

# 상호 간섭 Broadcast 채널을 위한 MIMO 간섭 정렬을 이용한 복잡도를 줄인 스케줄링

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## Reduced Complexity Scheduling Method with MIMO Interference Alignment for Mutually Interfering Broadcast Channels

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### 요 약

본 논문에서는, 다중 안테나 다중 사용자가 존재하는 3-cell 간섭 Broadcast 채널 (IFBC: interference broadcast channel) 에서 얻을 수 있는 공간 다중화 이득 (spatial multiplexing gain)에 대해서 소개한다. 이러한 공간 다중화 이득을 최대화하면서 총 수율을 높이기 위하여, 우리는 간섭 정렬 기법 (IA: interference alignment) 에 스케줄링 기법을 더한 방법을 제안하며, 이는 IFBC 환경에서 TDMA 기법보다 높은 수율 성능을 보인다. 최적의 스케줄링 방법은 다중 사용자 이득을 이용하여 총 수율을 최대화하는 전수 조사 알고리즘을 이용한다. 또한, 전수 조사의 계산 복잡도가 매우 높기 때문에, 이를 효과적으로 줄이는 coordinated ascent 방법을 이용한 부 최적 스케줄링 방법도 제안 한다.

**Key Words** : multiple-input multiple-output (MIMO), interference alignment (IA), spatial multiplexing gain, scheduling, broadcast channel

### ABSTRACT

In this paper, we first study the spatial multiplexing gain for the 3-cell interfering broadcast channels (IFBC) where all base stations and mobile users are equipped with multiple antennas. Then, we present the IA scheme in conjunction with user selection which outperforms the TDMA technique in the IFBC environment. The optimal scheduling method utilizes multiuser diversity to achieve a significant fraction of sum capacity by using an exhaustive search algorithm. To reduce the computational complexity, a suboptimal scheduling method is proposed based on a coordinate ascent approach.

### I. Introduction

In past years, there have been many researches on the capacity region of Gaussian interference

channels (IFCs). Although the capacity region of some special cases such as strong interference has been derived in [1] and [2], the complete characterization of the capacity for general IFCs

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is still an open problem<sup>[3,4]</sup>. Alternatively, studies on degrees of freedom (DOF) have received a large amount of attention, since the DOF determines the sum capacity slope in the high signal-to-noise ratio (SNR) region.

Especially, in [5], the authors introduced a novel algorithm called interference alignment (IA) and proved the achievability of the theoretical bound on the DOF for multi-input multi-output (MIMO) 3-user IFC systems with perfect channel state information (CSI) at all nodes. Note that the assumptions of the time variant nature and the symbol extension of the channel were adopted to derive the DOF for the general K-user IFC<sup>[5,6]</sup>. A key idea of the IA scheme is to design the precoding matrices which align all interference signals to be overlapped in almost the half of signal spaces at each receiver so that the dimension of interference-free space for the desired signal is maximized. Unfortunately, in the low and medium SNR range, the IA scheme shows the sum rate performance inferior to time division multiple access (TDMA) systems which serves one user with the largest channel gain at each time slot<sup>[7]</sup>. This rather poor performance is contributed to a fact that the IA scheme is originally optimized in terms of the DOF which becomes a dominant factor in high SNR, and cannot exploit multiuser diversity in MIMO IFC systems.

In order to alleviate this problem, we propose an IA scheme combined with user selection algorithms for 3-cell interference broadcast channels (IFBC) with linear transceiver where each base station (BS) equipped with  $M$  antennas transmits messages to its corresponding  $K_i$  mobile users ( $i = 1, 2, 3$ ) with  $M$  antennas. In this paper, we consider spatial multiplexing gain (SMG) for designing the IFBC, where the SMG is defined as the DOF obtained when only space dimensions are exploited. i.e., we do not consider symbol extension of the channel. The SMG for 2-cell multi-input single-output (MISO) and MIMO IFBC with linear processing have been derived in [8] and [9], respectively. First, we provide an

expression of SMG for this system configuration using the result of [5] and [9]. Surprisingly, the derived SMG is given by  $\frac{3M}{2}$  regardless of the number of users  $K_i$ 's, which means that before the application of the IA algorithm, selecting one active user for each cell not only maintains the optimal performance in the context of the SMG, but also allows exploiting additional multiuser diversity.

To efficiently utilize the multiuser diversity, we introduce both optimal and reduced complexity scheduling algorithms. The optimal method which maximizes the sum rate needs an exhaustive search for all possible user combinations, and thus the complexity may become prohibitive as  $K_i$  increases. Thus, we develop a reduced complexity suboptimal scheduling algorithm based on a coordinate ascent approach [10] which provides the sum rate close to the optimal one. Compared to the optimal scheduling which requires exponential complexity, the complexity of the proposed suboptimal method becomes linear with respect to the number of users. The simulation results show that the optimal scheduling algorithm provides 7.1dB and 5.8dB gains over the conventional IA scheme [5] and the TDMA scheme, respectively, at the sum rate of  $15 \text{ bps/Hz}$  with  $K_1 = K_2 = K_3 = 10$  and  $M = 2$ . Also, the reduced complexity scheduling method performs within 1dB of the optimal solution with substantially reduced search complexity.

The rest of the paper is organized as follows: In Section II, we describe a general system model for the B-cell IFBC. The SMG of the 3-cell MIMO IFBC is analyzed in Section III, and then Section IV briefly reviews the IA scheme and presents scheduling algorithms. Section V illustrates the simulation results and the paper is closed with conclusions in Section VI.

The following notations are used throughout the paper. We employ uppercase boldface letters for matrices, lowercase boldface for vectors and normal letters for scalar quantities.  $\mathbf{A}^T, \mathbf{A}^H$  and

$Tr(\mathbf{A})$  represent the transpose, the conjugate transpose and the trace for any matrix  $\mathbf{A}$ , respectively. A set of all complex matrices of size  $M \times N$  is denoted by  $C^{M \times N}$ . The Frobenius norm of a matrix  $\mathbf{A}$  is  $\|\mathbf{A}\|_F = \sqrt{Tr(\mathbf{A}\mathbf{A}^H)}$  and  $\|\cdot\|$  denotes the Euclidean 2-norm of a vector. Additionally,  $\mathbf{I}_d$  indicates an identity matrix of size  $d$  and  $E[\cdot]$  accounts for expectation.

## II. System Model

In this section, we present a general description of the IFBC illustrated in Figure 1. There are  $B$  BSs,  $BS^{(1)} \dots BS^{(B)}$ , which support its corresponding  $K_i$  users ( $i = 1, \dots, B$ ) where all nodes are equipped with  $M$  antennas. Here we assume that  $M$  is even. We refer to this configuration as the  $M \times B \times (K_1 \dots K_B)$  IFBC. For simplicity, we define user  $(i, j)$  as the  $j$ -th user in the  $i$ -th cell. Then, the channel output of user  $(i, j)$ ,  $\mathbf{y}_j^{(i)}$ , is represented as

$$\mathbf{y}_j^{(i)} = \mathbf{H}_i^{(i,j)} \mathbf{x}^{(i)} + \sum_{k \neq i, k=1}^B \mathbf{H}_k^{(i,j)} \mathbf{x}^{(k)} + \mathbf{n}_j^{(i)} \quad (1)$$

where  $i \in \{1, \dots, B\}$  is a cell index,  $j \in \{1, \dots, K_i\}$  denotes a user index,  $\mathbf{x}^{(i)} \in C^M$  stands for the transmitted signal vector from  $BS^{(i)}$ ,  $\mathbf{H}_k^{(i,j)} \in C^{M \times M}$  represents the channel matrix from  $BS^{(k)}$  to user  $(i, j)$ , and  $\mathbf{n}_j^{(i)}$  indicates the additive complex Gaussian noise vector with zero mean and covariance  $\sigma_n^2 \mathbf{I}_M$  for user  $(i, j)$ . It is assumed that the channel elements are sampled from independent and identically distributed (i.i.d.) complex Gaussian random variables with zero mean and unit variance. Thus the probability of event that the channel is rank-deficient converges to zero. Also, we assume that the channel information is globally available, i.e., all channel elements are perfectly known at all nodes. Although the desired

channel elements of  $\mathbf{H}_i^{(i,j)}$  generally have power larger than that of the interference channel coefficient  $\mathbf{H}_k^{(i,j)}$ , ( $k \neq i$ ), due to a path loss, we consider the most challenging case where users are located in cell boundaries so that the elements of both  $\mathbf{H}_i^{(i,j)}$  and  $\mathbf{H}_k^{(i,j)}$  have the same power. Also, we consider the total power constraint  $\sum_{i=1}^B E[\|\mathbf{x}^{(i)}\|^2] = P$  where  $P$  denotes the total

transmit power and thus the SNR is defined as  $\frac{P}{\sigma_n^2}$ .

For practical implementation issues, we consider IFBCs using linear precoding and no symbol extension of the channel in the B-cell environment. Then, the transmitted signal vector  $\mathbf{x}^{(i)}$  in (1) is related to  $\mathbf{s}_j^{(i)}$  as  $\mathbf{x}^{(i)} = \frac{1}{\sqrt{\gamma}} \sum_{j=1}^{K_i} \mathbf{V}_j^{(i)} \mathbf{s}_j^{(i)}$  where  $\mathbf{s}_j^{(i)} \in C^{L_j^{(i)}}$

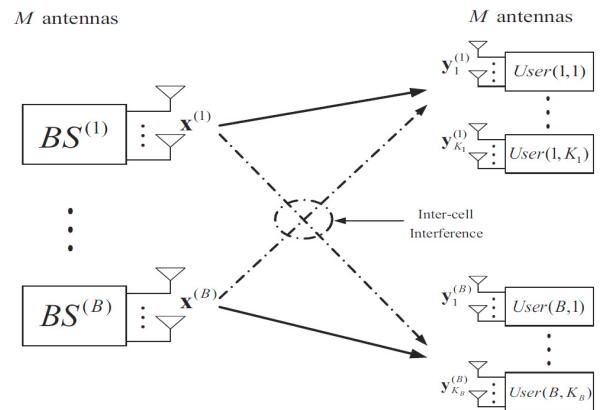


Fig. 1. Diagram of  $M \times B \times (K_1, \dots, K_B)$  IFBC Systems.

indicates the information signal vector for user  $(i, j)$  whose covariance matrix is given as  $\mathbf{I}_{L_j^{(i)}}$ ,  $\mathbf{V}_j^{(i)} \in C^{M \times L_j^{(i)}}$  represents the precoding matrix for user  $(i, j)$  and  $\gamma = \left( \sum_{i=1}^B \sum_{j=1}^{K_i} \|\mathbf{V}_j^{(i)}\|_F^2 \right) / P$  is introduced to satisfy the total power constraint. From the dimensions of  $\mathbf{V}_j^{(i)}$  and  $\mathbf{s}_j^{(i)}$ , we can see that user  $(i, j)$  is served with  $L_j^{(i)}$  data streams.

At user  $(i, j)$ , the receive filter  $\mathbf{U}_j^{(i)} \in C^{L_j^{(i)} \times M}$  is post-multiplied to  $\mathbf{y}_j^{(i)}$  to obtain  $\widehat{\mathbf{s}}_j^{(i)} = \mathbf{U}_j^{(i)} \mathbf{y}_j^{(i)}$  as

$$\begin{aligned} \widehat{\mathbf{s}}_j^{(i)} &= \frac{1}{\sqrt{\gamma}} \mathbf{U}_j^{(i)} \mathbf{H}_i^{(i,j)} \mathbf{V}_j^{(i)} \mathbf{s}_j^{(i)} + \\ &\frac{1}{\sqrt{\gamma}} \mathbf{U}_j^{(i)} \mathbf{H}_i^{(i,j)} \sum_{\substack{m \neq j, \\ m=1}}^{K_i} \mathbf{V}_m^{(i)} \mathbf{s}_m^{(i)} + \\ &\frac{1}{\sqrt{\gamma}} \mathbf{U}_j^{(i)} \sum_{k \neq i, k=1}^B \mathbf{H}_k^{(i,j)} \sum_{m=1}^{K_k} \mathbf{V}_m^{(k)} \mathbf{s}_m^{(k)} + \mathbf{n}_j^{(i)} \end{aligned} \quad (2)$$

where  $\widehat{\mathbf{s}}_j^{(i)}$  is the estimated vector for  $\mathbf{s}_j^{(i)}$  and  $\mathbf{n}_j^{(i)}$  represents the filtered noise with covariance  $\sigma_n^2 \mathbf{U}_j^{(i)} \mathbf{U}_j^{(i)H}$ . For given  $\{\mathbf{V}_j^{(i)}\}$  and  $\{\mathbf{U}_j^{(i)}\}$ , the achievable sum rate is then computed as

$$\begin{aligned} R_{\Sigma}(\{\mathbf{V}_j^{(i)}\}, \{\mathbf{U}_j^{(i)}\}) &= \sum_{i=1}^B \sum_{j=1}^{K_i} R_j^{(i)} \\ &= \sum_{i=1}^B \sum_{j=1}^{K_i} \sum_{l=1}^{L_j^{(i)}} \log_2 \left( 1 + \frac{|\mathbf{u}_{j,l}^{(i)} \mathbf{H}_i^{(i,j)} \mathbf{v}_{j,l}^{(i)}|^2}{\gamma \sigma_n^2 \|\mathbf{u}_{j,l}^{(i)}\|^2 + I_{i,j,l}} \right) \end{aligned} \quad (3)$$

where  $R_j^{(i)}$ ,  $\mathbf{u}_{j,l}^{(i)}$  and  $\mathbf{v}_{j,l}^{(i)}$  denote the individual rate for user  $(i, j)$ , the  $l$ -th row of  $\mathbf{U}_j^{(i)}$  and  $l$ -th column of  $\mathbf{V}_j^{(i)}$ ,

respectively, and  $I_{i,j,l} = |\mathbf{u}_{j,l}^{(i)} \mathbf{H}_k^{(i,j)} \mathbf{v}_{m,n}^{(k)}|^2$

Thus, the maximum sum rate  $R_{\Sigma}^{\max}(P)$  for given power constraint  $P$  is written as

$$R_{\Sigma}^{\max}(P) = \max_{(R_1^{(1)}, R_2^{(1)}, \dots, R_{K_B}^{(B)})} R_{\Sigma}(\{\mathbf{V}_j^{(i)}\}, \{\mathbf{U}_j^{(i)}\})$$

where the achievable rate region with linear processing and no symbol extension is defined as

$$\mathbb{R}(P) = \bigcup_{\substack{0 \leq L_j^{(i)} \leq M, \\ \mathbf{V}_j^{(i)}, \mathbf{U}_j^{(i)} \text{ for } \forall i, j, \\ \sum_{i=1}^B \sum_{j=1}^{K_i} \|\mathbf{V}_j^{(i)}\|_F^2 = P}} \{(R_1^{(1)}, R_2^{(1)}, \dots, R_{K_B}^{(B)})\}.$$

Finally, the SMG is defined as

$$\eta = \lim_{P \rightarrow \infty} \frac{R_{\Sigma}^{\max}(P)}{\log P}.$$

### III. Spatial Multiplexing Gain for $M \times B \times (K_1, \dots, K_B)$ IFBC

In this section, we derive the SMG for  $M \times 3 \times (K_1 \dots K_B)$  IFBC systems with linear precoding and no symbol extension. To this end, we utilize the result of [9] to confirm the converse argument. The IA scheme introduced in [5] is employed to show the achievability. Note that the authors in [5] proposed the MIMO IA scheme only for 3-user IFC under the assumption of linear precoding and no symbol extension, and to the best of our knowledge, there are no existing  $B$ -user MIMO IA scheme which obtains the maximum SMG for  $B > 3$ . Thus, in this paper, we assume  $B = 3$ .

Based on the derived result, we will show that one can maintain the optimal SMG while exploiting multiuser diversity by performing the IA scheme after selecting one best user within each cell. In the following, we will show that the SMG for the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC is given by

$$\eta_{IFBC}(M \times 3 \times (K_1, K_2, K_3)) = \frac{3M}{2}. \quad (4)$$

#### 3.1. Upper Bound

We first verify that the SMG larger than (4) cannot be achieved in the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC. For ease of explanation, let us denote  $\eta_i$  as the SMG associated with cell  $i$ . In [9], the authors derived the SMG of the  $(M, K_1 N, M, K_2 N)$  IFBC under the assumption of full CSI at all nodes as

$$\eta_{IFBC}(M, K_1 N, M, K_2 N) = \min\{2M, (K_1 + K_2)N, \max(M, N)\}. \quad (5)$$

Here, the  $(M, K_1 N, M, K_2 N)$  IFBC represents

the two-cell IFBC ( $B=2$ ) where  $BS^{(i)}$  ( $i=1,2$ ) equipped with  $M$  transmit antennas supports its corresponding  $K_i$  mobile users with  $N$  antennas.

Choosing any  $\eta_i$  and  $\eta_j$  ( $i \neq j$ ) from  $\{\eta_1, \eta_2, \eta_3\}$  gives the following bound:

$$\begin{aligned} \eta_i + \eta_j &\leq \eta_{IFBC}(M, K_1 M, M, K_2 M) \\ &= M \text{ for } \forall i, j \in \{1, 2, 3\}, i \neq j. \end{aligned} \tag{6}$$

Then we simply add up all possible combinations of an inequality (6). This gives us

$$\begin{aligned} \sum_{i, j \in \{1, 2, 3\}, i \neq j} (\eta_i + \eta_j) &\leq \sum_{i, j \in \{1, 2, 3\}, i \neq j} M \\ \Rightarrow \eta_1 + \eta_2 + \eta_3 &\leq \frac{3M}{2}. \end{aligned} \tag{7}$$

Consequently, we conclude that the SMG for  $M \times 3 \times (K_1, K_2, K_3)$  IFBC is upper bounded by  $\frac{3M}{2}$ .

### 3.2. Lower Bound

Now we show that the SMG of (4) can be achieved in the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC. In [5], the IA scheme was introduced to achieve the maximum SMG of  $\frac{3M}{2}$  in 3-user MIMO IFC systems. Since eliminating  $(K_i - 1)$  users does not increase the SMG, the SMG of the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC cannot be less than that of the 3-user MIMO IFC denoted by  $\eta_{IFC}(M \times 3)$ . Consequently, the following lower bound is obtained as

$$\begin{aligned} \eta_{IFBC}(M \times 3 \times (K_1, K_2, K_3)) &\geq \eta_{IFBC}(M \times 3 \times (1, 1, 1)) \\ &= \eta_{IFC}(M \times 3) = \frac{3M}{2}. \end{aligned} \tag{8}$$

From (7) and (8), in conclusion, the maximum

achievable SMG in the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC is exactly given by  $\frac{3M}{2}$  regardless of  $K_i$ 's.

Through the achievability proof, we can see that the IA scheme combined with scheduling attains the maximum SMG as well as multiuser diversity.

## IV. Scheduling Methods

From the SMG analysis in Section III, we learn that the optimal SMG is maintained when a single user per cell is supported with the IA algorithm in [5]. Moreover, if we choose the user with the best channel characteristics in each cell, the system can exploit the benefits of multiuser diversity. As the number of users  $K_i$  increases, the gain in spectral efficiency achieved by the multiuser diversity also grows. Thus, it is natural to consider a scheduling method combined with the IA algorithm for the improved sum rate performance. In this section, we first review the IA algorithm with our system configuration and present the optimal scheduling method using an exhaustive search. For an implementation issue, we also propose a reduced complexity scheduling algorithm.

### 4.1. Interference Alignment for

$M \times 3 \times (K_1, K_2, K_3)$  IFBC

Without loss of generality, we suppose that user  $(i, 1)$  is selected in cell  $i$  for  $i=1, 2, 3$ . A detailed description of user selection will be presented in the following subsections. To align the interference, we perform linear precoding. With scheduling, the received signal vector for user  $(i, 1)$  in (2) can be expressed as

$$\mathbf{y}_1^{(i)} = \frac{1}{\sqrt{\gamma}} \sum_{k=1}^3 H_k^{(i,1)} V_1^{(k)} \mathbf{s}_1^{(k)} + \mathbf{n}_1^{(i)}. \tag{9}$$

Here, the length of the information signal vector  $\mathbf{s}_1^{(i)}, L_1^{(i)}$ , is  $\frac{M}{2}$  for  $i=1, 2, 3$  [5].

To decode the desired information signal vector from (9), the space of interference signals should

have at most  $\frac{M}{2}$  dimension and be linearly independent with the desired signal space. Thus, each precoder has to be designed to satisfy the three interference aligning constraints described as

$$\begin{aligned} span(\mathbf{H}_2^{(1,1)} \mathbf{V}_1^{(2)}) &= span(\mathbf{H}_3^{(1,1)} \mathbf{V}_1^{(3)}), \\ span(\mathbf{H}_3^{(2,1)} \mathbf{V}_1^{(3)}) &= span(\mathbf{H}_1^{(2,1)} \mathbf{V}_1^{(1)}), \\ span(\mathbf{H}_2^{(3,1)} \mathbf{V}_1^{(2)}) &= span(\mathbf{H}_1^{(3,1)} \mathbf{V}_1^{(1)}) \end{aligned} \quad (10)$$

where  $span(\mathbf{X})$  indicates the vector space spanned by the column vectors of  $\mathbf{X}$ .

The precoding matrices are obtained by restricting the above constraints (10) as

$$\begin{aligned} span(\mathbf{H}_2^{(1,1)} \mathbf{V}_1^{(2)}) &= span(\mathbf{H}_3^{(1,1)} \mathbf{V}_1^{(3)}), \\ (\mathbf{H}_3^{(2,1)} \mathbf{V}_1^{(3)}) &= (\mathbf{H}_1^{(2,1)} \mathbf{V}_1^{(1)}), \\ (\mathbf{H}_2^{(3,1)} \mathbf{V}_1^{(2)}) &= (\mathbf{H}_1^{(3,1)} \mathbf{V}_1^{(1)}). \end{aligned} \quad (11)$$

Equation (11) can be equivalently expressed as

$$\begin{aligned} span(\mathbf{V}_1^{(2)}) &= span(\mathbf{E} \mathbf{V}_1^{(3)}), \\ \mathbf{V}_1^{(2)} &= (\mathbf{H}_2^{(3,1)})^{-1} \mathbf{H}_1^{(3,1)} \mathbf{V}_1^{(1)}, \\ \mathbf{V}_1^{(3)} &= (\mathbf{H}_3^{(2,1)})^{-1} \mathbf{H}_1^{(2,1)} \mathbf{V}_1^{(1)} \end{aligned} \quad (12)$$

where

$$\mathbf{E} = (\mathbf{H}_1^{(3,1)})^{-1} \mathbf{H}_2^{(3,1)} (\mathbf{H}_2^{(1,1)})^{-1} \times \mathbf{H}_3^{(1,1)} (\mathbf{H}_3^{(2,1)})^{-1} \mathbf{H}_1^{(2,1)}.$$

Finally, we can set  $\mathbf{V}_1^{(1)}$  as

$$\mathbf{V}_1^{(1)} = [e_1 e_2 \cdots e_{M/2}] \quad (13)$$

where  $e_1 e_2 \cdots e_{M/2}$  are the eigenvectors of  $\mathbf{E}$ . Once  $\mathbf{V}_1^{(1)}$  is determined,  $\mathbf{V}_1^{(2)}$  and  $\mathbf{V}_1^{(3)}$  are calculated by (12). Actually  $\mathbf{V}_1^{(1)}$  in (13) is not optimized in terms of the sum rate. The authors in [11] proposed a zero-forcing (ZF) based decoder design and precoder optimization method which improves the sum rate performance significantly.

#### 4.2. Optimal Scheduling Method

We employ the IA scheme in conjunction with scheduling for the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC. The main concern is how to select the best user in each cell to fully utilize the multiuser diversity. To this end, we will optimize the objective problem through the following algorithm. For simplifying explanation, we define  $(k_1, k_2, k_3) \in \Pi$  as the selected user set where  $k_i$  represents the selected user index in cell  $i$  for  $i=1,2,3$  and  $\Pi$  denotes the set of all user combinations as  $\{(1, 1, 1), \dots, (1, 1, K_1), \dots, (K_1, K_2, K_3)\}$ .

Also, we assume that the information signal vectors are decoded with a minimum mean squared error (MMSE) filter at each receiver. From (9), defining the  $M \times \frac{3M}{2}$  effective channel matrix of user  $(i, k_i)$ ,  $\mathbf{H}_{(i, k_i)}^e$ , as

$$\mathbf{H}_{(i, k_i)}^e = \begin{bmatrix} \mathbf{H}_1^{(i, k_i)} \mathbf{V}_{k_i}^{(1)} & \mathbf{H}_2^{(i, k_i)} \mathbf{V}_{k_i}^{(2)} & \mathbf{H}_3^{(i, k_i)} \mathbf{V}_{k_i}^{(3)} \end{bmatrix}. \quad (14)$$

the conventional MMSE filter for this effective channel is given by

$$\overline{\mathbf{U}}_{k_i}^{(i)} = (\mathbf{H}_{(i, k_i)}^{eH} \mathbf{H}_{(i, k_i)}^e + \sigma_n^2 \mathbf{I}_{3M/2})^{-1} \mathbf{H}_{(i, k_i)}^{eH} \quad (15)$$

From (14) and (15), the  $\frac{M}{2} \times M$  MMSE interference nulling matrix for user  $(i, k_i)$  can be expressed as

$$\mathbf{U}_{k_i}^{(i)} = \overline{\mathbf{U}}_{k_i}^{(i)T} \overline{\mathbf{U}}_{k_i}^{(i)} = [\mathbf{u}_{k_i, ((i-1)M/2+1)}^{(i)T} \mathbf{u}_{k_i, ((i-1)M/2+2)}^{(i)T} \cdots \mathbf{u}_{k_i, ((i-1)M/2+M/2)}^{(i)T}]^T \quad (16)$$

where  $\mathbf{u}_{k_i, l}^{(i)}$  represents the  $l$ -th row vector of  $\overline{\mathbf{U}}_{k_i}^{(i)}$ . By plugging (12),(13) and (16) to (3), the

optimal user set which maximize the sum rate achieved by the IA algorithm in conjunction with scheduling can be expressed as

$$(k_1^*, k_2^*, k_3^*) = \arg \max_{(k_1, k_2, k_3) \in \Pi R_\Sigma} \left( \{V_{k_i}^{(i)}\}, \{U_{k_i}^{(i)}\} \right). \quad (17)$$

From the above equation (17), we can find the optimal user set by employing an exhaustive searching method over all possible  $\prod_{i=1}^3 K_i$  combinations. Although the optimal performance is obtained in terms of the sum rate, the search complexity is prohibitive when  $K_i$  is large. In general, the problem of finding an efficient optimal scheduling algorithm is still an open issue and mainly heuristic algorithms have been introduced for multiuser wireless networks<sup>[12,13]</sup>.

### 4.3. Suboptimal Scheduling Method

In this subsection, we propose a reduced complexity scheduling algorithm. Obviously, our purpose is not only to reduce the complexity, but also to approach the optimal sum rate performance. To achieve this goal, we employ a coordinate ascent approach [10] with an initial user set. In this approach, we first select users with the maximum Frobenius norm of  $H_i^{(i, k_i)}, 1 \leq k_i \leq K_i$  for  $i = 1, 2, 3$ . The main idea of the coordinate ascent approach is to maximize the sum rate with respect to  $k_i$  while the others  $k_j$ 's for  $j \neq i$  are fixed to the most updated values. This operation is repeated for  $i = 1, 2, 3$  and the user set is obtained.

In what follows, we summarize the proposed suboptimal scheduling algorithm. In this algorithm,  $S_{i,j}$  represents a user set whose  $i$ -th component is changed to  $j$  from the original user set  $S = (k_1, k_2, k_3)$ . For example, we have  $S_{2,j} = (k_1, j, k_3)$ .

### Algorithm

#### Step1) Initialization

Step 1-1) Find a user  $k_i^0$  such that

$$k_i^0 = \arg \max_{k \in \{1, \dots, K_i\}} \|H_i^{(i, k)}\|_F^2 \quad \text{for } i = 1, 2, 3$$

Step 1-2) Initialize a user set

$$S = (k_1^0, k_2^0, k_3^0) \quad \text{and } i = 1.$$

#### Step2) Main Loop

Step 2-1)

For  $i = 1 : 3$

Find a user  $k_i^*$  such that

$$k_i^* = \arg \max_{k \in \{1, \dots, K_i\}} R_\Sigma(S_{i,k}).$$

Update  $S \leftarrow S_{i, k_i^*}$

End

The computation of the Frobenius norm for an  $M \times M$  matrix, which requires  $2M^2$  real multiplications and  $2M^2 - 1$  real additions, is relatively trivial comparing with the computation of the sum rate in equation (3). Neglecting the complexity related to the Frobenius norm computation, the number of search candidates of the proposed scheduling method is significantly reduced to  $\sum_{i=1}^3 K_i$  in comparison to the optimal scheduling method which requires  $\prod_{i=1}^3 K_i$ .

## V. Simulation Results

In this section, we present simulation results to demonstrate the efficacy of the proposed IA scheme in conjunction with scheduling for the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC. For simplicity, we set  $K_1 = K_2 = K_3 = K$ . Since each BS has its own power amplifier, we enforce the per-BS power constraint as  $E[\|x^{(i)}\|^2] = \frac{P}{3}$  for  $i = 1, 2, 3$

in our computer simulations.

Figures 2 and 3 present the average sum rate performance comparison between the TDMA and the IA scheme combined with the optimal scheduling for various  $K$  with  $M=2$  and  $M=4$ , respectively. The TDMA scheme serves one user with the maximum rate out of  $3K$  users at each time slot and total  $M$  data streams are transmitted

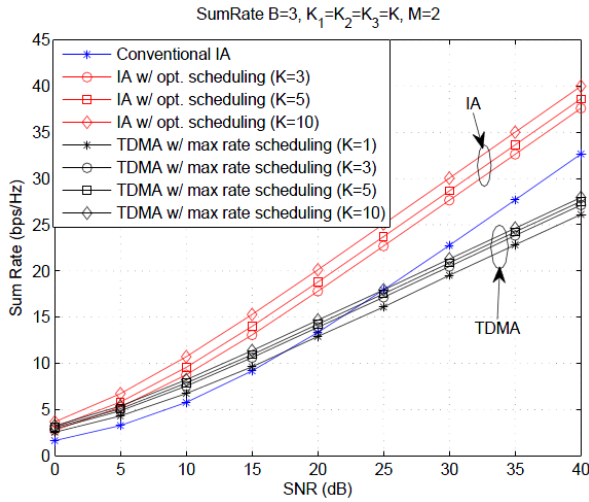


Fig. 2. Sum rate comparison between TDMA and IA with  $M=2$ .

through well known water-filling solution [14] at each transmission so that the SMG is given as  $M$ . Although the IA scheme achieves the optimal SMG at the high SNR regime as shown in the plot, the conventional IA scheme provides lower sum rate performance than that of the TDMA scheme in the low and medium SNR range where the conventional IA represents the IA scheme with a Round-Robin scheduling. However, observing the cross-over points between the IA and the TDMA scheme curves, we learn that the sum rate performance can be significantly improved by the optimal scheduling. By comparing  $M=2$  and  $M=4$  cases, we also observe an increase in the sum rate gap between the conventional IA and the TDMA scheme for a larger  $M$  in the low and medium SNR region, and the required multiuser diversity gain to alleviate this problem becomes larger as  $M$  increase.

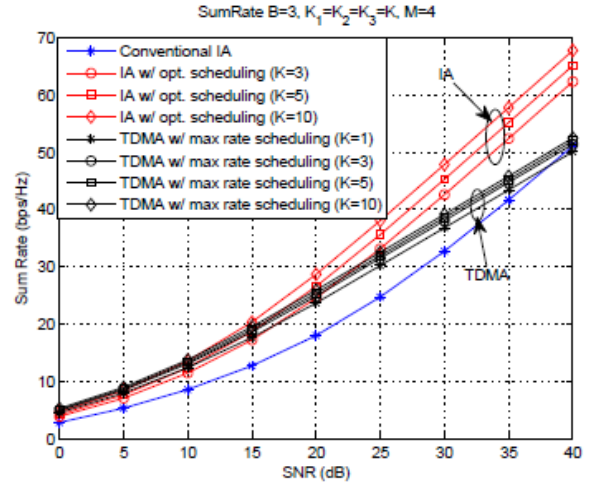


Fig. 3. Sum rate comparison between TDMA and IA with  $M=4$ .

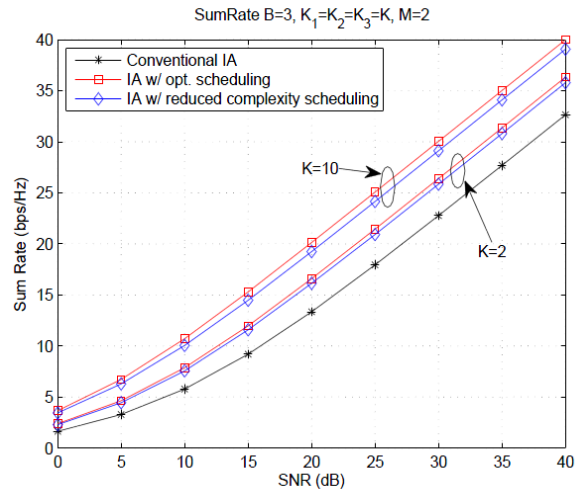


Fig. 4. Sum rate performance comparison of the optimal and suboptimal comparison of the optimal and suboptimal scheduling method with  $M=2$ .

In Figure 4, we compare the performance between the optimal and reduced complexity scheduling algorithms with various  $K$  and  $M=2$  in terms of the sum rate. We can see that optimal and reduced complexity scheduling algorithms provide 7.1dB and 6.3dB gains at the sum rate of 15 bps/Hz over the conventional IA scheme [5], respectively, with  $K=10$ . The sum rate performance for the optimal and reduced complexity scheduling algorithm is almost identical when  $K$  is small. In contrast, the sum rate gap between two algorithms



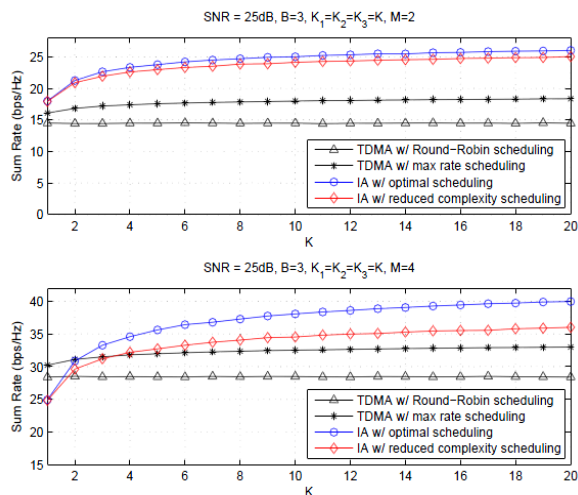


Fig. 5. Sum rate of the TDMA scheme and IA scheme combined with scheduling for  $M=2$  and  $M=4$ .

becomes larger as  $K$  increases. Note that the proposed reduced complexity scheduling method has linear complexity with respect to  $K$ , whereas the complexity of the optimal exhaustive search method grows exponentially. This leads to a significant reduction in the search complexity while approaching the sum rate performance close to the optimal one.

Figure 5. illustrates the multiuser diversity for the TDMA scheme and the IA scheme combined with scheduling algorithms at SNR=25dB. It is observed that the gain of the IA scheme obtained from exploiting multiuser diversity is larger than that of the TDMA scheme. This is because of TDMA scheme already takes advantage of the multiuser diversity even  $K=1$ . Also, we can see that the multiuser diversity becomes saturated as  $K$  grows.

## VI. Conclusion

In this paper, we have first studied an exact expression of the SMG for the  $M \times 3 \times (K_1, K_2, K_3)$  IFBC. To obtain higher sum rate performance compared to the TDMA scheme in the low and medium SNR region while maintaining the optimal SMG, we have proposed an IA scheme in conjunction with scheduling algorithms. The optimal scheduling algorithm can be obtained by an exhaustive search

which requires exponential complexity as  $K_i$  increases. In contrast, a reduced complexity scheduling algorithm based on a coordinate ascent approach provides the sum rate close to the optimal one with the significantly reduced number of search candidates. Through the simulation results, we have illustrated that the proposed IA scheme combined with scheduling outperforms the TDMA scheme in terms of the sum rate and performs only 1dB away from the optimal system with substantially reduced complexity.

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