Journal of the Korean Data & Information Science Society
2012, **23**(4), 843–850

http://dx.doi.org/10.7465/jkdi.2012.23.4.843
한국데이터정보과학회지

Optimal three step stress accelerated life tests under periodic inspection and type I censoring

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Received 22 June 2012, revised 16 July 2012, accepted 21 July 2012

Abstract

The inferences of data obtained from periodic inspection and type I censoring for the three step stress accelerated life test are studied in this paper. The failure rate function that a log-quadratic relation of stress and the tampered failure rate model are considered under the exponential distribution. The optimal stress change times which minimize the asymptotic variance of maximum likelihood estimators of parameters is determined and the maximum likelihood estimators of the model parameters are estimated. A numerical example will be given to illustrate the proposed inferential procedures.

Keywords: Asymptotic variance, exponential distribution, Fisher information, optimum plan, periodic inspection, tampered failure rate model, three step stress accelerated life test.

1. Introduction

The life testing time under environment conditions may be very long and it is difficult for extremely reliable units to make life testing at use stress. The accelerated life testings (ALTs) are used to overcome this problem. ALTs are done on greater stresses than use stress and then ALTs quickly yield informations on test units. Three types of models have been mostly used on step stress ALTs. They are the tampered random variable (TRV) model by DeGroot and Goel (1979), the cumulative exposure (CE) model by Nelson (1980) and the tampered failure rate (TFR) model by Bhattacharyya and Soejoeti (1989).

The lifetimes of test units can be examined continuously or intermittently in the step stress ALTs. The periodic inspection of life testing time is often used due to further reduction in time and cost, on the other hand earlier studies assumed continuous inspection. The data from periodic inspection consists of only the number of failures in the inspection intervals. Yum and Choi (1989) first studied asymptotic optimal ALTs plans for periodic inspection and type I censoring. Bai *et el.* (1989) extended the results of Miller and Nelson (1983) that considered optimal plans for simple step stress accelerated life test to the case of Type I censoring under periodic observation. Seo and Yum (1993) proposed several approximate maximum likelihood estimators (MLEs) for the mean of exponential distribution and compared them by a Monte Carlo simulation. Islam and Ahmad (1994) studied

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the optimal ALTs plans for Weibull distribution under the constant stress ALTs with the periodic inspection and type I censoring. Xiong and Ji (2004) studied the statistical inference of model parameters and optimum test plans using only grouped and Type I censored data from a step stress ALTs. Ahmad *et el.* (2006) generalized the previous works on the design for periodic inspection and Type I censoring under the constant stress ALTs. Moon and Kim (2006) studied parameter estimation of the two-parameter exponential distribution under three step-stress accelerated life test. Moon (2008) considered the estimation of model parameters and optimum plans based on grouped and Type I censored data from three step stress ALTs for exponential distribution under the TFR model. Moon and Park (2009) studied the optimum plan and the estimation of model parameters on periodic inspection with Type I censoring under two step stress ALTs for exponential distribution under the TFR model.

In this paper, the results of Moon and Park (2009) are extended to the case of three step stress ALTs based on periodic inspection with type I censoring. The MLEs of model parameters and optimum plan searching the optimal stress changing times are studied, assuming that the lifetime of test units follows an exponential distribution under the TFR model. In Section 2, the model and some necessary assumptions are described. In Section 3, MLEs of the parameters and the optimum plan that minimizes the asymptotic variance of the MLE of logarithm of the mean lifetime at use stress are obtained. A numerical example is presented for the proposed inferential procedures in Section 4.

2. Model and assumptions

Suppose that there are four level stresses $y_0 < y_1 < y_2 < y_3$, where y_0 is the use stress. In the presentation of our results and without loss of generality, the following notation is used.

$$x_i = \frac{y_i - y_0}{y_3 - y_0},\ i = 0, 1, 2, 3.$$

All test units n are simultaneously put on stress x_1 and inspections are conducted at preset times $t_{11}, t_{12}, \cdots, t_{1K(1)}$, but if all units do not fail before time $t_{1K(1)}$ $(= \tau_1)$, the surviving units are subjected to the stronger stress x_2 and observed at pre-set times $t_{21}, t_{22}, \cdots, t_{2K(2)}$, but if all units on stress x_2 do not fail before time $t_{2K(2)}$ $(= \tau_2)$, the surviving units are subjected to the stronger stress x_3 and observed at pre-set times $t_{31}, t_{32}, \cdots, t_{3K(3)}$ and surviving units at time $t_{3K(3)}$ $(= \tau_c)$ are censored, where $K(i)$ is the number of inspections at stress x_i, $i = 1, 2, 3$. At stress x_i, the number of failures n_{ij} are recorded corresponding p_{ij}, probability of failures in the interval $(t_{ij-1}, t_{ij}]$, $i = 1, 2, 3,\ j = 1, 2 \cdots, K(i)$.

Some useful notations are given as follows.

(1) n_{ij} is the number of failed units during the inspection time interval $(t_{i,j-1}, t_{ij}]$ at stress x_i, $i = 1, 2, 3$, $j = 1, 2, \cdots, K(i)$,

(2) n_c is the censored units at a censoring time τ_c,

(3) $n_i = \sum_{j=1}^{K(i)} n_{ij}, i = 1, 2, 3$ and $n_c = n - (n_1 + n_2 + n_3)$,

(4) $p_{ij} = P(t_{ij-1} < T \le t_{ij}), i = 1, 2, 3, j = 1, 2, \cdots, K(i)$ and $p_c = P(\tau_c < T < \infty)$, where $t_{10} = \tau_0 = 0$, $t_{1K(1)} = \tau_1 = t_{20}$, $t_{2K(2)} = \tau_2 = t_{30}$, $t_{3K(3)} = \tau_c$.

Suppose that stress response relationship of each test unit has the log-quadratic function with the stress variable x_i, which is given by

$$\log\theta_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2,\ i = 1, 2, 3, \tag{2.1}$$

where β_0, β_1 and β_2 are unknown model parameters.

The numbers of failed units $n_{ij},\ i = 1, 2, 3,\ j = 1, 2\cdots, K(i)$ are used to estimate the model parameters β_0, β_1 and β_2, and then the model is extrapolated to make statistical inferences under the use stress.

The probability distribution function $f(t)$ for a test unit lifetime T at stress $x_i, i = 1, 2, 3$ is given by

$$f(t) = \begin{cases} \dfrac{1}{\theta_1}\exp\left(-\dfrac{t}{\theta_1}\right), & t \le \tau_1, \\ \dfrac{1}{\theta_2}\exp\left(-\dfrac{t-\tau_1}{\theta_2} - \dfrac{\tau_1}{\theta_1}\right), & \tau_1 < t \le \tau_2, \\ \dfrac{1}{\theta_3}\exp\left(-\dfrac{t-\tau_2}{\theta_3} - \dfrac{\tau_2-\tau_1}{\theta_2} - \dfrac{\tau_1}{\theta_1}\right), & \tau_2 < t. \end{cases} \tag{2.2}$$

3. Maximum likelihood estimators and optimum plan

In this section, MLEs of the model parameters β_0, β_1 and β_2 are obtained by Newton-Rapshon method and the optimum plan for searching the optimal stress change times τ_1 and τ_2, which minimize the asymptotic variance of the MLE of logarithm of mean lifetime at the use stress x_0.The following notations to simplify equations are used.

$$\begin{aligned} u_{ij-1}^{(m)}(\beta_0,\beta_1,\beta_2) = u_{ij-1}^{(m)} &= (t_{ij-1} - \tau_{i-1})\, x_i^m \exp(-\beta_0 - \beta_1 x_i - \beta_2 x_i^2) \\ &\quad + (\tau_{i-1} - \tau_{i-2}) x_{i-1}^m \exp(-\beta_0 - \beta_1 x_{i-1} - \beta_2 x_{i-1}^2) \\ &\quad + \tau_{i-2} x_{i-2}^m \exp\left(-\beta_0 - \beta_1 x_{i-2} - \beta_2 x_{i-2}^2\right), \end{aligned}$$

$$\begin{aligned} u_{ij}^{(m)}(\beta_0,\beta_1,\beta_2) = u_{ij}^{(m)} &= (t_{ij} - \tau_{i-1})\, x_i^m \exp(-\beta_0 - \beta_1 x_i - \beta_2 x_i^2) \\ &\quad + (\tau_{i-1} - \tau_{i-2}) x_{i-1}^m \exp(-\beta_0 - \beta_1 x_{i-1} - \beta_2 x_{i-1}^2) \\ &\quad + \tau_{i-2} x_{i-2}^m \exp\left(-\beta_0 - \beta_1 x_{i-2} - \beta_2 x_{i-2}^2\right), \end{aligned}$$

$$\begin{aligned} u_{3K(3)}^{(m)}(\beta_0,\beta_1,\beta_2) = u_{3K(3)}^{(m)} &= (\tau_c - \tau_2) x_3^m \exp(-\beta_0 - \beta_1 x_3 - \beta_2^2 x_3^2) \\ &\quad + (\tau_2 - \tau_1) x_2^m \exp(-\beta_0 - \beta_1 x_2 - \beta_2^2 x_2^2) \\ &\quad + \tau_1 x_1^m \exp(-\beta_0 - \beta_1 x_1 - \beta_2^2 x_1^2) \end{aligned}$$

for $i = 1, 2, 3,\ j = 1, 2, \cdots, K(i)$ and $m = 0, 1, \cdots, 4$ where $x_0 = 0$ and $\tau_0 = 0$.

The likelihood function is given by

$$L \propto \prod_{i=1}^{3} \prod_{j=1}^{K(i)} p_{ij}^{n_{ij}} \cdot p_c^{n_c}$$

for $i=1,2,3,\ j=1,2,\cdots,K(i)$, where

$$p_{ij}=P\left(t_{ij-1}<T\le t_{ij}\right)=\exp\left(-u_{ij-1}^{(0)}\right)-\exp\left(-u_{ij}^{(0)}\right),\ p_c=P\left(T>\tau_c\right)=\exp\left(-u_{3K(3)}^{(0)}\right).$$

Thus, the log likelihood function which is a function of unknown parameters β_0,β_1 and β_2 is given by as follows.

$$\log L(\beta_0,\beta_1,\beta_2)\propto\sum_{i=1}^{3}\sum_{j=1}^{K(i)}n_{ij}\log p_{ij}+n_c\log p_c.$$

The MLEs for the model parameters β_0,β_1 and β_2 can be obtained by solving the following equation in (3.1).

$$\frac{\partial}{\partial\beta_k}\log L(\beta_0,\beta_1,\beta_2)=\sum_{i=1}^{3}\sum_{j=1}^{K(i)}n_{ij}\cdot\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial\beta_k}\right)+n_c\cdot\frac{1}{p_c}\left(\frac{\partial p_c}{\partial\beta_k}\right)=0 \tag{3.1}$$

for $k=0,1,2$ where

$$\frac{\partial p_{ij}}{\partial\beta_k}=u_{ij-1}^{(k)}\exp(-u_{ij-1}^{(0)})-u_{ij}^{(k)}\exp(-u_{ij}^{(0)}),\ i=1,2,3,\ \frac{\partial p_c}{\partial\beta_k}=u_{3K(3)}^{(k)}\exp\left(-u_{3K(3)}^{(0)}\right).$$

The Fisher information matrix F is defined as $F=(f_{kl}),\ k,l=0,1,2$ and can be obtained by taking the expected value of the second partial and mixed partial derivatives of $\log L(\beta_0,\beta_1,\beta_2)$ with respect to β_0,β_1 and β_2 as follows.

$$\frac{\partial^2\log L(\beta_0,\beta_1,\beta_2)}{\partial\beta_k^2}=\sum_{i=1}^{3}\sum_{j=1}^{K(i)}\frac{n_{ij}}{p_{ij}}\left[\frac{\partial^2 p_{ij}}{\partial\beta_k^2}-\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial\beta_k}\right)^2\right]+\frac{n_c}{p_c}\left[\frac{\partial^2 p_c}{\partial\beta_k^2}-\frac{1}{p_c}\left(\frac{\partial p_c}{\partial\beta_k}\right)^2\right],$$

$$\begin{aligned}\frac{\partial^2\log L(\beta_0,\beta_1,\beta_2)}{\partial\beta_k\partial\beta_l}=&\sum_{i=1}^{3}\sum_{j=1}^{K(i)}\frac{n_{ij}}{p_{ij}}\left[\frac{\partial^2 p_{ij}}{\partial\beta_k\partial\beta_l}-\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial\beta_k}\right)\left(\frac{\partial p_{ij}}{\partial\beta_l}\right)\right]\\&+\frac{n_c}{p_c}\left[\frac{\partial^2 p_c}{\partial\beta_k\partial\beta_l}-\frac{1}{p_c}\left(\frac{\partial p_c}{\partial\beta_k}\right)\left(\frac{\partial p_c}{\partial\beta_l}\right)\right],\end{aligned}$$

for $k\ne l=0,1,2$, where for $i=1,2,3$,

$$\begin{aligned}\frac{\partial^2 p_{ij}}{\partial\beta_k^2}&=\left((u_{ij-1}^{(k)})^2-u_{ij-1}^{(2k)}\right)\exp(-u_{ij-1}^{(0)})-\left((u_{ij}^{(k)})^2-u_{ij}^{(2k)}\right)\exp(-u_{ij}^{(0)}),\\\frac{\partial^2 p_{ij}}{\partial\beta_k\partial\beta_l}&=\left(u_{ij-1}^{(k)}u_{ij-1}^{(l)}-u_{ij-1}^{(k+l)}\right)\exp(-u_{ij-1}^{(0)})-\left(u_{ij}^{(k)}u_{ij}^{(l)}-u_{ij}^{(k+l)}\right)\exp(-u_{ij}^{(0)}),\\\frac{\partial^2 p_c}{\partial\beta_k^2}&=\left((u_{3K(3)}^{(k)})^2-u_{3K(3)}^{(2k)}\right)\exp\left(-u_{3K(3)}^{(0)}\right),\\\frac{\partial^2 p_c}{\partial\beta_k\partial\beta_l}&=\left(u_{3K(3)}^{(k)}u_{3K(3)}^{(l)}-u_{3K(3)}^{(k+l)}\right)\exp\left(-u_{3K(3)}^{(0)}\right).\end{aligned}$$

The expected value of the second partial and mixed partial derivatives of $\log L(\beta_0,\beta_1,\beta_2)$ with respect to β_0,β_1 and β_2 are given by

$$\begin{aligned} f_{kk} = -E\left(\frac{\partial^2 \log L}{\partial \beta_k^2}\right) &= n\left\{\sum_{i=1}^{3}\sum_{j=1}^{K(i)}\left[\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial \beta_k}\right)^2 - \left(\frac{\partial^2 p_{ij}}{\partial \beta_k^2}\right)\right] + \frac{1}{p_c}\left(\frac{\partial p_c}{\partial \beta_k}\right)^2 - \frac{\partial^2 p_c}{\partial \beta_k^2}\right\} \\ &= n\left\{\sum_{i=1}^{3} Q_{ikk} + Q_{ckk}\right\}, \\ f_{kl} = -E\left(\frac{\partial^2 \log L}{\partial \beta_k \partial \beta_l}\right) &= n\sum_{i=1}^{3}\sum_{j=1}^{K(i)}\left[\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial \beta_k}\right)\left(\frac{\partial p_{ij}}{\partial \beta_l}\right) - \left(\frac{\partial^2 p_{ij}}{\partial \beta_k \partial \beta_l}\right)\right] \\ &\quad + n\left[\frac{1}{p_c}\left(\frac{\partial p_c}{\partial \beta_k}\right)\left(\frac{\partial p_c}{\partial \beta_l}\right) - \left(\frac{\partial^2 p_c}{\partial \beta_k \partial \beta_l}\right)\right] \\ &= n\left\{\sum_{i=1}^{3} Q_{ikl} + Q_{ckl}\right\} \end{aligned}$$

where

$$Q_{ikl} = \sum_{j=1}^{K(i)}\left[\frac{1}{p_{ij}}\left(\frac{\partial p_{ij}}{\partial \beta_k}\right)\left(\frac{\partial p_{ij}}{\partial \beta_l}\right) - \frac{\partial^2 p_{ij}}{\partial \beta_k \partial \beta_l}\right],\ Q_{ckl} = \frac{1}{p_c}\left(\frac{\partial p_c}{\partial \beta_k}\right)\left(\frac{\partial p_c}{\partial \beta_l}\right) - \frac{\partial^2 p_c}{\partial \beta_k \partial \beta_l}$$

for $k,\ l = 0,1,2$ and $i = 1,2,3$.

The optimum plan for determining optimal stress change times τ_1^* and τ_2^* under three step stress ALTs is presented, which minimize the asymptotic variance of $\log\widehat{\theta}_0$, MLE of logarithm of mean lifetime at the use stress x_0.

The asymptotic covariance matrix, V of MLEs, $\widehat{\beta}_0,\widehat{\beta}_1$ and $\widehat{\beta}_2$ for model parameters β_0,β_1 and β_2 is given by

$$V = F^{-1} = (f_{kl})^{-1}$$

for $k,l = 0,1,2$ and the asymptotic variance of $\log\widehat{\theta}_0$ is given by

$$\begin{aligned} Avar(\log\widehat{\theta}_0) &= (1,x_0,x_0^2)V(1,x_0,x_0^2)' \\ &= \frac{f_{11}f_{22} - f_{12}^2}{f_{00}\left(f_{11}f_{22} - f_{12}^2\right) + f_{01}\left(f_{02}f_{12} - f_{01}f_{22}\right) + f_{02}\left(f_{01}f_{12} - f_{11}f_{02}\right)}. \end{aligned} \tag{3.2}$$

Then the optimal change times τ_1^* and τ_2^* minimizing the asymptotic variance, $Avar(\log\widehat{\theta}_0)$, in (3.2) are unique solutions of the equations given by

$$f_1(\tau_1,\tau_2) = \frac{\partial Avar(\log\widehat{\theta}_0)}{\partial \tau_1} = \frac{1}{A}\left(f_{11}^{'}f_{22} + f_{11}f_{22}^{'} - 2f_{12}f_{12}^{'}\right) - \frac{A^{'}}{A^2}\left(f_{11}f_{22} - f_{12}^2\right),$$

$$f_2(\tau_1,\tau_2)=\frac{\partial Avar(\log\widehat{\theta}_0)}{\partial\tau_2}=\frac{1}{A}(f^o_{11}f_{22}+f_{11}f^o_{22}-2f_{12}f^o_{12})-\frac{A^o}{A^2}\left(f_{11}f_{22}-f^2_{12}\right)\tag{3.3}$$

where

$$A=|F|=f_{00}(f_{11}f_{22}-f^2_{12})+f_{01}(f_{02}f_{12}-f_{01}f_{22})+f_{02}(f_{01}f_{12}-f_{02}f_{11}),$$
$$A^{'}=\frac{\partial A}{\partial\tau_1},A^o=\frac{\partial A}{\partial\tau_2},$$
$$f^{'}_{kl}=\frac{\partial f_{kl}}{\partial\tau_1}=n\left[\sum_{i=1}^{3}\frac{\partial Q_{ikl}}{\partial\tau_1}+\frac{\partial Q_{ckl}}{\partial\tau_1}\right],\ f^o_{kl}=\frac{\partial f_{kl}}{\partial\tau_2}=n\left[\sum_{i=1}^{3}\frac{\partial Q_{ikl}}{\partial\tau_2}+\frac{\partial Q_{ckl}}{\partial\tau_2}\right]$$

with

$$\frac{\partial Q_{1kl}}{\partial\tau_1}=0,\ \frac{\partial Q_{ikl}}{\partial\tau_1}=\left(\frac{1}{\theta_2}-\frac{1}{\theta_1}\right)Q_{ikl}-\left(\frac{x_2^{k+l}}{\theta_2}-\frac{x_1^{k+l}}{\theta_1}\right)\sum_{j=1}^{K(i)}p_{ij},\ i=2,3,$$
$$\frac{\partial Q_{ckl}}{\partial\tau_1}=\left(\frac{1}{\theta_2}-\frac{1}{\theta_1}\right)Q_{ckl}-\left(\frac{x_2^{k+l}}{\theta_2}-\frac{x_1^{k+l}}{\theta_1}\right)p_c,$$
$$\frac{\partial Q_{1kl}}{\partial\tau_2}=\frac{\partial Q_{2kl}}{\partial\tau_2}=0,\ \frac{\partial Q_{3kl}}{\partial\tau_2}=\left(\frac{1}{\theta_3}-\frac{1}{\theta_2}\right)Q_{3kl}-\left(\frac{x_3^{k+l}}{\theta_3}-\frac{x_2^{k+l}}{\theta_2}\right)\sum_{j=1}^{K(i)}p_{3j},$$
$$\frac{\partial Q_{ckl}}{\partial\tau_2}=\left(\frac{1}{\theta_3}-\frac{1}{\theta_2}\right)Q_{ckl}-\left(\frac{x_3^{k+l}}{\theta_3}-\frac{x_2^{k+l}}{\theta_2}\right)p_c$$

for $k,\ l=0,1,2$.

4. Examples

The data from periodic inspections in three step stress ALTs consists of only the number of failures in each inspection interval $(t_{ij-1},t_{ij}]$, $i=1,2,3$, $j=1,2,\cdots,K(i)$ where $K(i)$ is the number of inspection in each stress level. In practice, to find the optimal stress change times τ_1^* and τ_2^*, the parameters must be approximated by experience, similar data or preliminary test.

It is assumed that the numbers of inspections on each stress are $K(1)=3,K(2)=2,K(3)=1$ and the probabilities of failure, p_{ij} in inspection intervals $(t_{ij-1},t_{ij}]$, $i=1,2,3$, $j=1,2,\cdots,K(i)$ are $p_{11}=0.2$, $p_{12}=0.15$, $p_{13}=0.1$ on stress x_1, $p_{21}=0.15$, $p_{22}=0.15$ on stress x_2, $p_{31}=0.15$ on stress x_3, $p_c=0.1$ and three stress levels are $x_1=0.3$, $x_2=0.6$, $x_3=1.0$, and model parameters are $\beta_0=1.0$, $\beta_1=-2.0$, $\beta_2=-5.0$, and the stress change times are $\tau_1=0.56868$, $\tau_2=0.67539$, $\tau_c=0.67766$.

The optimal stress change times τ_1^* and τ_2^* minimizing the asymptotic variance, $Avar(\log\widehat{\theta}_0)$ of MLE of logarithm of mean lifetime at the use stress x_0 in (3.2) were obtained as $\tau_1^*=0.44869$ and $\tau_2^*=0.67677$ by (3.3).

Now, the MLEs for model parameters β_0, β_1 and β_2 using the optimal stress change times τ_1^* and τ_2^* based on $\beta_0 = 1.0$, $\beta_1 = -2.0$ and $\beta_2 = -5.0$ are obtained for $n = 40$, $n = 30$ and $n = 25$ to examine the behavior of MLEs due to the sample size change.

All test units are simultaneously put on stress $x_1 = 0.3$ and inspections are conducted three times at pre-set times $t_{11} = 0.21226$, $t_{12} = 0.40977$ and $t_{13} = \tau_1^* = 0.44869$, but if all units do not fail before time τ_1^*, the surviving units are subjected to a stronger stress $x_2 = 0.6$ and also observed at specified times $t_{21} = 0.61178$, $t_{22} = \tau_2^* = 0.67677$, but if all units do not fail before time τ_2^*, the surviving units are subjected to a stronger stress $x_3 = 1.0$ and observed until censoring time.

For $n = 40$, the number of failed test units at each inspection interval $(t_{ij-1}, t_{ij}]$, $i = 1, 2, 3$, $j = 1, 2, \cdots, K(i)$ were $n_{11} = 4$, $n_{12} = 9$, $n_{13} = 3$ on stress x_1, $n_{21} = 17$, $n_{22} = 6$ on stress x_2, $n_{31} = 1$ on stress x_3 and the number of censoring units was $n_c = 0$, where $t_{10} = 0$, $t_{20} = \tau_1^*$ and $t_{30} = \tau_2^*$. By Newton-Raphson method, the MLEs of model parameters β_0, β_1 and β_2 were obtained as $\widehat{\beta}_0 = 1.14961$, $\widehat{\beta}_1 = -1.96114$ and $\widehat{\beta}_2 = -4.86005$.

For $n = 30$, $n_{11} = 5$, $n_{12} = 3$, $n_{13} = 4$ on stress x_1, $n_{21} = 11$, $n_{22} = 6$ on stress x_2, $n_{31} = 1$ on stress x_3, $n_c = 0$ and the MLEs of model parameters β_0, β_1 and β_2 were $\widehat{\beta}_0 = 1.12180$, $\widehat{\beta}_1 = -1.91964$ and $\widehat{\beta}_2 = -4.86936$.

For $n = 25$, $n_{11} = 3$, $n_{12} = 8$, $n_{13} = 0$ on stress x_1, $n_{21} = 9$, $n_{22} = 3$ on stress x_2, $n_{31} = 2$ on stress x_3, $n_c = 0$ and the MLEs of model parameters β_0, β_1 and β_2 were $\widehat{\beta}_0 = 1.25757$, $\widehat{\beta}_1 = -2.09730$ and $\widehat{\beta}_2 = -4.85060$.

As test units n changes, the behaviors of MLEs of model parameters are not likely to be remarkable but, the MLEs of β_1 and β_2 except β_0 are closer to the true values of model parameters as n increases.

The optimum plan is presented and maximum likelihood estimators of model parameters are obtained by periodic inspection and type I censored data from the step-stress accelerated life tests. This method will be very helpful in the situation that the intermittent inspection is the only practicable way of checking the status of test units under a step stress test. This results will be extended to the research associated with periodic inspection and type I censoring for multiple step stress accelerated life tests.

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