - \frac{1}{\sigma_{1,m}^2}\right) x_m^2[n] - \omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right)\right)\right) p_{0,m}(x_m)\, dx_m. \tag{A2}$$

Hence, we have:

$$E_{H_0}\{e^{h_0 \tilde{z}}\} = \prod_{m=1}^{M} \frac{e^{h_0 \omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right)}}{\sqrt{1 - h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)}} \tag{A3}$$

with the condition that $h_0 < \min_{\omega_m, m=1..M} \left\{ \frac{1}{h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)} \right\} = h_{0\max}$.

We have that $E_{H_0}\{e^{h_0 \tilde{z}}\}=1$ is equivalent to the equation as given,

$$\prod_{m=1}^{M} \frac{e^{h_0 \omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right)}}{\sqrt{1 - h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)}} = 1$$
$$\Leftrightarrow g(h_0) = 0 \tag{A4}$$

where

$$h_0 \sum_{m=1}^{M} \omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right) - \frac{1}{2} \sum_{m=1}^{M} \ln\left(1 - h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)\right) = 0 \tag{A5}$$

Taking the 1st and 2nd order of derivation, we achieve:

$$\frac{\partial g(h_0)}{\partial h_0} = \frac{1}{2} \sum_{m=1}^{M} \omega_m \left(\frac{1 - \sigma_{0,m}^2 / \sigma_{1,m}^2}{1 - h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)} + 2 \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right) \right) \tag{A6}$$

$$\frac{\partial^2 g(h_0)}{(\partial h_0)^2} = \sum_{m=1}^{M} \frac{\omega_m^2 \left(1 - \sigma_{0,m}^2 / \sigma_{1,m}^2\right)^2}{\left(1 - h_0 \omega_m \left(1 - \frac{\sigma_{0,m}^2}{\sigma_{1,m}^2}\right)\right)^2} \tag{A7}$$

We have $\frac{\partial^2 g(h_0)}{(\partial h_0)^2} > 0$ because tha t $\sigma_{0,m}^2 < \sigma_{1,m}^2$. Therefore, $\frac{\partial g(h_0)}{\partial h_0}$ monotonically increases for $h_0 \in (-\infty, h_{0\max})$. As $h_0 \to -\infty$, the $\frac{\partial g(h_0)}{\partial h_0}$ approaches to $-\sum_{m=1}^{M} \omega_m \ln\left(\frac{\sigma_{0,m}}{\sigma_{1,m}}\right)$ and as $h_0 \to h_{0\max}$, $\frac{\partial g(h_0)}{\partial h_0}$ approaches to $+\infty$. Therefore, a value $\tilde{h}_0$ always exists at which $\frac{\partial g(h_0)}{\partial h_0}$ vanishes. We can see that $g(h_0)$ monotonically decreases from $+\infty$ to $g(\tilde{h}_0)$ in the range $(-\infty, \tilde{h}_0)$ and monotonically increases from $g(\tilde{h}_0)$ to $+\infty$ in the range $(\tilde{h}_0, h_{0\max})$. On the other hand, we also have $g(0)=0$, so this is always $g(\tilde{h}_0) \le g(0)$. Furthermore, we always have $g(0) \neq 0$, so a value of $h_0 \neq 0$ exists when $g(\tilde{h}_0) < g(0)$ where $g(h_0)=0$. Hence, we always find a value $h_0 \neq 0$ which satisfies $E_{H_0}\{e^{h_0 \tilde{z}}\}=1$.

Similarly, we can achieve the same conclusion for the hypothesis H_1.

Hence, the proposition is completely proved.

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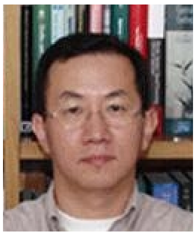

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