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Power Allocation Schemes For Downlink Cognitive Radio Networks With Opportunistic Sub-channel Access

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Abstract

This paper considers a downlink cognitive radio (CR) network where one secondary user (SU) and one primary user (PU) share the same base station (BS). The spectrum of interest is divided into a set of independent, orthogonal subchannels. The communication of the PU is of high priority and the quality of service (QoS) is guaranteed by the minimum rate constraint. On the other hand, the communication of the SU is of low priority and the SU opportunistically accesses the subchannels that were previously discarded by the PU during power allocation. The BS assigns fractions α and $1-\alpha$ of the total available transmit power to the PU and the SU respectively. Two power allocation schemes with opportunistic subchannel access are proposed, in which the optimal values of α's are also obtained. The objective of one scheme is to maximize the rate of the SU, and the objective of the other scheme is to maximize the sum rate of the SU and the PU, both under the PU minimum rate constraint and the total transmit power constraint. Extensive simulation results are obtained to verify the effectiveness of the proposed schemes.

Keywords: Cognitive radio, power allocation, opportunistic subchannel access, minimum rate constraint

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1. Introduction

Radio spectrum is a limited and highly valuable resource for wireless communications. Therefore, how to efficiently utilize the spectrum is extremely important. However, many actual measurements have shown that most of the licensed spectrum is largely underutilized under the current inflexible approaches to spectrum regulation [1]. Cognitive radio (CR), a term first coined by Mitola [2], has emerged as a promising way to improve spectrum efficiency. CR allows the secondary user (SU) to exploit the underutilized spectrum originally allocated to the primary user (PU) as long as the quality of service (QoS) of the PU is not unduly affected.

Basically, there are two main paradigms for the SU to share the spectrum with the PU: opportunistic spectrum access (OSA) [3] and spectrum sharing (SS) [4]. In the OSA paradigm, the SU is allowed to opportunistically operate in the PU spectrum bands when the PU is inactive. On the other hand, in the SS paradigm, simultaneous transmissions of the SU and the PU are permitted as long as the interference caused by the SU to the PU is guaranteed to be within a tolerable range. Accordingly, the concept of interference temperature/power has been proposed to measure the tolerable interference level at the PU [4].

Power allocation is an important operation for the SU to guarantee that the QoS of the PU is not affected improperly. In this respect, a lot of valuable work on the problem of power allocation has been done for both OSA and SS based CR networks. The related work on the problem of power allocation in OSA based CR network include [5][6][7][8][[9]. Specifically, in [5], the authors investigated the problem of downlink power allocation in an OSA based CR network, and proposed power allocation schemes to maximize the SU capacity under interference constraints while considering imperfect spectrum sensing. In [6], using soft sensing information, the authors proposed power allocation schemes to maximize the signal to noise ratio (SNR) and capacity of the SU respectively in an OSA based CR network. By adjusting the transmit power at the SU transmitter to maintain a constant output SNR to the SU receiver, the authors in [7] proposed a power allocation scheme that maximizes the output SNR and limits the interference to a PU within an acceptable level. In [8], using the spectrum sensing information (SSI) gathered during the sensing period, the authors proposed two power allocation schemes that maximize the average data rate and minimize the outage probability respectively, while keeping the probability of detection and average transmit power constrained. By designing a cognitive receiver and frame structure that allows simultaneous spectrum sensing and data transmission, the authors in [9] proposed the optimal power allocation scheme that maximizes the ergodic capacity of the SU.

For SS based CR networks, the related work on the problem of power allocation include [10][11][12][13][14][15]. The authors in [10] proposed the optimal power allocation strategy to maximize the rate of the SU under the PU rate loss constraint, and in their further work [11], the optimal power allocation strategies to maximize the SU ergodic/outage capacity under the PU outage constraint were derived. In [12], the authors exploited the bi-directional nature of the primary network and proposed a distributed power control scheme that is based on the observation of PU communications, which can achieve higher spectrum usage while limiting the interference to the PU. Assuming that the SU knows the PU's power policy and channel state information (CSI) of the entire network, the authors in [13] studied the optimal power allocation problem of maximizing the ergodic capacity of the SU under the PU's outage probability constraint, the SU's outage probability constraint and the average transmit power constraint. In our previous work, [14] obtained the optimal power allocation to maximize the rate of the SU under the PU signal to interference plus noise ratio (SINR) constraint with limited PU's cooperation, and [15] obtained the optimal power/rate allocation schemes to minimize the weighted aggregate outage probability of the SUs in a CR multicast network.

In this paper, unlike existing work in literature [5][6][7][8][9][10][11][12][13][14][15], we consider a different spectrum sharing scenario, i.e., a downlink CR network with opportunistic subchannel access, where one SU shares the same base station (BS) with one PU. The communication of the SU is of low priority while the communication of the PU is of high priority. The QoS of the PU is guaranteed by the minimum rate constraint. The spectrum of interest is divided into a set of independent, orthogonal subchannels. Considering the fact that channel fadings between the BS and the PU on some subchannels are inferior to those on the other subchannels, these subchannels may be thus discarded by the PU during power allocation. However, considering the fact that the subchannels are not fully correlated for the PU and the SU in most cases[1], the discarded subchannels may be reused for the SU to communicate. The key feature of the opportunistic subchannel access is that the SU opportunistically accesses the subchannels that were previously discarded by the PU. The total available transmit power of the BS is shared between the PU and the SU, and a fraction α where $0 \leq \alpha \leq 1$, of the total available transmit power of the BS is allocated to the PU, with the remaining transmit power assigned to the SU. Two power allocation schemes are proposed, in which the optimal values of α's are also obtained. One power allocation scheme is to maximize the rate of the SU, and the other scheme is to maximize the sum rate of the SU and the PU, both under the PU minimum rate constraint and the total transmit power constraint. The main contribution of our work lies in proposing two power allocation schemes that do not require spectrum sensing as in [5]-[9] and can guarantee the QoS of the PU as well as provide certain communication opportunities for the SU.

The closest work related to our paper is in [17][18]. The authors in [17][18] studied an uplink scenario with opportunistic scheduling, where the SUs share the same BS with the PU under the interference outage constraint. It is noted that, [17][18] focused on performance analysis of opportunistic scheduling for the uplink scenario, while, this paper investigates the problem of power allocation with opportunistic subchannel access for the downlink scenario. In addition, unlike this paper, [17][18] adopted interference power constraint to protect the PU and focused on single channel scenario.

The rest of the paper is organized as follows. The system model is described in Section 2. The power allocation scheme under the PU minimum rate constraint and the total transmit power constraint that maximizes the rate of the SU is proposed in Section 3, while the power allocation scheme that maximizes the sum rate of the SU and the PU is proposed in Section 4. Simulation results are presented in Section 5. Finally, Section 6 concludes the paper.

2. System Model

For clarity of exposition, we consider a downlink CR network where one SU and one PU share the same BS. The spectrum of interest is divided into N independent, orthogonal channels. The channel power gains on channel n from the BS to the SU, and the BS to the PU are denoted by g_{ss}^n and g_{sp}^n, respectively. All the channels are assumed to be flat and block fading

[1] It has been recognized that channel fading even for two very closed locations may vary greatly, and the coherence distance is typically smaller than ten times the weavelength [16].

channels. Furthermore, we assume $g_{ss}^n \neq g_{pp}^n$, which means that the channel fadings on channel n for the SU and the PU are not fully correlated, in other words, the correlation coefficient, ρ_n, which is given by

$$\rho_n = \frac{\mathrm{cov}(g_{ss}^n, g_{pp}^n)}{\sqrt{\mathrm{var}(g_{ss}^n)\mathrm{var}(g_{pp}^n)}}, \tag{1}$$

is less than one, i.e., $0 \leq \rho < 1$. The white Gaussian noise power at both the SU and the PU is denoted by σ^2.

We assume that the SU and the PU can utilize up to N channels at a time while each channel can only be occupied by one SU or one PU at a time. The minimum rate constraint at the PU is adopted to guarantee the QoS of the PU as follows

$$\sum_{n=1}^{N} \ln\left(1 + \frac{P_p^n g_{pp}^n}{\sigma^2}\right) \geq R_p, \tag{2}$$

where P_p^n denotes the transmit power of the PU on channel n, and R_p denotes the required minimum rate at the PU. The PU allows the SU to opportunistically utilize the channel n only if that channel is discarded by the power allocation of the PU, i.e., $P_p^n = 0$. Since the correlation coefficient, ρ_n, is less than one, the channel discarded by the PU may experience less severe fading by the SU and can be thus reused by the SU to communicate. In addition, the total transmit power allocated to the SU and the PU is also restricted as

$$\sum_{n=1}^{N} (P_s^n + P_p^n) \leq P_{total}, \tag{3}$$

where P_s^n denotes the transmit power of the SU on channel n, and P_{total} denotes the total transmit power limit of the BS. We assume that the PU needs at most the available transmit power P_{total} to achieve the required minimum rate R_p. In other words, the available transmit power is assumed to be sufficient for achieving the required minimum rate of the PU[2]. Denote the fraction α where $0 \leq \alpha \leq 1$ as the ratio of the transmit power allocated to the PU to the total transmit power limit P_{total}. Thus the SU has opportunities to use the remaining transmit power $(1-\alpha)P_{total}$ to communicate.

We assume CSI on the channel power gains g_{ss}^n and g_{pp}^n is available at the BS. Thus the BS then makes the power allocation decisions for the PU and the SU respectively. In practice, the CSI may be obtained by classic channel estimation, training, or feedback mechanisms.

3. Power Allocation to Maximize the Rate of the SU

In this section, the power allocation scheme under the PU minimum rate constraint and the total transmit power constraint, to maximize the rate of the SU is proposed. In this scheme, power allocation is divided into two steps. Firstly, the PU performs power allocation to achieve its required minimum rate. Then, the channels discarded by the PU and the optimal fraction α can be determined. The SU thus performs a second power allocation according to the discarded channels and the remaining available transmit power. The detailed power allocation scheme is discussed in the following.

[2] Admission control of the PU can be used to guarantee that the required minimum rate is achieved with total transmit power P_{total}, which is beyond the scope of this paper.

3.1 Power allocation of the PU

The problem of power allocation to achieve the required minimum rate of the PU can be formulated as follows

$$\mathbf{P1}: \min_{P_p^n \geq 0} \sum_{n=1}^{N} P_p^n \tag{4}$$

$$\text{s.t.} \sum_{n=1}^{N} \ln\left(1 + \frac{P_p^n g_{pp}^n}{\sigma^2}\right) \geq R_p. \tag{5}$$

It is easy to verify that P1 is convex and can be thus solved by convex optimization [19]. The Lagrangian function of P1 is given by

$$L_1(P_p^n, \lambda) = \sum_{n=1}^{N} P_p^n - \lambda\left(\sum_{n=1}^{N} \ln\left(1 + \frac{P_p^n g_{pp}^n}{\sigma^2}\right) - R_p\right), \tag{6}$$

where λ is the Lagrange multiplier associated with the constraint (5). According to the Karush-Kuhn-Tucker (KKT) conditions [19], the optimal P_p^n should satisfy the following

$$\frac{\partial L_1(P_p^n, \lambda)}{\partial P_p^n} = 1 - \lambda \frac{\frac{g_{pp}^n}{\sigma^2}}{1 + \frac{P_p^n g_{pp}^n}{\sigma^2}} = 0. \tag{7}$$

Solving (7) with the constraint $P_p^n \geq 0$ yields

$$P_p^n = \left(\lambda - \frac{\sigma^2}{g_{pp}^n}\right)^+, \tag{8}$$

where $(.)^+$ denotes $\max\{., 0\}$. After inserting (8) into (5) set at equality, we have

$$\prod_{n=1}^{N} \left(1 + \left(\frac{\lambda g_{pp}^n}{\sigma^2} - 1\right)^+\right) = e^{R_p}. \tag{9}$$

It is observed that the left-hand side of (9) is a monotonically non-decreasing function of λ, thus λ can be easily found from (9) by the bisection search method [19]. Moreover, it is observed from (8) that the transmit power allocated to the PU on channel n, P_p^n, is zero if $\lambda \leq \frac{\sigma^2}{g_{pp}^n}$. That is to say the channel n is discarded by the PU if $\lambda \leq \frac{\sigma^2}{g_{pp}^n}$, and that channel can be thus used by the SU. Define $\mathbb{N}_p$ and $\mathbb{N}_s$ as the sets of channels utilized by the PU and being available to the SU respectively as

$$\mathbb{N}_p \triangleq \{\{n\} : \lambda > \frac{\sigma^2}{g_{pp}^n}\}, \tag{10}$$

and

$$\mathbb{N}_s \triangleq \{\{n\} : \lambda \leq \frac{\sigma^2}{g_{pp}^n}\}. \tag{11}$$

The total transmit power allocated to the PU is given by inserting (8) into the objective function in (4) as

$$\sum_{n=1}^{N} P_p^n = |\mathbb{N}_p|\lambda - \sigma^2 \sum_{n \in \mathbb{N}_p} \frac{1}{g_{pp}^n}, \tag{12}$$

where $|\mathbb{N}_p|$ is the cardinality of the set $\mathbb{N}_p$. Thus the fraction α is obtained as

$$\alpha = \frac{\sum_{n=1}^{N} P_p^n}{P_{total}} = \frac{|\mathbb{N}_p|\lambda}{P_{total}} - \frac{\sigma^2}{P_{total}} \sum_{n\in\mathbb{N}_p} \frac{1}{g_{pp}^n}. \tag{13}$$

Recall that we assume that the PU needs at most the available transmit power P_{total} to achieve its required minimum rate, thus gives SU opportunities to utilize the remaining transmit power to communicate, i.e., $0 \le \alpha \le 1$.

3.2 Power allocation of the SU

After having obtained $\mathbb{N}_s$ and α according to (11) and (13) respectively, the problem of power allocation of the SU is then formulated as

$$\mathbf{P2}: \max_{P_s^n \ge 0} \sum_{n\in\mathbb{N}_s} \ln\left(1 + \frac{P_s^n g_{ss}^n}{\sigma^2}\right) \tag{14}$$

$$\text{s.t.} \sum_{n\in\mathbb{N}_s} P_s^n \le (1-\alpha)P_{total}. \tag{15}$$

It is seen that P2 is convex. The Lagrangian function of P2 is written as

$$L_2(P_s^n, \mu(\alpha)) = \sum_{n\in\mathbb{N}_s} \ln\left(1 + \frac{P_s^n g_{ss}^n}{\sigma^2}\right) - \mu(\alpha)\left(\sum_{n\in\mathbb{N}_s} P_s^n - (1-\alpha)P_{total}\right), \tag{16}$$

where $\mu(\alpha)$ is the Lagrange multiplier associated with the constraint (15). By using the KKT conditions, the optimal P_s^n should satisfy the following

$$\frac{\partial L_2(P_s^n, \mu(\alpha))}{\partial P_s^n} = \frac{\frac{g_{ss}^n}{\sigma^2}}{1 + \frac{P_s^n g_{ss}^n}{\sigma^2}} - \mu(\alpha) = 0. \tag{17}$$

Solving (17) with the constraint $P_s^n \ge 0$ yields the optimal P_s^n as follows

$$P_s^n = \left(\frac{1}{\mu(\alpha)} - \frac{\sigma^2}{g_{ss}^n}\right)^+, n \in \mathbb{N}_s \tag{18}$$

Inserting (18) into (15) set at equality we have

$$\sum_{n\in\mathbb{N}_s} \left(\frac{1}{\mu(\alpha)} - \frac{\sigma^2}{g_{ss}^n}\right)^+ = (1-\alpha)P_{total}. \tag{19}$$

It is observed that the left-hand side of (19) is a monotonically non-increasing function of $\mu(\alpha)$, thus $\mu(\alpha)$ can be easily found from (19) by the bisection search method.

The overall proposed two-step power allocation scheme to maximize the rate of the SU, under the PU minimum rate constraint and the total transmit power constraint, is summarized as follows:

Algorithm 1: Power allocation to maximize the rate of the SU with opportunistic subchannel access

The PU performs power allocation as follows:

1: Calculate P_p^n, $\forall n \in \{1, \ldots, N\}$ by $\left(\lambda - \frac{\sigma^2}{g_{pp}^n}\right)^+$, where λ is obtained from (9) by the bisection search.

Determine available subchannels and fraction α:

1: Calculate $\mathbb{N}_p$, $\mathbb{N}_s$ and α by

$\mathbb{N}_p = \{\{n\} : \lambda > \frac{\sigma^2}{g_{pp}^n}\}$,

$\mathbb{N}_s = \{\{n\} : \lambda \leq \frac{\sigma^2}{g_{pp}^n}\}$,

$\alpha = \frac{|\mathbb{N}_p|\lambda}{P_{total}} - \frac{\sigma^2}{P_{total}} \sum_{n \in \mathbb{N}_p} \frac{1}{g_{pp}^n}$.

The SU performs power allocation as follows:

1: Calculate P_s^n, $\forall n \in \mathbb{N}_s$ by $P_s^n = \left(\frac{1}{\mu(\alpha)} - \frac{\sigma^2}{g_{ss}^n}\right)^+$, where $\mu(\alpha)$ is obtained from (19) by the bisection search.

Remark 1: The computational complexity of Algorithm 1 is analyzed in what follows. For the bisection search with guaranteed error tolerance of ε, according to [20], $O\left(N \log_2\left(\varepsilon^{-1}\right)\right)$ operations are needed to calculate P_p^n. Besides, calculating $\mathbb{N}_p$, $\mathbb{N}_s$ and α requires $N-1$ operations. Likewise, the bisection search for calculating P_s^n guarantees error tolerance of ε after $O\left(|\mathbb{N}_s| \log_2\left(\varepsilon^{-1}\right)\right)$ operations. Therefore, the total computational complexity of Algorithm 1 is approximately $O\left(N \log_2\left(\varepsilon^{-1}\right)\right)$.

4. Power Allocation to Maximize the Sum Rate of the SU and the PU

In this section, the power allocation scheme under the PU minimum rate constraint and the total transmit power constraint, to maximize the sum rate of the SU and the PU is proposed. In this scheme, we need to determine the optimal value of the fraction α. Let α_{min} as the minimum value of α in order to satisfy the PU minimum rate constraint, which is calculated according to (13). Then, the problem of power allocation to maximize the sum rate of the SU and the PU can be stated as

$$\mathbf{P3}: \max_{P_p^n \geq 0, P_s^n \geq 0, \alpha} \omega_1 \sum_{n=1}^{N} \ln\left(1 + \frac{P_p^n g_{pp}^n}{\sigma^2}\right) + \omega_2 \sum_{n \in \mathbb{N}_s(\alpha)} \ln\left(1 + \frac{P_s^n g_{ss}^n}{\sigma^2}\right) \tag{20}$$

$$\text{s.t.} \sum_{n=1}^{N} P_p^n \leq \alpha P_{total}, \tag{21}$$

$$\sum_{n \in \mathbb{N}_s(\alpha)} P_s^n \leq (1-\alpha) P_{total}, \tag{22}$$

$$\alpha_{min} \leq \alpha \leq 1, \tag{23}$$

where weight factors ω_1 and ω_2 represent the priorities designated to the PU and the SU, respectively, and $\omega_1 + \omega_2 = 1$. It is observed that if variable α was fixed, then $\mathbb{N}_p(\alpha)$ and $\mathbb{N}_s(\alpha)$ would be obtained, thus P3 would be decoupled into two subproblems. Therefore, we separate the above problem into two levels of optimization.

At the lower level, for a fixed value of α, P3 decouples into two independent subproblems as follows

$$\mathbf{P4}: \max_{P_p^n \geq 0} \sum_{n=1}^{N} \ln\left(1+\frac{P_p^n g_{pp}^n}{\sigma^2}\right) \tag{24}$$

$$\text{s.t.} \sum_{n=1}^{N} P_p^n \leq \alpha P_{total}, \tag{25}$$

and

$$\mathbf{P5}: \max_{P_s^n \geq 0} \sum_{n\in\mathbb{N}_s(\alpha)} \ln\left(1+\frac{P_s^n g_{ss}^n}{\sigma^2}\right) \tag{26}$$

$$\text{s.t.} \sum_{n\in\mathbb{N}_s(\alpha)} P_s^n \leq (1-\alpha) P_{total}. \tag{27}$$

It is observed that P4 has the similar structure as P2. Hence, by applying the convex optimization in a similar way as in Section 3.2, the optimal P_p^n for P4 is given by

$$P_p^n = \left(\frac{1}{\nu(\alpha)} - \frac{\sigma^2}{g_{pp}^n}\right)^+. \tag{28}$$

where $\nu(\alpha)$ is the Lagrange multiplier associated with the constraint (25). Inserting (28) into (25) set at equality we have

$$\sum_{n=1}^{N} \left(\frac{1}{\nu(\alpha)} - \frac{\sigma^2}{g_{pp}^n}\right)^+ = \alpha P_{total}. \tag{29}$$

The value of $\nu(\alpha)$ is obtained from (29) by the bisection search. Then, the sets of channels $\mathbb{N}_p(\alpha)$ and $\mathbb{N}_s(\alpha)$ are obtained respectively as

$$\mathbb{N}_p(\alpha) = \{\{n\} : \nu(\alpha) < \frac{g_{pp}^n}{\sigma^2}\}, \tag{30}$$

and

$$\mathbb{N}_s(\alpha) = \{\{n\} : \nu(\alpha) \geq \frac{g_{pp}^n}{\sigma^2}\}. \tag{31}$$

For P5, it is seen that it has the same structure as P2 and can be thus solved as in Section 3.2.

At the higher level, by substituting (18) and (28) into the objective function in (20), we have the master problem in charge of updating variable α by solving

$$\mathbf{P7}: \max_{\alpha} f(\alpha) \tag{32}$$

$$\text{s.t. } \alpha_{min} \leq \alpha \leq 1, \tag{33}$$

where

$$f(\alpha) = \omega_1 \sum_{n\in\mathbb{N}_p(\alpha)} \ln\left(\frac{g_{pp}^n}{\nu(\alpha)\sigma^2}\right) + \omega_2 \sum_{n\in\mathbb{N}_s(\alpha)} \ln\left(1+\left(\frac{g_{ss}^n}{\mu(\alpha)\sigma^2}-1\right)^+\right). \tag{34}$$

Considering the fact that α lies within the interval $[\alpha_{min}, 1]$, it can be easily obtained by one-dimension exhaustive search.

The overall proposed power allocation scheme to maximize the sum rate of the SU and the PU, under the PU minimum rate constraint and the total transmit power constraint, is summarized as follows:

Algorithm 2: Power allocation to maximize the rate of the SU and the PU with opportunistic subchannel access

Determine optimal fraction α^* *and available subchannels:*

1: **for** $\alpha = \alpha_{min}$ to **1** **do**

2: Calculate $\mathbb{N}_p(\alpha)$ and $\mathbb{N}_s(\alpha)$ by

$\mathbb{N}_p(\alpha) = \{\{n\} : \nu(\alpha) < \frac{g_{pp}^n}{\sigma^2}\}$,

$\mathbb{N}_s(\alpha) = \{\{n\} : \nu(\alpha) \geq \frac{g_{pp}^n}{\sigma^2}\}$,

where $\nu(\alpha)$ is obtained from (29) by the bisection search.

3: Calculate $f(\alpha)$ by

$$f(\alpha) = \omega_1 \sum_{n\in\mathbb{N}_p(\alpha)} \ln\left(\frac{g_{pp}^n}{\nu(\alpha)\sigma^2}\right) + \omega_2 \sum_{n\in\mathbb{N}_s(\alpha)} \ln\left(1 + \left(\frac{g_{ss}^n}{\mu(\alpha)\sigma^2} - 1\right)^+\right).$$

4: **end for**

5: Obtain the optimal α^*, $\mathbb{N}_p$, $\mathbb{N}_s$ as:

$\alpha^* = \arg\max f(\alpha)$,

$\mathbb{N}_p(\alpha^*) = \{\{n\} : \nu(\alpha) < \frac{g_{pp}^n}{\sigma^2}\}$,

$\mathbb{N}_s(\alpha^*) = \{\{n\} : \nu(\alpha) \geq \frac{g_{pp}^n}{\sigma^2}\}$.

The PU performs power allocation as follows:

1: Calculate P_p^n, $\forall n \in \{1,\ldots,N\}$ by $P_p^n = \left(\frac{1}{\nu(\alpha^*)} - \frac{\sigma^2}{g_{pp}^n}\right)^+$.

The SU performs power allocation as follows:

1: Calculate P_s^n, $\forall n \in \mathbb{N}_s$ by $P_s^n = \left(\frac{1}{\mu(\alpha^*)} - \frac{\sigma^2}{g_{ss}^n}\right)^+$, where $\mu(\alpha^*)$ is obtained from (19) by the bisection search.

Remark 2: The computational complexity of Algorithm 2 is analyzed in what follows. It has been shown in Section 3 that approximately $O\left(N\log_2\left(\varepsilon^{-1}\right)\right)$ operations are required to obtain α_{min}. As for obtaining α^*, the one-dimension exhaustive search guarantees error tolerance of ϵ for α after $(1-\alpha_{min})\epsilon^{-1}$ calculations. Thus, totally $O\left(N\epsilon^{-1}\log_2\left(\varepsilon^{-1}\right)\right)$ operations are required to calculate α^*. Besides, according to [20], the complexity of the bisection search to obtain P_p^n and P_s^n is $O\left(N\log_2\left(\varepsilon^{-1}\right)\right)$. Therefore, the total computational complexity of Algorithm 2 is approximately $O\left(N\epsilon^{-1}\log_2\left(\varepsilon^{-1}\right)\right)$.

5. Simulation Results

In this section, we present the simulation results to evaluate the effectiveness of the proposed schemes in Algorithm 1 (referred to as scheme 1) and Algorithm 2 (referred to as scheme 2). A conservative scheme in which the PU uses all the available transmit power P_{total} to maximize its rate is used as benchmark to measure the performance of the proposed schemes. Without loss of generality, the noise power σ^2 is assumed to be 1, weight factors ω_1 and ω_2 are assumed to be 0.5 respectively, the number of subchannels N is assumed to be 32, and all the channels involved are assumed to be Rayleigh fading. Accordingly, the channel power gains for these channels, i.e., g_{ss}^n and g_{pp}^n are exponentially distributed. The average channel power gains for the SU link and the PU links are all assumed to be 1, i.e., $E(g_{ss}^n) = 1$, $E(g_{pp}^n) = 1$. For simplicity, the correlation coefficients for all N subchannels are assumed to be identical and denoted as ρ, i.e., $\rho_n = \rho$. Moreover, corresponding simulation results are obtained by 1000 simulation runs. Two performance metrics are used in this section to verify the effectiveness of the proposed power allocation schemes, namely, average rate and outage probability. The average rate is a good performance metric suitable for the SU that carries delay-tolerant services and is obtained by averaging rate over multiple runs. The outage probability is a good performance metric suitable for the SU that carries delay-sensitive services which has been used widely as the performance metric in the literature such as [15] and is defined as the probability that the rate is lower than a predefined threshold. Considering the fact that the minimum rate of the PU is guaranteed, thus only the outage probability of the SU is given in the following results, and the predefined threshold is assumed to be 3.5 nats/s/Hz.

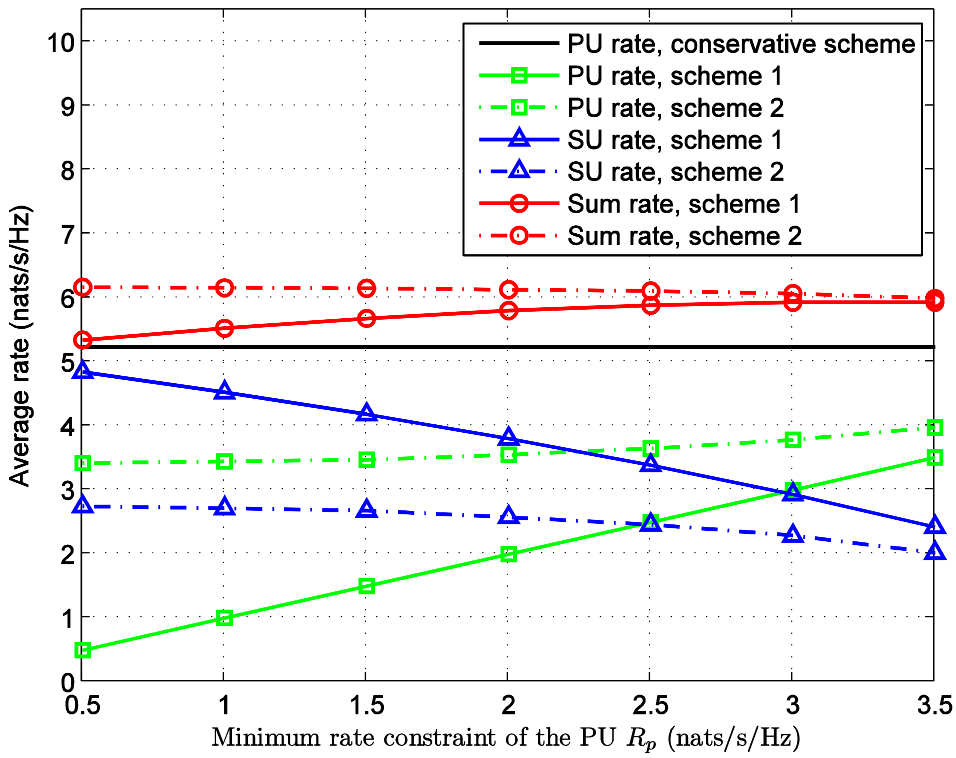

Fig. 1. Average rate vs. R_p ($P_{total} = 5$ dB and $\rho = 0.3$).

Fig. 1 shows the performance of the proposed power allocation schemes in terms of average rate against the required minimum rate of the PU R_p. As expected the rate of the PU achieved by scheme 1 is exactly the same as R_p, while the rate of the PU achieved by scheme 2 is higher than R_p and increases slowly with the increase of R_p. It can be seen that, as R_p increases, the rate of the SU achieved by scheme 1 or scheme 2 decreases, while the rate of the PU achieved by scheme 1 or scheme 2 increases. This is due to the fact that higher $\bar{R}_p$ results in less available subchannels for the SU and larger α, which causes the rate of the SU to decrease and the rate of the PU to increase. In addition, it can be seen that the rate of the SU achieved by scheme 1 is higher than that achieved by scheme 2, while the rate of the PU achieved by scheme 1 is lower than that achieved by scheme 2. This indicates that the SU prefers scheme 1 while the PU prefers scheme 2 especially when $\bar{R}_p$ is low. Furthermore, it is observed that the sum rate achieved by scheme 1 or scheme 2 is higher than the rate achieved by the conservative scheme. In addition, in high $\bar{R}_p$ regime, it is seen that the performance difference between scheme 1 and scheme 2 is small. Considering the fact that the complexity of scheme 2 is much higher that scheme 1 (scheme 2 requires an exhaustive search), scheme 1 is more favorable than scheme 2 in high $\bar{R}_p$ regime.\

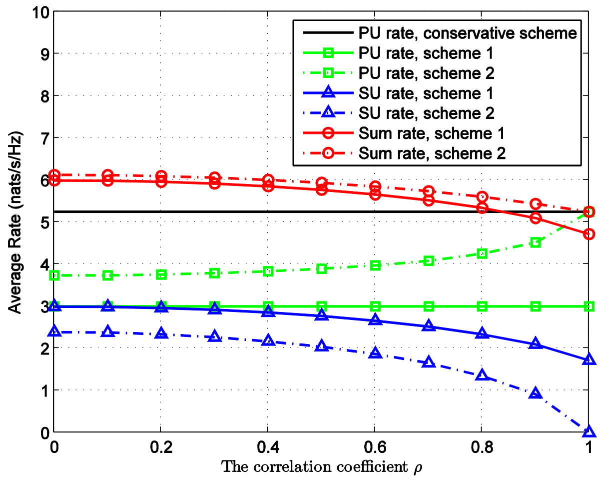

Fig. 2. Average rate vs. ρ ($\bar{R}_p = 3$ nats/s/Hz and $P_{total} = 5$ dB).

Fig. 2 plots the performance of the proposed power allocation schemes in terms of average rate against the correlation coefficient ρ. It is seen that the rate of the SU and the sum rate achieved by scheme 1 decrease as ρ increases. This is as expected since the subchannels discarded by the PU experience inferior channel fadings at the PU compared to other subchannels, and high value of ρ will cause these subchannels experience similar inferior channel fadings at the SU. Besides, it is shown that the rate of the SU achieved by scheme 1 decreases much slower in low ρ regime compared to that in high ρ regime. This indicates that the rate of the SU achieved by scheme 1 is insensitive to ρ in low ρ regime. For scheme 2, it is also observed that scheme 2 is insensitive to ρ in low ρ regime, and, as ρ increases, the rate of the SU decreases and the rate of the PU increases especially when ρ is high. Furthermore, it is observed that, as ρ increases from 0 to 1, the rate of the SU achieved by scheme 2 decreases to zero, while the rate of the PU achieved by scheme 2 increases to the rate achieved by the conservative scheme. This indicates that scheme 2 allocates more transmit power to the PU as ρ increases, and when ρ is equal to 1, there will be no transmit power allocated to the SU. Thus, for high ρ regime, scheme 2 becomes invalid for the SU and scheme 1 is preferred from the SU's perspective. In addition, it is seen that the performance gap between scheme 1 and scheme 2 increases with the increases of ρ especially when ρ is high. This indicates that the SU prefers scheme 1 while the PU prefers scheme 2 especially when ρ is high.

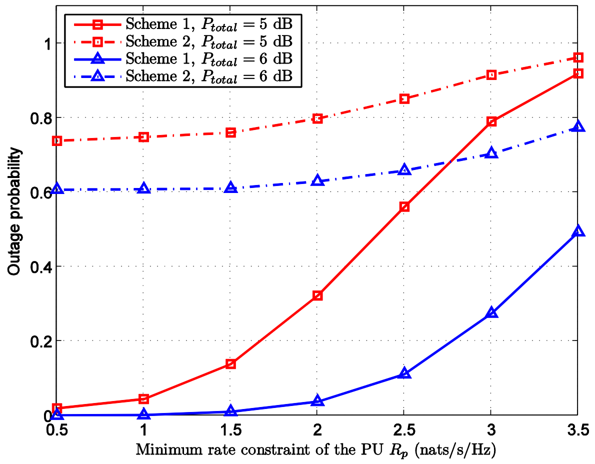

Fig. 3. Outage probability of the SU vs. R_p ($\rho = 0.3$).

Fig. 3 shows the performance of the proposed power allocation schemes in terms of outage probability of the SU against the required minimum rate of the PU R_p. As expected the outage probability increases with the increase of R_p. It is seen that the outage probability achieved by scheme 1 is much lower than that achieved by scheme 2 especially for low R_p. It is also observed that the performance gap between the two schemes decreases with the increases of R_p. This indicates that scheme 1 is more favorable for the SU than scheme 2 to achieve lower outage probability especially for low R_p. Furthermore, it is seen that the outage probability achieved by scheme 2 is more steady compared to that achieved by scheme 1 as R_p increases. This indicates that scheme 2 is more robust to R_p in terms of outage probability of the SU compared to scheme 1.

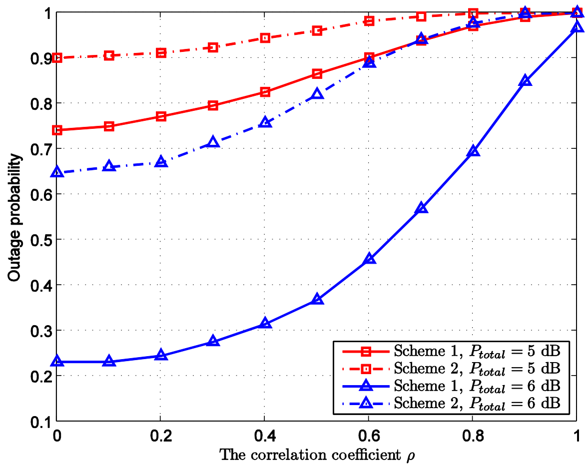

Fig. 4. Outage probability of the SU vs. ρ ($R_p = 3$ nats/s/Hz).

Finally, in **Fig. 4**, we plot the performance of the proposed power allocation schemes in terms of outage probability of the SU against the correlation coefficient ρ. It is seen that the outage probability increases as ρ increases. It is also seen that scheme 1 achieves much lower outage probability than scheme 2 especially for low ρ. Besides, it is observed that the performance gap in terms of outage probability of the SU between the two schemes decreases with the increase of ρ. This indicates that scheme 1 is more favorable for the SU than scheme 2 to achieve lower outage probability especially for low ρ.

6. Conclusion

This paper considers a downlink CR network with opportunistic subchannel access, where one SU shares the same BS with the PU. The QoS of the PU, whose communication is of high priority, is guaranteed by the minimum rate constraint. The SU, whose communication is of low priority, opportunistically accesses the subchannels that were previously discarded by the PU during power allocation. The total transmit power of the BS is shared between the PU and the SU. Two power allocation schemes are proposed, the objective of which are to maximize the rate of the SU and the sum rate of the SU and the PU respectively, both under the PU minimum rate constraint and the total transmit power constraint. Simulation results are obtained to verify the effectiveness of the proposed schemes. It is shown that the proposed schemes not only can guarantee the QoS of the PU, but also can provide certain communication opportunities for the SU.

It is noted that one SU and one PU is considered in the paper for brevity of expositions. For the case of multiple SUs and multiple PUs, not only power allocation but also channel allocation shall be considered. Therefore, our future work will emphasize on extending the current work to a more complicated scenario where multiple SUs and multiple PUs coexist and investigating the problem of joint power and channel allocation.

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