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FUZZY r-MINIMAL β-OPEN SETS ON FUZZY MINIMAL SPACES

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Abstract. We introduce the concept of fuzzy r-minimal β-open set on a fuzzy minimal space and basic some properties. We also introduce the concept of fuzzy r-M β-continuous mapping which is a generalization of fuzzy r-M continuous mapping and fuzzy r-M semicontinuous mapping, and investigate characterization for the continuity.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [5]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [2], Ramadan introduced the concept of smooth topological space, which is a generalization of fuzzy topological space. We introduced the concept of fuzzy r-minimal space [4] which is an extension of the smooth fuzzy topological space. The concepts of fuzzy r-open sets and fuzzy r-M continuous mappings are also introduced and studied. We introduced the concepts of fuzzy r-minimal semiopen sets [3] and fuzzy r-M semicontinuous mappings, and investigate properties of such concepts. In this paper, we introduce the concept of fuzzy r-minimal β-open set on a fuzzy minimal space and basic some properties. We also introduce the concept of fuzzy r-M β-continuous mapping which is a generalization of fuzzy r-M continuous mapping and fuzzy r-M semicontinuous mapping, and investigate characterization for the continuity.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* [5] of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1,

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respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{1} - A$. All other notations are standard notations of fuzzy set theory.

An *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha \text{ if } y = x \\ 0 \text{ if } y \neq x. \end{cases}$$

A *smooth topology* [2] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
(2) $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
(3) $\mathcal{T}(\cup A_i) \geq \wedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space*.

Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* [4] if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{0}$ and $\tilde{1}$.

Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* [4] (simply r-FMS). Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure of A, denoted by $mC(A, r)$, is defined as

$$mC(A, r) = \cap\{B \in I^X : B^c \in \mathcal{M}_r \text{ and } A \subseteq B\}.$$

The fuzzy r-minimal interior of A, denoted by $mI(A, r)$, is defined as

$$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A\}.$$

Theorem 2.1 ([4]). *Let $(X, \mathcal{M})$ be an r-FMS and $A, B \in I^X$.*

(1) *$mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.*
(2) *$A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.*
(3) *If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.*
(4) *$mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.*
(5) *$mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.*
(6) *$\tilde{1} - mC(A, r) = mI(\tilde{1} - A, r)$ and $\tilde{1} - mI(A, r) = mC(\tilde{1} - A, r)$.*

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal semiopen set* [3] in X if

$$A \subseteq mC(mI(A, r), r).$$

A fuzzy set A is called a *fuzzy r-minimal semiclosed* set if the complement of A is fuzzy r-minimal semiopen.

Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then $f : X \to Y$ is said to be *fuzzy r-M continuous* function if for every $A \in \mathcal{N}_r$, $f^{-1}(A)$ is in $\mathcal{M}_r$.

3. Fuzzy r-minimal β-open Sets

In this section, we introduce and study the concept of fuzzy r-minimal β-open sets. The two operators $m\beta C(A, r)$ and $m\beta I(A, r)$ are introduced and investigated.

Definition 3.1. Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal β-open set* in X if

$$A \subseteq mC(mI(mC(A, r), r), r).$$

A fuzzy set A is called a *fuzzy r-minimal β-closed* set if the complement of A is fuzzy r-minimal β-open.

Remark 3.2. From definitions of fuzzy r-minimal semiopen set and fuzzy r-minimal β-open set, the following implications are obtained but the converses are not true in general.

fuzzy r-minimal open $\Rightarrow$ fuzzy r-minimal semiopen $\Rightarrow$ fuzzy r-minimal β-open

Example 3.3. Let $X = I = [0, 1]$ and let A and B be fuzzy sets defined as follows

$$A(x) = \begin{cases} -x + \dfrac{1}{2}, & \text{if } 0 \leq x \leq \dfrac{1}{4}, \\ \dfrac{1}{3}(x-1) + \dfrac{1}{2}, & \text{if } \dfrac{1}{4} \leq x \leq 1; \end{cases}$$

$$B(x) = \frac{1}{4}(x+3), \quad \text{if } 0 \leq x \leq 1.$$

Let us consider a fuzzy minimal structure

$$\mathcal{M}(\mu) = \begin{cases} \dfrac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, A, \\ 0, & \text{otherwise.} \end{cases}$$

Then the fuzzy set B is a fuzzy $\frac{2}{3}$-minimal β-open set but not fuzzy $\frac{2}{3}$-minimal semiopen.

Lemma 3.4. *Let* $(X, \mathcal{M})$ *be an* r*-FMS. Then a fuzzy set* A *is fuzzy* r*-minimal* β*-closed if and only if* $mI(mC(mI(A,r),r),r) \subseteq A$.

Theorem 3.5. *Let* $(X, \mathcal{M})$ *be an* r*-FMS. Any union of fuzzy* r*-minimal* β*-open sets is fuzzy* r*-minimal* β*-open.*

Proof. Let A_i be a fuzzy r-minimal β-open set for $i \in J$. Then from Theorem 2.1,

$$A_i \subseteq mI(mC(A_i,r),r) \subseteq mI(mC(\cup A_i,r),r).$$

This implies $\cup A_i \subseteq mI(mC(\cup A_i,r),r)$ and so $\cup A_i$ is fuzzy r-minimal β-open. $\square$

Remark 3.6. In general, the intersection of two fuzzy r-minimal β-open sets may not be fuzzy r-minimal β-open as shown in the next example.

Example 3.7. Let $X = I = [0,1]$ and let A, B and C be fuzzy sets defined as follows

$$A(x) = -\frac{1}{2}(x-1), \quad \text{if } x \in I;$$

$$B(x) = \frac{1}{2}x, \text{ if } x \in I;$$

$$C(x) = \frac{3}{4}x, \quad x \in I.$$

Let us consider a fuzzy minimal structure

$$\mathcal{N}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, A, B, A \cup B \\ 0, & \text{otherwise.} \end{cases}$$

Then the fuzzy sets A and B are fuzzy $\frac{2}{3}$-minimal β-open. But $A \cap B$ is not fuzzy $\frac{2}{3}$-minimal β-open, because of $mI(mC(A \cap B, \frac{2}{3}), \frac{2}{3}) = \tilde{0}$.

Definition 3.8. Let $(X, \mathcal{M})$ be an r-FMS. For $A \in I^X$, $m\beta C(A,r)$ and $m\beta I(A,r)$, respectively, are defined as the following:

$$m\beta C(A,r) = \cap\{F \in I^X : A \subseteq F, \quad F \text{ is fuzzy } r\text{-minimal } \beta\text{-closed}\}$$

$$m\beta I(A,r) = \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open }\}.$$

Theorem 3.9. *Let* $(X, \mathcal{M})$ *be an* r*-FMS and* $A \in I^X$*. Then*

(1) $m\beta I(A,r) \subseteq A$.

(2) *If* $A \subseteq B$*, then* $m\beta I(A,r) \subseteq m\beta I(B,r)$.

(3) A *is* r*-minimal* β*-open iff* $m\beta I(A,r) = A$.

(4) $m\beta I(\beta mI(A,r),r) = m\beta I(A,r)$.

(5) $m\beta C(\tilde{1} - A, r) = \tilde{1} - m\beta I(A,r)$ *and* $m\beta I(\tilde{1} - A, r) = \tilde{1} - m\beta C(A,r)$.

Proof. (1), (2), (3) and (4) are clear from Theorem 3.5.

(5) For $A \in I^X$,

$$\begin{aligned}\tilde{1} - m\beta I(A,r) &= \tilde{1} - \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}\\ &= \cap\{\tilde{1} - U : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}\\ &= \cap\{\tilde{1} - U : \tilde{1} - A \subseteq \tilde{1} - U, U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}\\ &= m\beta C(\tilde{1} - A, r).\end{aligned}$$

Similarly, we can show that $m\beta I(\tilde{1} - A, r) = \tilde{1} - m\beta C(A, r)$. □

Theorem 3.10. *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then*

(1) *$A \subseteq m\beta C(A, r)$.*

(2) *If $A \subseteq B$, then $m\beta C(A, r) \subseteq m\beta C(B, r)$.*

(3) *F is r-minimal β-closed iff $m\beta C(F, r) = F$.*

(4) *$m\beta C(m\beta C(A, r), r) = m\beta C(A, r)$.*

Proof. It is similar to the proof of Theorem 3.9. □

Lemma 3.11. *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then*

(1) *$x_\alpha \in m\beta C(A, r)$ if and only if $A \cap V \neq \tilde{0}$ for every r-minimal β-open set V containing x_α.*

(2) *$x_\alpha \in m\beta I(A, r)$ if and only if there exists a fuzzy r-minimal β-open set G such that $G \subseteq A$.*

Proof. (1) If there is a fuzzy r-minimal β-open set V containing x_α such that $A \cap V = \tilde{0}$, then $\tilde{1} - V$ is a fuzzy r-minimal β-closed set such that $A \subseteq \tilde{1} - V$, $x_\alpha \notin \tilde{1} - V$. From this fact, $x_\alpha \notin m\beta C(A, r)$.

The converse is easily proved by definition of the operator of $m\beta C(A, r)$.

(2) Obvious. □

4. Fuzzy r-M β-continuity and Fuzzy r-$M(M^*)$ β-open Mappings

In this section, we introduce the concepts of fuzzy r-M β-continuous mapping, fuzzy r-M β-open mapping and fuzzy r-M^* β-open mapping, and investigate characterization for such mappings.

Definition 4.1. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be r-FMS's. Then a mapping $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is said to be *fuzzy r-M β-continuous* if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal β-open set U containing x_α such that $f(U) \subseteq V$.

Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be r-FMS's. Then a mapping $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ is said to be *fuzzy r-M semicontinuous* [3] if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal semiopen set U containing x_α such that $f(U) \subseteq V$.

Remark 4.2. It is obvious that every fuzzy r-M semicontinuous mapping is fuzzy r-M β-continuous but the converse may not be true as shown in the next example.

fuzzy r-M continuous $\Rightarrow$ fuzzy r-M semicontinuous $\Rightarrow$ fuzzy r-M β-continuous

Example 4.3. For $X = [0, 1]$, consider two fuzzy minimal structures $\mathcal{M}$ and $\mathcal{N}$ defined in Example 3.3 and Example 3.7, respectively. The identity mapping $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ is fuzzy r-M β-continuous but not fuzzy r-M semicontinuous.

Theorem 4.4. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following statements are equivalent:*

(1) *f is fuzzy r-M β-continuous.*

(2) *$f^{-1}(V)$ is a fuzzy r-minimal β-open set for each fuzzy r-minimal open set V in Y.*

(3) *$f^{-1}(B)$ is a fuzzy r-minimal β-closed set for each fuzzy r-minimal closed set B in Y.*

(4) *$f(m\beta C(A, r)) \subseteq mC(f(A), r)$ for $A \subseteq X$.*

(5) *$m\beta C(f^{-1}(B), r) \subseteq f^{-1}(mC(B, r))$ for $B \in I^Y$.*

(6) *$f^{-1}(mI(B, r)) \subseteq m\beta I(f^{-1}(B), r)$ for $B \in I^Y$.*

Proof. (1) $\Rightarrow$ (2) Let V be any fuzzy r-minimal open set in Y and $x_\alpha \in f^{-1}(V)$. By hypothesis, there exists a fuzzy r-minimal β-open set U containing x_α such that $f(U) \subseteq V$. This implies that $\cup U = f^{-1}(V)$ and hence from Theorem 3.5, $f^{-1}(V)$ is fuzzy r-minimal β-open.

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (4) For $A \in I^X$,

$$\begin{aligned} f^{-1}(mC(f(A), r)) &= f^{-1}(\cap\{F \in I^Y : f(A) \subseteq F \text{ and } F \text{ is fuzzy } r\text{-minimal closed}\}) \\ &= \cap\{f^{-1}(F) \in I^X : A \subseteq f^{-1}(F) \text{ and} \\ &\qquad f^{-1}(F) \text{ is fuzzy } r\text{-minimal } \beta\text{-closed}\} \\ &\supseteq \cap\{K \in I^X : A \subseteq K \text{ and } K \text{ is fuzzy } r\text{-minimal } \beta\text{-closed}\} \\ &= m\beta C(A, r). \end{aligned}$$

Hence $f(m\beta C(A, r)) \subseteq mC(f(A), r)$.

(4) $\Rightarrow$ (5) For $B \in I^Y$,

$$f(m\beta C(f^{-1}(B), r)) \subseteq mC(f(f^{-1}(B)), r) \subseteq mC(B, r).$$

So $m\beta C(f^{-1}(B), r) \subseteq f^{-1}(mC(B, r))$.

(5) $\Rightarrow$ (6) For $B \subseteq Y$, from Theorem 2.1 and Theorem 3.9, it follows

$$\begin{aligned} f^{-1}(mI(B, r)) &= f^{-1}(\tilde{1} - mC(\tilde{1} - B, r)) \\ &= \tilde{1} - f^{-1}(mC(\tilde{1} - B, r)) \\ &\subseteq \tilde{1} - m\beta C(f^{-1}(\tilde{1} - B), r) \\ &= m\beta I(f^{-1}(B), r). \end{aligned}$$

This implies $f^{-1}(mI(B, r)) \subseteq m\beta I(f^{-1}(B), r)$.

(6) $\Rightarrow$ (1) Let V be any fuzzy r-minimal open set containing $f(x_\alpha)$ for a fuzzy point x_α. By hypothesis, $x_\alpha \in f^{-1}(V) = f^{-1}(mI(V, r)) \subseteq m\beta I(f^{-1}(V), r)$. Since $x_\alpha \in m\beta I(f^{-1}(V), r)$, by Lemma 3.11, there exists a fuzzy r-minimal β-open set U containing x_α such that $U \subseteq f^{-1}(V)$. This implies $f^{-1}(V)$ is fuzzy r-minimal β-open, and hence f is fuzzy r-M β-continuous. $\square$

Definition 4.5. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then f is said to be *fuzzy r-M^*-β-open* if for every fuzzy r-minimal β-open set A in X, $f(A)$ is fuzzy r-minimal open in Y.

Theorem 4.6. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$.*

(1) *f is fuzzy r-M^*-β-open.*

(2) *$f(m\beta I(A, r)) \subseteq mI(f(A), r)$ for $A \in I^X$.*

(3) *$m\beta I(f^{-1}(B), r) \subseteq f^{-1}(mI(B, r))$ for $B \in I^Y$.*

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $A \in I^X$,

$$\begin{aligned} f(m\beta I(A, r)) &= f(\cup\{B \in I^X : B \subseteq A, B \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\}) \\ &= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r\text{-minimal open}\} \\ &\subseteq \cup\{U \in I^Y : U \subseteq f(A), U \text{ is fuzzy } r\text{-minimal open}\} \\ &= mI(f(A), r) \end{aligned}$$

Hence $f(m\beta I(A, r)) \subseteq mI(f(A), r)$.

(2) $\Rightarrow$ (3)

For $B \in I^Y$, from (3),

$$f(m\beta I(f^{-1}(B),r)) \subseteq mI(f(f^{-1}(B)),r) \subseteq mI(B,r).$$

Similarly, we have the implication (3) ⇒ (2).

□

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy r-minimal structure $\mathcal{M}_r$ is said to have the property $(\mathcal{U})$ [4] if for $A_i \in \mathcal{M}_r$ $(i \in J)$,

$$\mathcal{M}_r(\cup A_i) \geq \wedge \mathcal{M}_r(A_i).$$

Theorem 4.7 ([4])**.** *Let $(X,\mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then $mI(A,r) = A$ if and only if A is fuzzy r-minimal open for $A \in I^X$.*

From the above Theorem 4.7, obviously the following corollary is obtained:

Corollary 4.8. Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$. If $(Y,\mathcal{N})$ has the property $(\mathcal{U})$, then the following are equivalent:
(1) f is fuzzy r-M^*-β-open.
(2) $f(m\beta I(A,r)) \subseteq mI(f(A),r)$ for $A \in I^X$.
(3) $m\beta I(f^{-1}(B),r) \subseteq f^{-1}(mI(B,r))$ for $B \in I^Y$.

Definition 4.9. Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$. Then f is said to be *fuzzy r-M-β-open*if for fuzzy r-minimal open set A in X, $f(A)$ is fuzzy r-minimal β-open in Y.

Theorem 4.10. *Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$. Then the following are equivalent:*
(1) f is fuzzy r-M-β-open.
(2) $f(mI(A,r)) \subseteq m\beta I(f(A),r)$ for $A \in I^X$.
(3) $mI(f^{-1}(B),r) \subseteq f^{-1}(m\beta I(B,r))$ for $B \in I^Y$.

Proof. (1) ⇒ (2) For $A \in I^X$,

$$\begin{aligned} f(mI(A,r)) &= f(\cup\{B \in I^X : B \subseteq A, B \text{ is fuzzy } r\text{-minimal open}\}) \\ &= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\} \\ &\subseteq \cup\{U \in I^X : U \subseteq f(A), U \text{ is fuzzy } r\text{-minimal } \beta\text{-open}\} \\ &= m\beta I(f(A),r) \end{aligned}$$

Hence $f(mI(A,r)) \subseteq m\beta I(f(A),r)$.
(2) ⇒ (3)
For $B \in I^Y$, from (3) it follows that

$$f(mI(f^{-1}(B), r)) \subseteq m\beta I(f(f^{-1}(B)), r) \subseteq m\beta I(B, r).$$

Hence we get (3).

(3) ⇒ (2) It is similar to the proof of the implication (2) ⇒ (3).

(2) ⇒ (1) Let A be a fuzzy r-minimal open set in X. Then $A = mI(A, r)$. By (2), $f(A) = m\beta I(f(A), r)$ and hence by Theorem 3.9 (3), $f(A)$ is fuzzy r-minimal β-open.

□

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