논리연구 15-1(2012) pp. 1～15

Kripke-style Semantics for UL*

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【Abstract】 This paper deals with Kripke-style semantics for fuzzy logics. As an example we consider a Kripke-style semantics for the uninorm based fuzzy logic **UL**. For this, first, we introduce **UL**, define the corresponding algebraic structures UL-algebras, and give algebraic completeness results for it. We next introduce a Kripke-style semantics for **UL**, and connect it with algebraic semantics.

【Key Words】 UL (Uninorm logic), Kripke-style semantics, Algebraic semantics, Many-valued logic, Fuzzy logic

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* 접수일자: 2011.09.05. 심사 및 수정 완료일: 2011.10.03. 게재확정일: 2011.12.29.
** I must thank the anonymous referees for their helpful comments.

1. Introduction

This paper is a contribution to the study of Kripke-style semantics, i.e., semantics with binary accessibility relations, for *substructural fuzzy* logics: *substructural* logics lacking structural rules such as weakening and contraction, and *fuzzy* logics dealing with vagueness. For this, recall first some historical facts associated to Kripke-style semantics for many-valued logics. A lot of Kripke-style semantics have been provided for three- and four-valued logics. For instance, Thomason [12] gave a three-valued Kripke-style semantics for the Nelson's system **N** of constructible falsity by allowing partial evaluations ("gaps" (N)). Dunn [3, 4] provided a three-valued Kripke-style semantics for the **R** of Relevance with mingle (**RM**) by allowing non-functional evaluations ("gluts" (B)). He [4] especially gave several three- and four-valued Kripke-style semantics for logics such as $\mathbf{Bc}_1$, $\mathbf{N}_{1,0}$, $\mathbf{BNc}_{1,0}$, etc., by allowing non-functional and/or partial evaluations, i.e., either B or N, and both B and N. Furthermore, Yang [15, 16] has provided Kripke-style semantics for three- and four-valued logics, which can be regarded as the three-valued Dummett-Gödel logic $\mathbf{G}_3$ and neighbors of the relevance logics **R**, **E** of Entailment, and **T** of Ticket entailment. In particular, several Kripke-style semantics have been recently provided for infinite-valued logics based on t-norms (so called, t-norm based logics) by Montagna and Ono [9], Montagna and Sacchetti [10, 11], and Diaconescu and Georgescu [2].

For these semantics, there are at least the following two interesting points to state. One interesting point is that Kripke-style semantics for the t-norm based logics are quite different from those for the three- and four-valued logics mentioned above: while (Kripke) frames for the latter logics are given *set-theoretically*, frames for the former logics are provided on *algebraic structures*. (Note that while frames for the latter logics are defined just by means of linearly ordered (arbitrary) sets (as states of information or possible worlds) or by means of linearly ordered (arbitrary) structures based on such sets, frames for the former logics are defined as linearly ordered integral commutative monoids, i,e., (reducts of) algebras for t-norm based logics.) The other point is that while algebraic semantics for weakening-free fuzzy logics based on uninorms (so called, uninorm based logics) have been introduced (see e.g. [5, 6, 7, 8]), Kripke-style semantics for such logics have not yet been introduced. (Uninorms are functions introduced by Yager and Rybalov [14] as a generalization of t-norms where the identity can lie anywhere in [0, 1].)

Let us call Kripke-style semantics whose frames are defined only set-theoretically, i.e., based on possible worlds or states of information but not algebraic structures, *set-theoretical* Kripke-style semantics; call Kripke-style semantics whose frames are defined algebraically, i.e., based on algebras, *algebraic* Kripke-style semantics. The above two points raise the following interesting question:

● Can we introduce (algebraic or set-theoretical) Kripke-style semantics for uninorm based logics?

The answer to the question is positive in a sense because: we can introduce *algebraic* Kripke-style semantics for uninorm based logics, although not *set-theoretical* ones. This paper verifies it by introducing an algebraic Kripke-style semantics for **UL**. For this, first, in Section 2 we introduce **UL** and the corresponding algebraic semantics as the necessary notions for treating the question. In Section 3 we introduce an algebraic Kripke-style semantics for **UL**, and connect them with algebraic semantics.

For convenience, we shall adopt the notation and terminology similar to those in [1, 4, 7, 10, 11], and assume familiarity with them (together with results found in them).

2. The logic UL and its algebraic semantics

We base **UL** on a countable propositional language with formulas *FOR* built inductively as usual from a set of propositional variables *VAR*, binary connectives $\rightarrow$, &, $\wedge$, $\vee$, and constants **T**, **F**, **f**, **t**, with defined connectives:

df1. $\sim\phi := \phi \rightarrow \mathbf{f}$, and
df2. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

We moreover define ϕ^n_t as $\phi_t \,\&\, \cdots \,\&\, \phi_t$, n factors, where $\phi_t := \phi \wedge \mathbf{t}$. For the remainder we shall follow the customary notation and terminology. We use the axiom systems to provide a consequence relation.

We start with the following axiomatization of **UL** as the most basic (substructural) fuzzy logic introduced here.

Definition 2.1 **UL** consists of the following axiom schemes and rules:

A1. $\phi \rightarrow \phi$ (self-implication, SI)

A2. $(\phi \wedge \psi) \rightarrow \phi$, $(\phi \wedge \psi) \rightarrow \psi$ ($\wedge$-elimination, $\wedge$-E)

A3. $((\phi\rightarrow\psi)\wedge (\phi\rightarrow\chi)) \rightarrow (\phi\rightarrow(\psi\wedge\chi))$ ($\wedge$-introduction, $\wedge$-I)

A4. $\phi \rightarrow (\phi \vee \psi)$, $\psi \rightarrow (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)

A5. $((\phi\rightarrow\chi)\wedge (\psi\rightarrow\chi)) \rightarrow ((\phi\vee\psi)\rightarrow\chi)$ ($\vee$-elimination, $\vee$-E)

A6. $\phi \rightarrow \mathbf{T}$ (verum ex quolibet, VE)

A7. $\mathbf{F} \rightarrow \phi$ (ex falso quadlibet, EF)

A8. $(\phi \,\&\, \psi) \rightarrow (\psi \,\&\, \phi)$ (&-commutativity, &-C)

A9. $(\phi \,\&\, \mathbf{t}) \leftrightarrow \phi$ (push and pop, PP)

A10. $(\phi \rightarrow \psi) \rightarrow ((\psi \rightarrow \chi) \rightarrow (\phi \rightarrow \chi))$ (suffixing, SF)

A11. $(\phi \rightarrow (\psi \rightarrow \chi)) \leftrightarrow ((\phi \,\&\, \psi) \rightarrow \chi)$ (residuation, RE)

A12. $(\phi \rightarrow \psi)_{\mathbf{t}} \vee (\psi \rightarrow \phi)_{\mathbf{t}}$ (**t**-prelinearity, $PL_{\mathbf{t}}$)

$\phi \rightarrow \psi, \phi \vdash \psi$ (modus ponens, mp)

$\phi, \psi \vdash \phi \wedge \psi$ (adjunction, adj).

Note that **MAILL** (Multiplicative additive intuitionistic linear logic) is the **UL** omitting A12.

An easy computation shows the following.

Proposition 2.2 **UL** proves:

(1) $(\phi \,\&\, (\psi \,\&\, \chi)) \leftrightarrow ((\phi \,\&\, \psi) \,\&\, \chi)$ (&-associativity, AS)

(2) $(\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow (\psi \rightarrow (\phi \rightarrow \chi))$ (permutation, RE)

(3) $(\phi \rightarrow \psi))^{n}_{\mathbf{t}} \vee (\psi \rightarrow \phi)^{n}_{\mathbf{t}}$, for each n ($PL^{n}_{\mathbf{t}}$)

(4) $(\phi \to \psi) \vee (\psi \to \phi)$ (PL).

In **UL**, **f** can be defined as $\sim$**t**. A *theory* over **UL** is a set T of formulas. A *proof* in a sequence of formulas whose each member is either an axiom of **UL** or a member of T or follows from some preceding members of the sequence using the two rules in Definition 2.1. T $\vdash \phi$, more exactly T $\vdash_{\mathbf{UL}} \phi$, means that ϕ is *provable* in T w.r.t. **UL**, i.e., there is a **UL**-proof of ϕ in T. The local deduction theorem ($LDT_{\mathbf{t}}$) for **UL** is as follows:

Proposition 2.3 Let T be a theory, and ϕ, ψ formulas.
($LDT_{\mathbf{t}}$) $T \cup \{\phi\} \vdash \psi$ iff there is n such that $T \vdash \phi^{n}_{\mathbf{t}} \to \psi$.

A theory T is *inconsistent* if T $\vdash$ **F**; otherwise it is *consistent.*
For convenience, "$\sim$", "$\wedge$", "$\vee$", and "$\to$" are used ambiguously as propositional connectives and as algebraic operators, but context should make their meaning clear.
The algebraic counterpart of **UL** is the class of the so-called *UL-algebras*. Let $x_t := x \wedge t$. They are defined as follows.

Definition 2.4 (i) (MAILL-algebra) A *pointed bounded commutative residuated lattice* is a structure **A** = (A, $\top$, $\bot$, t, f, $\wedge$, $\vee$, *, $\to$) such that:
(Ⅰ) (A, $\top$, $\bot$, $\wedge$, $\vee$) is a bounded lattice with top element $\top$ and bottom element $\bot$.
(Ⅱ) (A, *, t) is a commutative monoid.
(Ⅲ) $y \le x \to z$ iff $x * y \le z$, for all x, y, z $\in$ A

(residuation).

(Ⅳ) $t \leq (x \rightarrow y)_t \vee (y \rightarrow x)_t$ (pl_t).

Additional (unary) negation and (binary) equivalence operations are defined as follows: $\sim x := x \rightarrow f$ and $x \leftrightarrow y := (x \rightarrow y) \wedge (y \rightarrow x)$.

The class of all UL-algebras is a variety which will be denoted by UL.

UL-algebra is said to be *linearly ordered* if the ordering of its algebra is linear, i.e., $x \leq y$ or $y \leq x$ (equivalently, $x \wedge y = x$ or $x \wedge y = y$) for each pair x, y.

Definition 2.5 Let K be a class of UL-algebras. We define consequence relation $\vDash_K$ in the following way: $T \vDash_K \phi$ iff for each $\mathcal{A} \in$ K and $\mathcal{A}$-evaluation v, we have $v(A) \geq t$ whenever $v(\psi) \geq t$ for each $\psi \in T$.

We write $\vDash_K \phi$ instead of $\varnothing \vDash_K \phi$, and $T \vDash_{\mathcal{A}} \phi$ instead of $T \vDash_{\{\mathcal{A}\}} \phi$.

That UL is the proper algebraic semantics for **UL** is witnessed by the following completeness result.

Theorem 2.6 ([7]) Let T be a theory over **UL**, and ϕ a formula. $T \vdash_{\mathbf{UL}} \phi$ iff $T \vDash_{UL} \phi$.

This completeness result can be refined by taking into account the following representation of UL-algebras related to the prelinearity property of UL-algebras.

Proposition 2.7 ([13]) Each UL-algebra is a subdirect product of linearly ordered UL-algebras.

This leads to the completeness of **UL** w.r.t. the class of chains of UL.

Corollary 2.8 ([7]) Let T be a theory over **UL**, and ϕ a formula. T $\vdash_{\mathbf{UL}} \phi$ iff T $\vDash^{1}_{\mathbf{UL}} \phi$.

An **A** algebra is said to be *standard* iff its lattice reduct is [0,1]. It is further proved that

Theorem 2.9 ([7]) For a theory T over **UL**, and a formula ϕ, the following are equivalent:

(1) T $\vdash_{\mathbf{UL}} \phi$.

(2) For every standard UL-algebra and evaluation v, $v(\phi) \geq t$ whenever $v(\psi) \geq t$ for each $\psi \in T$.

3. Kripke-style semantics for **UL**

We consider here algebraic Kripke-style semantics for **UL**.

Definition 3.1 (Algebraic Kripke frame) An *algebraic Kripke frame* is a structure $\mathbf{X} = (X, \top, \bot, t, f, \leq, *, \rightarrow)$ such that $(X, \top, \bot, t, f, \leq, *, \rightarrow)$ is a linearly ordered residuated pointed bounded commutative monoid. The elements of **X** are called *nodes*.

Definition 3.2 (UL frame) A *UL frame* is an algebraic Kripke frame, where $*$ is conjunctive (i.e., $\bot * \top = \bot$) and left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$), and so its residuum $\rightarrow$ is defined as $x \rightarrow y := \sup\{z: x * z \leq y\}$ for all $x, y \in X$.

Definition 3.2 ensures that a UL frame has a supremum w.r.t. $*$, i.e., for every $x, y \in X$, the set $\{z: x * z \leq y\}$ has the supremum. **X** is said to be *complete* if $\leq$ is a complete order.

An *evaluation* or *forcing* on an algebraic Kripke frame is a relation $\Vdash$ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(ABHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$;
(min) $\bot \Vdash p$; and

for arbitrary formulas,

(t) $x \Vdash \mathbf{t}$ iff $x \leq t$;
(f) $x \Vdash \mathbf{f}$ iff $x \leq f$;
($\bot$) $x \Vdash \mathbf{F}$ iff $x = \bot$;
($\wedge$) $x \Vdash \phi \wedge \psi$ = iff $x \Vdash \phi$ and $x \Vdash \psi$;
($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;

(&) $x \Vdash \phi \& \psi$ iff there are $y, z \in X$ such that $y \Vdash \phi$, $z \Vdash \psi$, and $x \leq y * z$;
(→) $x \Vdash \phi \to \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $x * y \Vdash \psi$.

An evaluation or forcing on a UL frame is an evaluation or forcing further satisfying that (max) for every atomic sentence p, $\{x : x \Vdash p\}$ has a maximum.

Definition 3.3 (i) (Algebraic Kripke model) An *algebraic Kripke model* is a pair $(\mathbf{X}, \Vdash)$, where $\mathbf{X}$ is an algebraic Kripke frame and $\Vdash$ is a forcing on $\mathbf{X}$.
(ii) (UL model) A *UL model* is a pair $(\mathbf{X}, \Vdash)$, where $\mathbf{X}$ is a UL frame and $\Vdash$ is a forcing on $\mathbf{X}$. A UL model $(\mathbf{X}, \Vdash)$ is said to be *complete* if $\mathbf{X}$ is a complete frame and $\Vdash$ is a forcing on $\mathbf{X}$.

Definition 3.4 (Cf. [11]) Given an algebraic Kripke model $(\mathbf{X}, \Vdash)$, a node x of $\mathbf{X}$ and a formula ϕ, we say that *x forces ϕ* to express $x \Vdash \phi$. We say that ϕ is *true* in $(\mathbf{X}, \Vdash)$ if $t \Vdash \phi$, and that ϕ is *valid* in the frame $\mathbf{X}$ (expressed by $\mathbf{X}$ models ϕ) if ϕ is true in $(\mathbf{X}, \Vdash)$ for every forcing $\Vdash$ on $\mathbf{X}$.

Remark 3.5 The definitions of a UL frame, a forcing on a UL frame, and a UL-model corresponds to those of a residuated Kripke frame, an r-forcing on a residuated Kripke frame, and a residuated Kripke model, respectively, in [10]. To the present author, it seems that we need not introduce forcing distinguished from r-forcing because the monoidal t-norm logics considered in [10] are complete w.r.t. frames based on r-forcing but not forcing.

For soundness and completeness for **UL**, let $\vdash_{\mathbf{UL}} \phi$ be the theoremhood of ϕ in **UL**. First we note the following lemma.

Lemma 3.6 (i) (Backward Hereditary Lemma, BHL) Let **X** be an algebraic Kripke frame. For any sentence ϕ and for all nodes $x, y \in \mathbf{X}$, if $x \Vdash \phi$ and $y \leq x$, then $y \Vdash \phi$.

(ii) Let $\Vdash$ be a forcing on a UL frame, and ϕ a sentence. Then the set $\{x \in X : x \Vdash \phi\}$ has a maximum.

Proof: (i) Easy. (ii) See Lemma 2.11 in [10]. □

Proposition 3.7 (Soundness) If $\vdash_{\mathbf{UL}} \phi$, then ϕ is valid in every UL frame.

Proof: We prove the validity of A12 as an example: it suffices to show that either $t \Vdash (\phi \to \psi) \wedge \mathbf{t}$ or $t \Vdash (\psi \to \phi) \wedge \mathbf{t}$. As mentioned in proof of Lemma 2.11 in [10], for every α, the set $\alpha^\circ = \{x : x \Vdash \alpha\}$ is downwards closed, therefore either $\phi^\circ \subseteq \psi^\circ$ or $\psi^\circ \subseteq \phi^\circ$. Thus $t \Vdash \phi \to \psi$ or $t \Vdash \psi \to \phi$. Let $t \Vdash \phi \to \psi$. Then, since $t \Vdash \mathbf{t}$, we can obtain that $t \Vdash (\phi \to \psi) \wedge \mathbf{t}$ by $(\wedge)$. Let $t \Vdash \psi \to \phi$. Analogously we can obtain that $t \Vdash (\psi \to \phi) \wedge \mathbf{t}$, as wished.

The proof for the other cases is left to the interested reader. □

By a *chain*, we mean a linearly ordered algebra. The next proposition connects algebraic Kripke semantics and algebraic semantics for **UL** (cf. see [11]).

Proposition 3.8 (i) The $\{\top, \bot, t, f, \leq, *, \rightarrow\}$ reduct of a UL-chain $\mathcal{A}$ is a UL frame, which is complete iff $\mathcal{A}$ is complete.

(ii) Let $\mathbf{X} = (X, \top, \bot, t, f, \leq, *, \rightarrow)$ be a UL frame. Then the structure $\mathbf{A} = (X, \top, \bot, t, f, max, min, *, \rightarrow)$ is a UL-algebra (where *max* and *min* are meant w.r.t. $\leq$).

(iii) Let $\mathbf{X}$ be the $\{\top, \bot, t, f, \leq, *, \rightarrow\}$ reduct of a UL-chain $\mathcal{A}$, and let v be an evaluation in $\mathcal{A}$. Let for every atomic formula p and for every $x \in \mathcal{A}$, $x \Vdash p$ iff $x \leq v(p)$. Then $(\mathbf{X}, \Vdash)$ is a UL model, and for every formula ϕ and for every $x \in \mathcal{A}$, we obtain that: $x \Vdash \phi$ iff $x \leq v(\phi)$.

(iv) Let $(\mathbf{X}, \Vdash)$ be a UL model, and let $\mathcal{A}$ be the UL-algebra defined as in (ii). Define for every atomic formula p, $v(p) = max\{x \in X : x \Vdash p\}$. Then for every formula ϕ, $v(\phi) = max\{x \in X : x \Vdash \phi\}$.

Proof: The proof for (i) and (ii) is easy. Since (iv) follows almost directly from (iii) and Lemma 3.1.6 (ii), we prove (iii). As regards to claim (iii), we consider the induction steps corresponding to the cases where $\phi = \psi \& \chi$ and $\phi = \psi \rightarrow \chi$. (The proof for the other cases are trivial.)

Suppose $\phi = \psi \& \chi$. By the condition (&), $x \Vdash \psi \& \chi$ iff there are $y, z \in X$ such that $y \Vdash \psi$, $z \Vdash \chi$, and $x \leq y * z$, hence by the induction hypothesis, $y \Vdash \psi$ and $z \Vdash \chi$ iff $y \leq$

$v(\psi)$ and $z \leq v(\chi)$. Then, it holds true that $x \leq y * z \leq v(\psi) * v(\chi) = v(\psi \& \chi)$. Conversely, if $x \leq v(\psi) * v(\chi) = v(\psi \& \chi)$, then take $y = v(\psi)$ and $z = v(\chi)$. Then we have $x \leq y * z$, $y \Vdash \psi$, and $z \Vdash \chi$, therefore $x \Vdash \psi \& \chi$.

Suppose $\phi = \psi \to \chi$. By the condition ($\to$), $x \Vdash \psi \to \chi$ iff for all $y \in X$, if $y \Vdash \psi$, then $x * y \Vdash \chi$, hence by the induction hypothesis, $y \Vdash \psi$ only if $x * y \Vdash \chi$ iff $y \leq v(\psi)$ only if $x * y \leq v(\chi)$, therefore iff $x * v(\psi) \leq v(\chi)$, therefore by residuation, iff $x \leq v(\psi) \to v(\chi) = v(\psi \to \chi)$, as desired. □

Theorem 3.9 (Strong completeness)

(i) **UL** is strongly complete w.r.t. the class of all UL-frames.

(ii) **UL** is strongly complete w.r.t. the class of complete UL-frames.

Proof: (i) and (ii) follow from Proposition 3.8 and Corollary 2.8, and from Proposition 3.8 and Theorem 2.9, respectively. □

4. Concluding remark

We investigated algebraic Kripke-style semantics for substructural relevance logics. As an example we introduced an algebraic Kripke-style semantics for **UL**. We proved soundness and completeness theorems. But we did not provide algebraic Kripke-style semantics for axiomatic extensions of **UL**. We will investigate it in a subsequent paper.

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부록	Abstracts

ARTICLE ABSTRACTS

UL을 위한 크립키형 의미론

양 은 석

이 글에서 우리는 퍼지 논리들을 위한 크립키형 의미론을 다룬다. 이를 위한 한 예로 **UL**을 위한 크립키형 의미론을 다룬다. 이를 위하여 먼저 **UL** 체계를 소개하고 그에 상응하는 UL-대수를 정의한 후 **UL**이 대수적으로 완전하다는 것을 보인다. 다음으로 **UL**을 위한 크립키형 의미론을 소개하고 이를 대수적 의미론과 연관짓는다.

주요어: UL, 크립키형 의미론, 대수적 의미론, 다치 논리, 퍼지 논리