Journal of Magnetics 17(3), 196-199 (2012) http://dx.doi.org/10.4283/JMAG.2012.17.3.196

Calculation of Winding Inductances for a Single-Phase Brushless DC Machine

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(Received 30 May 2012, Received in final form 8 July 2012, Accepted 9 July 2012)

This paper presents the analytical calculation of winding inductance for a single-phase brushless DC machine based on the magnetic circuit concept. The machine is used in the low power range of applications, such as ventilation fans, due to its simplicity and low cost. Since flux linkage is proportional to inductance, the calculation of winding inductance is of central importance. By comparison with experimental and analytical values, it is shown that proposed analytical expression is able to effectively predict the winding inductance of single-phase brushless DC machines at the design stage.

Keywords : analytical calculation, brushless DC machine, single-phase, winding inductance

1. Introduction

Single-phase brushless DC (BLDC) motors are used in the low power range of applications, such as ventilation fans, due to their simplicity and low cost. For single-phase BLDC motors, the interaction between the flux components produced by the winding current and permanent-magnets results in the production of excitation torque. Since flux linkage is proportional to inductance, the ability to characterize winding distributions and utilize this characterization in the calculation of winding inductances is of central importance [1]. In the recently published papers [2-6], the inductances of AC machines were computed using the winding function theory. However, the traditional winding function method is limited to multi-phase winding configurations. Therefore, previous literature does not cover single-phase machines.

This paper presents a calculation of the winding inductance of an outer rotor single-phase BLDC machine. The magnetic circuit concept is used to calculate the inductance of the machine. Since it is not possible to calculate the inductance by the winding function, the turns and air-gap functions are used. This analytical approach is validated by experimental measurement.

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2. Inductance of a Single-Phase Brushless DC Machine

The calculation model is for an outer rotor machine. In such a machine, a single bonded magnet ring can be used in the outer rotor, as shown in Fig. 1. This machine has eight poles and a non-uniform air gap. Here, it is assumed that one continuous winding, with N_t turns, is concentrated on the eight slots within the stator, as shown. N_t and P are the total number of winding turns and number of

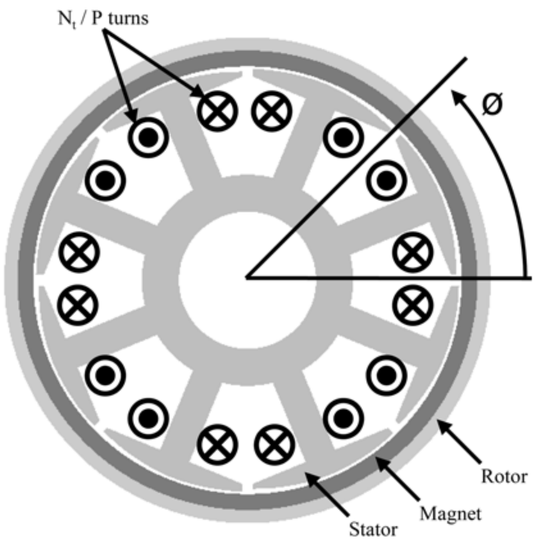

Fig. 1. Cross section of an outer rotor machine.

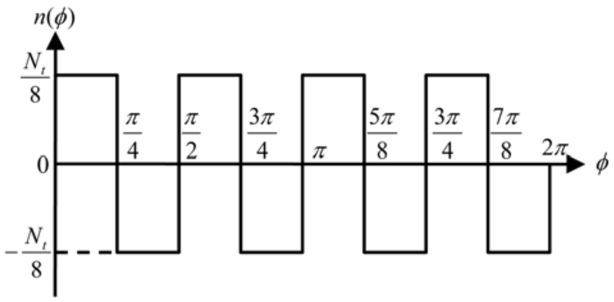

Fig. 2. Eight poles concentrated winding turns function.

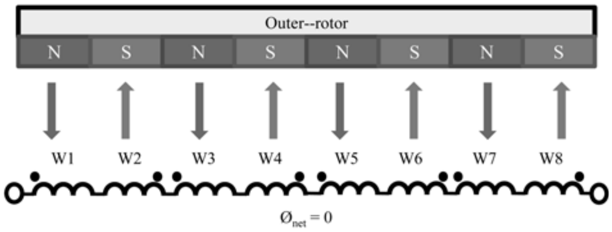

Fig. 3. Series connecting eight poles machine.

poles, respectively. The reference point ($\phi = 0$) for the angle ϕ is arbitrarily chosen to be the horizontal axis. The angle ϕ is assumed to increase in the anticlockwise direction from the reference point. Figure 2 shows the turns function $n(\phi)$ for any value of ϕ, $0 < \phi < 2\pi$. The winding of all the poles of the machine is assumed to be connected in series as illustrated in Fig. 3. The permanent-magnet does not have inductance. However, it does provide flux to link series connected windings. Using Gauss's law for magnetism, the net flux linkage of winding due to the permanent-magnet becomes zero. In this circuit, the winding inductance is independent of the permanent-magnet.

It is assumed that magnetomotive force (MMF) of a magnetic circuit can be written as

$$F = n(\phi)i \tag{1}$$

where i is the current flowing in a winding. Integrating Eq. (1) from 0 to 2π,

$$\int_0^{2\pi} F d\phi = \int_0^{2\pi} n(\phi) i d\phi. \tag{2}$$

Since F and i are independent of ϕ

$$F = \frac{1}{2\pi}\int_0^{2\pi} n(\phi) i d\phi = i\langle n(\phi)\rangle \tag{3}$$

where $\langle n(\phi)\rangle$ is the average value of the turns function $n(\phi)$. In this case, Eq. (3) becomes zero. Thus the turns function represents eight independent circuits as shown in

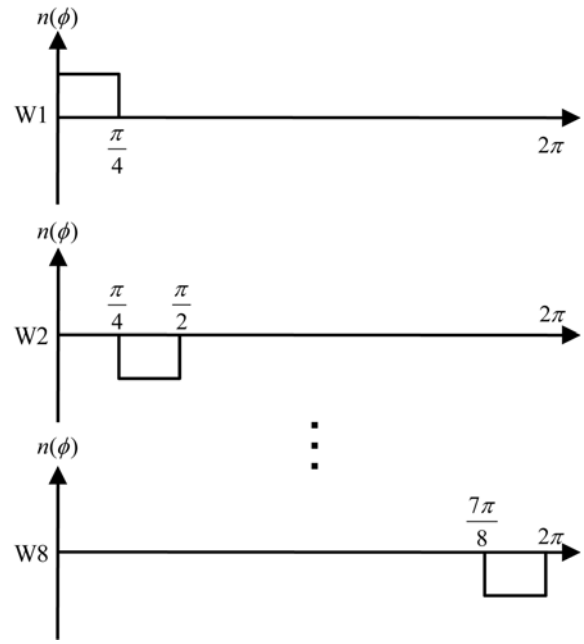

Fig. 4. Turns function with eight independent circuits.

Fig. 4. For this case, the average value of $n_{W1}(\phi)$ is

$$\langle n_{W1}\rangle = \frac{1}{2\pi}\int_0^{2\pi} \frac{N_t}{P} d\phi = \frac{N_t}{P^2}. \tag{4}$$

To develop a magnetic circuit model for this case, we write the flux produced by the W1 coil in the air gap as

$$\Phi = PF_{W1} \tag{5}$$

where

$$P = \frac{\mu_0 A}{g} \tag{6}$$

is defined as the permeance of the air gap having a cross-

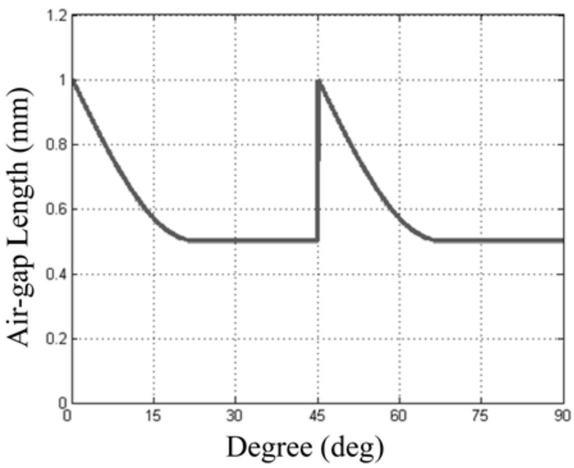

Fig. 5. Air gap function for non-uniform air gap.

sectional area A, air-gap length g, and the permeability of free space μ_0. Since the air gap is non-uniform, as shown in Fig. 5, the permeance of the air gap is

$$dP = \frac{\mu_0(Rd\phi)l}{g(\phi)} \tag{7}$$

where $(Rd\phi)l$ is the cross section A and $g(\phi)$ is the air gap function. The slot openings were ignored in this analysis. Thus total flux produced by the W1 coil can be written as the integral

$$\Phi_{W1} = F_{W1}\mu_0 Rl \int_0^{2\pi} \frac{1}{g(\phi)} d\phi. \tag{8}$$

The flux linkage of the $W2$ coil due to current in the $W1$ coil becomes

$$\lambda_{W2W1} = F_{W1}\mu_0 Rl \int_0^{2\pi} \frac{n_{W2}(\phi)}{g(\phi)} d\phi. \tag{9}$$

As a result, the mutual inductance L_{W2W1} is defined as

$$L_{W2W1} = \langle n_{W1} \rangle \mu_0 Rl \int_0^{2\pi} \frac{n_{W2}(\phi)}{g(\phi)} d\phi. \tag{10}$$

Since the turns function $n_{W2}(\phi)$ is non-zero only over a certain range, the mutual inductance is

$$L_{W2W1} = \langle n_{W1} \rangle \mu_0 Rl \int_{\frac{2\pi}{P}}^{\frac{4\pi}{P}} \frac{n_{W2}(\phi)}{g(\phi)} d\phi. \tag{11}$$

Combining the above expressions leads to

$$L_{W2W1} = -\mu_0 Rl \frac{2\pi N_t^2}{P^4} \int_0^{\frac{2\pi}{P}} \frac{1}{g(\phi)} d\phi \tag{12}$$

where it is assumed that the two coils produce the opposite magnetic polarity. If any two coils, i and j, have the same magnetic polarity then

$$L_{W2W1} = \mu_0 Rl \frac{2\pi N_t^2}{P^4} \int_0^{\frac{2\pi}{P}} \frac{1}{g(\phi)} d\phi. \tag{13}$$

In a similar manner the magnetizing inductance of all of the other coils is the same as Eq. (13). In this circuit, the flux linkage due to the W1 coil can be written as

$$\lambda_{W1} = L_{W1W1} i_{W1} - \sum_{j=2}^{P} L_{W1Wj} i_j. \tag{14}$$

Thus, the total flux linkage is

$$\lambda = \sum_{K=1}^{P} \left(L_{WkWk} i_{Wk} - \sum_{j=1(k \neq j)}^{P} L_{WkWj} i_j \right) \tag{15}$$

which leads to

$$L = \mu_0 Rl \frac{4\pi N_t^2}{P^3} \int_0^{\frac{2\pi}{P}} \frac{1}{g(\phi)} d\phi. \tag{16}$$

3. Inductance Calculation-A prototype

The specification of the single-phase BLDC machine is as given in Table 1. The air gap function is

$$\begin{cases} g(\phi) = g + \left(g - g \sin\frac{\phi P}{2} \right), & 0 \le \phi \le \frac{\pi}{P} \\ g(\phi) = g, & \frac{\pi}{P} \le \phi \le \frac{2\pi}{P} \end{cases}. \tag{17}$$

Using (16) and (17), the winding inductance is

$$L = 21.06 mH. \tag{18}$$

4. Experimental Results

The zero input response (ZIR) of the RL circuit shown in Fig. 6 is

$$i(t) = i(0)e^{-\frac{R}{L}t} = i(0)e^{-\frac{t}{\tau}} \tag{19}$$

Table 1. Prototype Machine Specifications

Number of poles, P	8	Stack length, l	30 [mm]
Number of turns, N_t	640 [turns]	Air gap length, g	0.5 [mm]
Stator radius, R	80 [mm]		

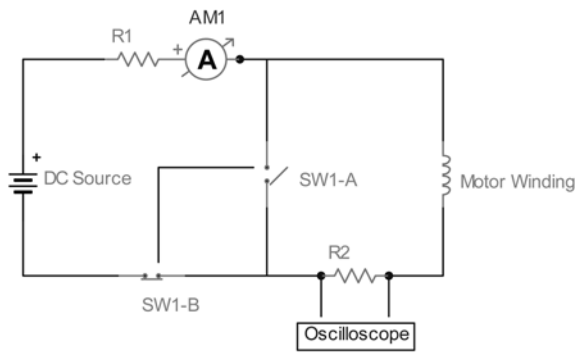

Fig. 6. RL circuit for ZIR.

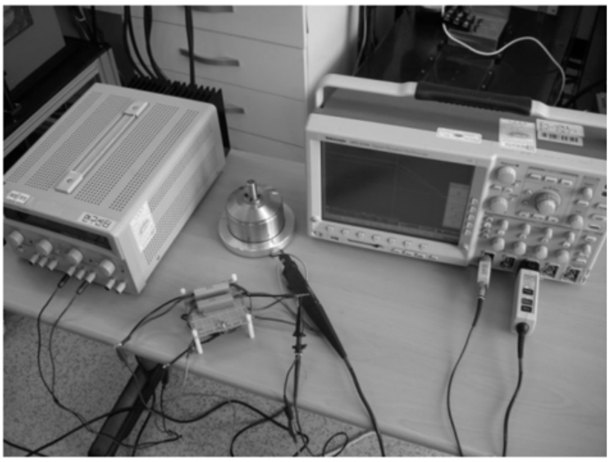

Fig. 7. Experimental set-up for ZIR.

where $i(0)$ is the initial current, R is the total resistance, L is the winding inductance and τ is the time constant. Let the switch be thrown at $t = 0$. When $t = \tau$

$$i(\tau) \approx 0.368 i(0). \quad (20)$$

In the RL circuit, the time constant τ is

$$\tau = \frac{L}{R1 + R2}. \quad (21)$$

Thus, the winding inductance is

Table 2. Measured Inductance Using ZIR

Outer rotor angle	With outer rotor		Without outer rotor	
	Time constant τ[ms]	Inductance L [mH]	Time constant τ[ms]	Inductance L [mH]
0	1.08	20.95		
90	1.02	19.79		
180	1.02	19.79	1.15	22.31
270	1.04	20.18		
Average	1.04	20.18		

$$L = (R1 + R2) \times \tau. \quad (22)$$

A ZIR test was performed to measure the time constant in a prototype machine, as shown in Fig. 7. Table 2 shows the measured time constant and calculated inductance using (22), both with an outer rotor and without an outer rotor.

5. Conclusion

This paper presents an analytical expression to calculate the winding inductance of a single-phase BLDC machine, based on the magnetic circuit concept. In this equation, it is assumed that the winding inductance is independent of the permanent-magnet outer rotor. The test results give, however, a minor difference between the inductance with an outer rotor and the inductance without an outer rotor because the permanent-magnet has an overhang. The analytical expression used to calculate winding inductance is a reasonable approximation of the experimental results. Thus, the proposed analytical expression is able to effectively predict the winding inductance of a single-phase BLDC machine at the design stage.

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