

Estimation of geometry-based manufacturing cost of complex offshore structures in early design stage

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ABSTRACT: *A scalar metric for the assessment of hull surface producibility was known to be useful in estimating the complexity of a hull form of ships or large offshore structures by looking at their shape. However, it could not serve as a comprehensive measuring tool due to its lack of important components of the hull form such as longitudinals, stiffeners, and web frames attached to the hull surface. To have a complete metric for cost estimation, these structural members must be included. In this paper, major inner structural members are considered by measuring the complexity of their geometric shape. The final scalar metric thus consists of the classes containing inner members with various curvature magnitudes as well as the classes containing curved plates with single and double curvature distribution. Those two distinct metrics are merged into a complete scalar metric that accounts for the total cost estimation of complex structural bodies.*

KEY WORDS: Scalar metric; Curvature; Geometric shape; Manufacturing cost; Offshore structures.

INTRODUCTION

Ships and offshore structures such as drillships, Floating Production Storage Offloading (FPSOs), and semi-submersibles have very complicated geometrical shapes. Without considering a number of small equipment on the topside, the hull comprising the outer body of the floating parts and the inner structural members that provide the strength of the body consists of curved and flat panels. The curved parts of the structure are hard to manufacture and thus result in longer manufacturing time, even by a skilled worker. Reducing the number of curved parts is desirable but unrealistic because of other engineering factors that affect the performance of offshore structures. If the designer is able to estimate the complexity of the body in advance, however, he or she can choose a better one from a set of various candidates. This guideline will be useful as it helps the designer to try as many different variations as possible without building a real body. Owners or clients who are going to build a structural body can also benefit from the guideline before they place an order.

A few publications about the cost-related index in hull form design have been released. A pioneering approach with a manufacturing point of view was published by Parsons, Nam and Singer (1999), who dealt with the curvature of hull plates and verified the usability of the metric by applying to a conventional tanker. The papers on the fabrication of curved part of hull using the thermo-elasto-plastic analysis (Shin, Kim and Nam, 2004; Sin, Ryu and Nam, 2004) provided technical background in understanding the difficulty of manufacturing curved objects. Although their results are directly linked to the manufacturing cost, they are useful in estimating the effort to analytically make the curved parts. Rigtterink and Singer (2009) investigated the manufacturing cost of stiffened members in term of the thickness of members and attached angles. They tried to include the

effect caused by the difficulty of welding. The feature-based costing method by Caprace and Rigo (2012) is an approach based on calculating the total cost of design during the early stage of ship design. To assess the cost for each individual operation, they suggest setting up a feature-based model. Given that the comprehensive nature of their model, immense time and effort are needed in advance.

In this work, a scalar metric that estimates the complexity of structural bodies with respect to their geometric shapes is introduced. A variety of new techniques and assumptions have been adopted to take more structural members into consideration, with the aim to produce more accurate cost estimation. Investigation on the manufacturing costs in shipbuilding industry is briefly introduced. Differential geometry used in the analysis of curved parts is reviewed. The concept of scalar metric is explained and followed by setting up a complete metric that enables the designer to estimate the approximate manufacturing cost based on the geometric shapes of complex hull forms.

MANUFACTURING COSTS

Whenever the designer determines the hull form of ships or offshore structures, he or she considers various requirements related to performance, strength, and other functional factors. Unlike the automobile industry, the shipbuilding industry does not identify aesthetic shape as a primary concern. But it is not uncommon to hear that a good shape generates a good performance from experienced designers, which implies that the shape of structures must be taken into account as well. The simpler and better-looking shape has been known to excel the complicated one in many applications. It is not surprising that the shape itself is directly associated with manufacturing costs. In this work, the shapes of ships or offshore structures are investigated in terms of how significantly these factors affect manufacturing costs.

The term, manufacturing cost, is so broad that it covers every possible expense that goes into the total cost of a product. In shipbuilding, it generally includes all costs incurred during the whole manufacturing process, shown in Fig. 1, such as pre-treatment, cutting, shaping, assembly, fabrication, and erection.

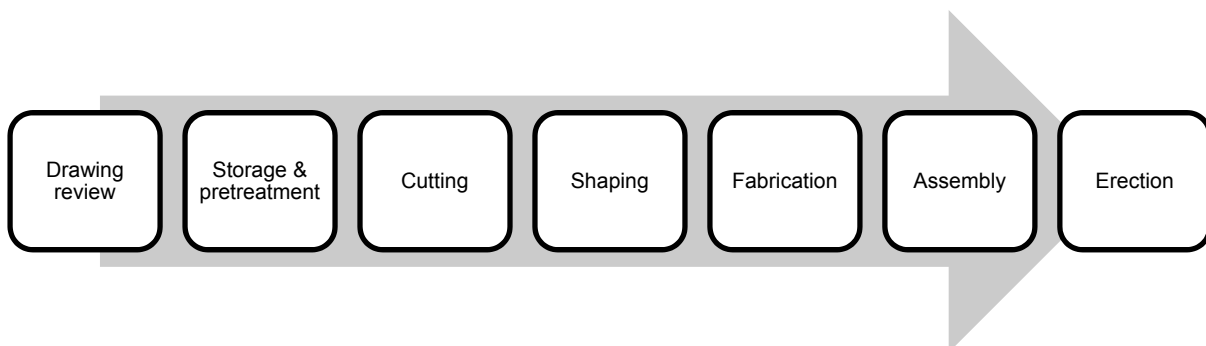


Fig. 1 Typical shipbuilding process.

In early stage ship design, it is not possible to figure out all the costs where only the rough shape of hull form is available. However, it is possible to create a tool that generates a cost guideline for the rapid estimation of cost impact in advance. Even though a design lacks detailed information needed to generate true cost estimations, one of the available properties at the initial design stage is the geometric information of structures and that can be used to develop “cost comparisons”.

The cost function presented that forms the basis of the scalar metric is solely based on the geometry of structures. Therefore, manufacturing cost discussed in this work refers to the costs involved in the process of making a desirable geometric shape. This metric, however, should not be underestimated because the subsequent processes such as welding and assembly are also strongly dependent on the geometric shapes.

PRELIMINERIES OF GEOMETRIC ENTITIES

In analyzing the geometric shape of complex structures where a number of curved parts exist, curves and surfaces need to be investigated. The Non Uniform Rational B-Spline (NURBS) representation is adopted to define those curves and surfaces in

this work. A brief review of NURBS and its geometric properties such as curve length, surface area, curvatures, and classification of a point on surface is summarized. For detailed descriptions, the reader is referred to classic differential geometry books (Struik, 1961; Millman and Parker, 1977) and NURBS books (Farin, 2002; Farouki, 2008).

NURBS representation

A three-dimensional curve $C(t)$ is defined by:

$$C(t) = \frac{\sum_{k=0}^N w_k V_k B_k^n(t)}{\sum_{k=0}^N w_k B_k^n(t)} \tag{1}$$

where $N + 1$ is the number of control points and $B_k^n(t)$ is the B-spline basis function of degree n on a given knot sequence $T=(t_0, t_1, \dots, t_{N+n+1})$.

A surface $S(u, v)$ is an extension of the curve. Similarly, a surface is defined by:

$$S(u, v) = \frac{\sum_{j=0}^M \sum_{k=0}^N w_{jk} V_{jk} B_j^m(u) B_k^n(v)}{\sum_{j=0}^M \sum_{k=0}^N w_{jk} B_j^m(u) B_k^n(v)} \tag{2}$$

where $M + 1$ control points in u and $N + 1$ in v define a control net V_{jk} , w_{jk} are corresponding weights, and $B_j^m(u)$ and $B_k^n(v)$ are the B-spline basis functions with knot vectors $U=(u_0, u_1, \dots, u_{M+n+1})$ and $V=(v_0, v_1, \dots, v_{M+m+1})$ in u and v parametric directions, respectively. Unless the conic sections such as circles or parabolas are seriously considered, the denominator may be set to unity by assigning the same weights in both curve and surface representations for simplicity.

Curve length and curvature

The length L of a curve for region $[a, b]$ is represented by:

$$L = \int_a^b |C'(t)| dt \tag{3}$$

The curvature of the three-dimensional, generalized curve $C(t)$ is defined by:

$$\kappa = \frac{|C'(t) \times C''(t)|}{|C'(t)|^3} \tag{4}$$

where the curvature is expressed in terms of magnitude. This curvature represents the curved amount of a space curve.

Fundamental forms in differential geometry

A surface has an infinite number of curves lying on it. A curve on a surface $S(u, v)$ can be represented in parametric form:

$u = u(t)$ and $v = v(t)$. The vector $\frac{d}{dt}$ at a point p of the surface can be written in a form independent of the choice of parameter: $dS = S_u \cdot du + S_v \cdot dv$.

The distance between two points on a curve is obtained by $ds = \sqrt{dS \cdot dS}$ along the curve. Substituting ds for the values in dS results in I , the first fundamental form:

$$I = Edu^2 + 2Fdudv + Gdv^2 \tag{5}$$

where $E = E(u, v) = S_u \cdot S_u$, $F = F(u, v) = S_u \cdot S_v$, and $G = G(u, v) = S_v \cdot S_v$.

The second fundamental form, II , is defined by dS and dn :

$$II = -d\mathbf{S} \cdot d\mathbf{n} = Ldu^2 + 2Mdudv + Ndv^2 \tag{6}$$

where dn is derived from the unit normal $n = n(u, v)$ to S at p : $dn = \mathbf{n}_u du + \mathbf{n}_v dv$ and L , M , and N are

$$L = L(u, v) = -\mathbf{S}_u \cdot \mathbf{n}_u = \mathbf{S}_{uu} \cdot \mathbf{n}$$

$$M = M(u, v) = -\frac{\mathbf{S}_u \cdot \mathbf{n}_v + \mathbf{S}_v \cdot \mathbf{n}_u}{2} = \mathbf{S}_{uv} \cdot \mathbf{n}$$

$$N = N(u, v) = -\mathbf{S}_v \cdot \mathbf{n}_v = \mathbf{S}_{vv} \cdot \mathbf{n} \tag{7}$$

Surface area, curvature, and shape

The area A of a region R on the surface is calculated by using the first fundamental form:

$$A = \iint_R \sqrt{EG - F^2} dudv \tag{8}$$

For bivariate surface representation, the area of a subdivided parallelogram, whose four corners are defined by (u_i, v_j) , (u_i, v_{j+1}) , (u_{i+1}, v_{j+1}) , and (u_{i+1}, v_j) , can be approximated by:

$$A_{ij} = \sqrt{EG - F^2} \Delta u \Delta v \tag{9}$$

where $\Delta u = u_{i+1} - u_i$ and $\Delta v = v_{j+1} - v_j$. Then, the total area is simply the sum of all parallelograms.

When the osculating plane of a curve on a surface is perpendicular to the tangent plane of the surface at a point p , the curvature of the curve is called the normal curvature of the surface in the direction of the tangent vector of the curve at p . The normal curvature κ_n , which determines the curved amount of a surface, is expressed in terms of the first and second fundamental forms shown in Eq. (10).

$$\kappa_n = \frac{Ldu^2 + 2Mdudv + Ndv^2}{Edu^2 + 2Fdudv + Gdv^2} \tag{10}$$

The normal curvature κ_n varies and has a maximum and a minimum value in two perpendicular directions. These two extreme values are called the principal curvatures denoted as κ_1 and κ_2 .

The product of the two principal curvatures is called the Gaussian curvature K and has very important geometric interpretation. From the Gaussian curvature K , a point p on a surface can be classified into three different categories depending on the sign of K , as summarized in Table 1. It can be readily deduced that the manufacturing cost of a surface is dependent on K .

Table 1 Classification of a surface point.

K	Geometric form	Surface shape
Positive	Elliptic	Convex or concave
Zero	Parabolic or flat	Ruled or planar
Negative	Hyperbolic	Saddle shaped

SCALAR METRICS

The total form of scalar metric introduced in this work is composed of two sub components: one for hull plate and the other for structural members attached to the hull plate. In the work presented, those separate metrics are combined into an integrated metric for cost estimation.

Scalar metric for hull plates

The scalar metric for outer hull plate of ships or complex offshore structures is assumed to be fundamentally proportional to the difficulties based on the amount of curved shape of a hull plate. For the development of various classes of curved hull plates, the following four assumptions were made by applying a group technology to ship production: 1) the plates have no twist; 2) only normal dimensional control is required; 3) large shipyard technology is used as a basis for expert opinion; and 4) each product is made with a specified process. Based on those assumptions, Lamb (1994, 1995) classified the forming of hull plates based upon the non-dimensional backset of the plating, where the backset was defined as the rise of the plate above a flat plane divided by the length of the plate, as depicted in Fig. 2. The curvature is then conveniently expressed in terms of non-dimensional backset.

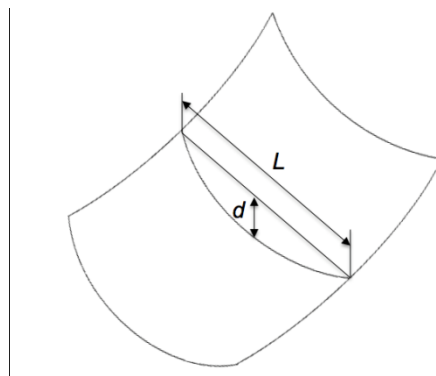


Fig. 2 Definition of backset (d/L).

In the author’s earlier work, the relative costs of hull plate forming were categorized as eight principal types (Parsons, Nam and Singer, 1999). For the work presented in this paper, the unit cost for each type has been revised and updated through consulting with some experts in charge of bending and line heating processes in Korean large shipyards, based on the basic concept of the previous work. New proposed cost values adjusted to compensate the characteristics of modern large shipyards are tabulated in Table 2. The backset ratios, b_1 and b_2 , are defined along two perpendicular directions of a plate, where b_1 is always taken as the larger.

Table 2 Relative manufacturing costs for curved hull plates.

Type	Parameter	b_1	b_2	Normalized Relative Cost
Flat plate		$0.0 \leq b_1 \leq 0.01$	$0.0 \leq b_2 \leq 0.003$	0.083
Simple low curvature in one direction		$0.01 \leq b_1 \leq 0.15$	$0.0 \leq b_2 \leq 0.003$	0.166
Simple high curvature in one direction		$0.15 \leq b_1$	$0.0 \leq b_2 \leq 0.003$	0.250
Moderate curvature in one direction and small backset in the other		$0.01 \leq b_1 \leq 0.075$	$0.003 \leq b_2 \leq 0.025$	0.250
Moderate double curvature		$0.075 \leq b_1 \leq 0.15$	$0.025 \leq b_2 \leq 0.05$	0.500
Moderate reverse double curvature		$0.075 \leq b_1 \leq 0.15$	$-0.05 \leq b_2 \leq -0.01$	0.750
High double curvature		$0.15 \leq b_1$	$0.05 \leq b_2$	0.666
High reverse double curvature		$0.15 \leq b_1$	$b_2 \leq -0.05$	1.000

To apply the eight types, a surface is subdivided into a finite number of small pieces. Taking advantage of the NURBS representation, the surface can be easily divided by assigning the total number of pieces along two parametric directions, u and v , from Eq. (2). The principal curvatures are computed at the center of each piece and the amount of bending is converted into backsets in the directions of principal curvatures. A scalar metric for a piece is mathematically represented by the product of cost value and area. Then the total cost of the surface is the sum of the cost of all pieces. The overall procedure to find the scalar metric is depicted in Fig. 3.

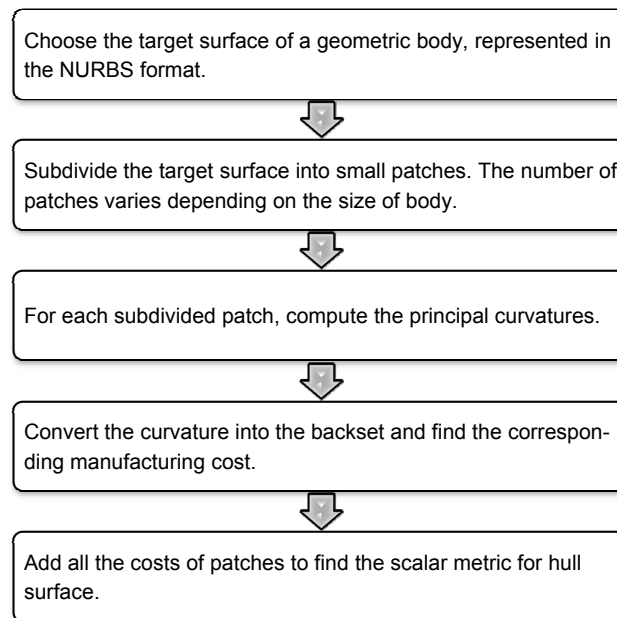


Fig. 3 Overall procedure to find scalar metric for hull surface.

The scalar metric for hull plate (SM_H) is normalized to exclude the influence of the size of surface, as shown in Eq. (11), where V_{ij} is the cost value found from Table 2, ΔA_{ij} is the area of corresponding piece, and i and j represent the indices along two parametric directions, respectively.

$$SM_H = \frac{\sum_j \sum_i V_{ij} \cdot \Delta A_{ij}}{\sum_j \sum_i \Delta A_{ij}} \quad (11)$$

The goal of this research is to provide a metric to assess the possible cost impact of one hull form to another, rather than simply suggesting a cost estimate of a hull form. Thus, the magnitude of the scalar metric itself has no special meaning. The scalar metric becomes significant when it is compared with that of other shapes. It implies that the higher the number is, the more expensive it becomes to manufacture.

It should be noted that the concept of the scalar metric is subjective and dependent on shipyard processes and equipment. It should also be stressed that the metric values may change between shipyards but they may also be different between shops within one shipyard. As variables can be altered according to manufacturing practice, this concept is believed to suggest a reasonable guideline for initial cost estimation.

Interpolation of manufacturing cost function for hull plates

The manufacturing costs investigated in our work are discrete in their values, jumping across the boundary of each range. Therefore, when the curvature of a hull plate falls into the two adjacent regions, it becomes ambiguous due to discontinuity across the boundary. In the author's earlier work, the fuzzy concept was utilized to resolve this problem (Parsons, Nam and Singer, 1999). Even though the use of the fuzzy theory can be powerful in handling the transition between two discrete values, it still possesses a problem of providing a 'right' membership function that guarantees the smooth transition. The linearized

transition pattern used in the earlier work can be replaced by an elegant mathematical method represented by an interpolated surface.

An interpolation scheme that facilitates the smooth transition across the boundary is adopted to resolve the ambiguous discontinuity issue. Within each range, the manufacturing cost is regarded as the mean value at the center of ranges. Then a three-dimensional surface that wraps all the manufacturing costs is generated. More values along the boundary of the surface are necessary for the complete interpolation of the costs. The interpolated cost function represented as a wrapping patch is displayed in Fig. 4. For the given backsets along two directions, the manufacturing cost is interpolated from the surface.

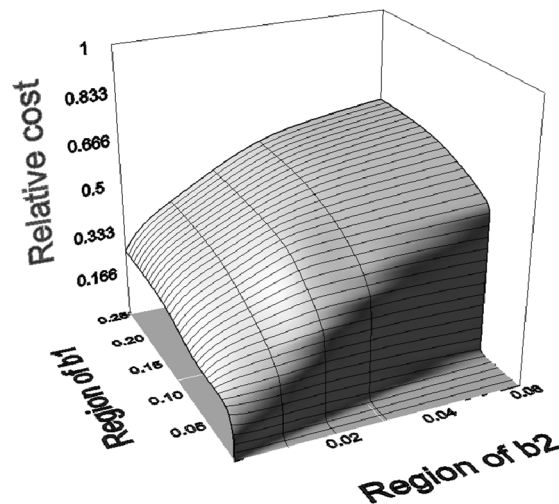


Fig. 4 Interpolated cost function.

Scalar metric for inner structural members

In general, ships or complex offshore structures contain a large quantity of inner members such as longitudinals, stiffeners, girders or so forth. Each inner member is designed to satisfy the strength requirements and other specific rules depending on the shipping class. Unfortunately, only rough layout, instead of detailed information such as thickness and width, of those members is available in the early design stage.

A typical fore part of a ship that has an initial arrangement of inner structural members is shown in Fig. 5. Due to the fact that these members must follow the contour of the hull plate, additional manufacturing costs are incurred and hence should be included in the estimation of final cost.

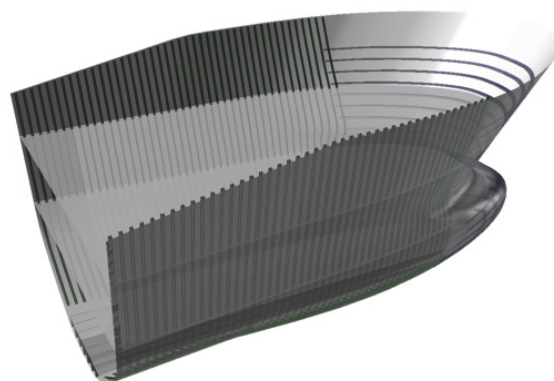


Fig. 5 Fore part of a ship with inner structural members attached.

A vertical structural member attached to the hull is shown in Fig. 6, where the right top inset depicts the way the member aligned with the hull shape and the right bottom shows a space curve representing the boundary of the member. Deriving the

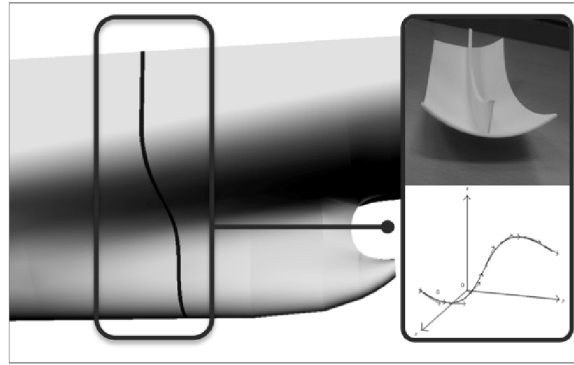


Fig. 6 A vertical structural member as a space curve.

manufacturing cost for inner structural members is a straightforward process and several methods have been evaluated (Rigterink and Singer, 2009). From the strong dependency of the inner structural members to the hull plate where the members attached, the concept of intersection curves is used in the formulation of curving amount of structural members. Each intersection curve is either planar or three-dimensional space curve that can be mathematically represented by the NURBS representation of Eq. (1).

The structural members considered in this work are of relatively long shape that has the large length-height ratio. The amount of curved shape is determined by examining the curving amount of a boundary edge attached to the hull.

A space curve possesses some geometrically significant characteristics such as bending, torsion, and inflection that affect the amount of curving. Thus, those characteristics should be factored in the manufacturing cost. Instead of applying the complicated mathematical consideration of factors, however, we computed the amount of curving of a space curve by looking at the curvature at a point of the curve. The curve is first subdivided into a series of substantially small pieces. This simplification enables us to estimate the degree of curving tendency of a curve without involving mathematically complicated formulas. For example, the inflection property can be eliminated if the infinitesimal pieces are divided at inflection points. The twisting property does not need to be similarly considered; however, further scrutiny is recommended for future research.

Similar to the surface case, the manufacturing costs of inner structural members were derived by consulting with the experts in shipyards. A simplification process that converted their manufacturing concept into a meaningful value was also accomplished. Table 3 summarizes the relative manufacturing costs for curved inner members according to the amount of curving.

Table 3 Relative manufacturing costs for inner structural members.

Type	Parameter	κ	Normalized Relative Cost
Flat		$0.0 \leq \kappa \leq 0.002$	0.25
Low curvature		$0.002 \leq \kappa \leq 0.1$	0.50
Moderate curvature		$0.1 \leq \kappa \leq 0.2$	0.75
High curvature		$\kappa > 0.2$	1.00

The scalar metric for inner members (SM_I) is defined as Eq. (12). Here, V_k is the cost value summarized in Table 3 and Δl_k is the length of a subdivided curve piece. Likewise, the product of value and sub-length is divided by the total curve length to be independent of curve length.

$$SM_I = \frac{\sum_k V_k \cdot \Delta l_k}{\sum_k \Delta l_k} \tag{12}$$

It should be pointed out that the manufacturing costs of the curved structural member are strongly dependent on the shape of curved hull plate where the member attaches. Their relationship must be simultaneously considered to derive the coupled cost.

Relationship between two metrics

Two separate scalar metrics for hull plates and inner members have been proposed. Each scalar metric is essential in estimating the cost of corresponding geometric parts. For the complete analysis of the cost estimation, a single metric that reflects the coupled influences of those two relevant metrics is required. Since coupling influences are difficult to determine with the information available in early stage design, a simple blending approach is used. Considering that the shape of inner members is positively dependent upon that of a hull form, it is reasonably deduced that those two metrics should be interrelated with respect to their manufacturing difficulties.

One approach is to assign the blending factors to the metrics to account for their contributions to the final metric. The blending factors, called the balancing weights, are subject to several causes or environmental reasons. The balancing weights, w_H and w_{IM} , are included in Eq. (13) where the formulation of total manufacturing metric consists of two separate costs: $w_H \cdot SM_H$ from a hull and $w_{IM} \cdot SM_{IM}$ from inner members. The subscripts H and IM represent the hull and inner members, respectively.

$$SM_{Total} = w_H \cdot SM_H + w_{IM} \cdot SM_{IM} \quad (13)$$

A major factor affecting the difference of the balancing weights is caused by the distinct manufacturing practice of each shipyard, which means different manufacturing processes for curved hull plates and inner members result in different costs after all. The relative cost values for the hull and inner members are simultaneously compared and analyzed. The best way to identify their relationship would be obtain work orders or schedule data that detail the amount of time for each manufacturing process. Most shipyards, unfortunately, have not released those numbers for security reasons, and some yards simply do not have written documentation. In this work, a direct approach was used: surveying workers on site on their relative working efforts to collect information of an on-going manufacturing practice. It should be stressed that every worker in the same shipyard can also have a different opinion from one another.

By analyzing the collected survey, averaged values are tentatively adopted. Our results suggested that the ratio of the manufacturing efforts for hull plates to inner members ranges from three to five.

APPLICATION EXAMPLE

A commercial vessel is used to illustrate the scalar metric. The ship is 260 m long and has a shape close to a fast container ship, as illustrated in Fig. 7. It should be noted that the proposed scalar metric could be applied to any shape of structural bodies.



Fig. 7 Hull form.

The distribution of curved hull patches of the ship is tabulated in Fig. 8. The bars show that each part (after, mid, and fore bodies) consists of certain percentages of curved types classified as four groups. The numbers in each group represent the range of normalized manufacturing cost. For instance, the group with the lower range is relatively inexpensive to manufacture.

Given that the fore part of the ship has typically the most complex curvature and internal structure, it has been used to investigate the validity of the proposed metric. To analyze the ship in its entirety, the aft and midship sections can be analyzed with the same methods used for the forward portion of the vessel. The computed scalar metric for the hull plates of the fore part is 0.3128, while the scalar metric for the inner members is 0.1144 when the ratio of balancing weights, $w_H/w_{InnerMembers}$, is four. If the designer wants to apply other balancing weight suggested at a different shipyard where a different manufacturing practice is performed, the contributions of the total scalar metric changes.

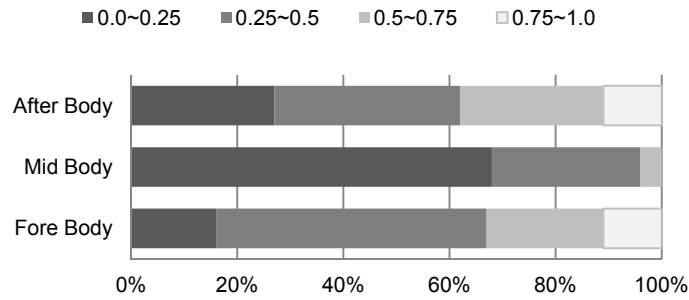


Fig. 8 Percentage of curved types in each part.

To investigate the trend of the scalar metric for different geometry, a slight variation of the fore part is made in Fig. 9. The local areas surrounded by two circles are intentionally distorted for this purpose. The Gaussian curvature plot indicates that the curvature changes its sign around the distorted regions and thus the curved amount increases after variation.

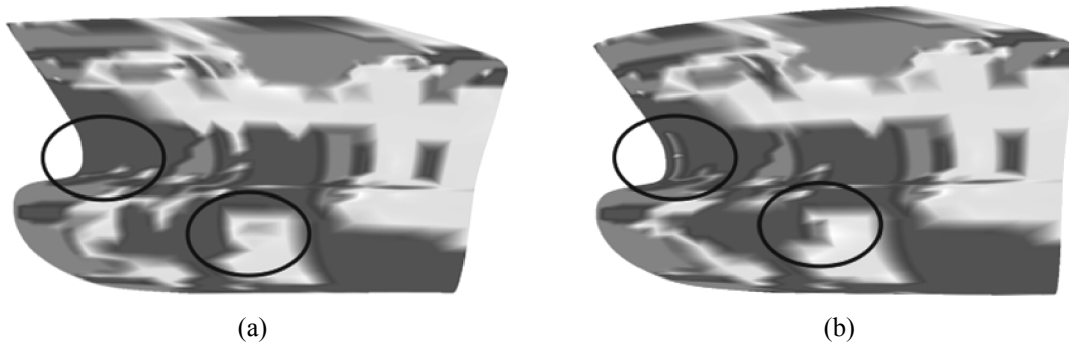


Fig. 9 Variation of fore part with Gaussian curvature plot, (a) before variation and (b) after variation.

The scalar metrics for the two shapes are compared in Table 4. The comparison clearly confirms that the modified fore part causes the rise of manufacturing cost due to the high curved regions, as expected. It also makes it hard to fabricate the attached inner structural members near the distorted regions.

Table 4 Scalar metrics for fore part.

	$(w * SM)_{Hull}$	$(w * SM)_{Inner\ members}$	SM_{Total}
Before	0.3128	0.1144	0.4272
After	0.3169	0.1151	0.4320

In this example, the difference of the two scalar metrics is not significant, owing to the minor variation. It is obvious that the difference becomes great if we compare two different models or even after heavy modification. The scalar metric proves to be an excellent tool to estimate the geometry-related manufacturing cost.

CONCLUSIONS

An updated scalar metric for the assessment of manufacturing cost of complex offshore structural bodies is introduced. By including the major inner structural members as well as the outer hull plate, this metric can help the designer to estimate manufacturing costs in the early design stage. Cost estimation becomes useful when the designer wants to try different sets of variations before building the actual structures.

The metric is formulated as a function of geometric shape of structures. To turn the scalar metric into a complete index for the total cost, the curved plates and inner structural members have been categorized into different types that affect the manu-

facturing costs. These subjective and shipyard dependent values have been obtained by consulting with the experts in industry. The scalar metric introduced has demonstrated to work properly for different shapes.

To make the proposed metric more practical, other manufacturing processes such as welding, assembling or even human factors should be included, as shown in the preliminary work done by Righeterink, Collette and Singer (2012). Even though the detailed information of such process is not available in early design stage, rough layout of stiffened members can offer more cost-related factors. Integration with the cost index associated with those expanded factors is currently being explored.

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