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FUZZY r-MINIMAL α-OPEN SETS ON FUZZY MINIMAL SPACES

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Abstract. We introduce the concept of fuzzy r-minimal α-open set on a fuzzy minimal space and some basic properties. We also introduce the concepts of fuzzy r-M α-continuous and fuzzy r-$M(M^*)$ α-open mappings, and investigate characterization for such mappings.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [5]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [3], Ramadan introduced the concept of smooth topological space, which is a generalization of fuzzy topological space. We introduced the concept of fuzzy r-minimal space [4] which is an extension of the smooth fuzzy topological space. The concepts of fuzzy r-open sets and fuzzy r-M continuous mappings are also introduced and studied. In this paper, we introduce the concept of fuzzy r-minimal α-open set on a fuzzy minimal space and some basic properties. We also introduce the concepts of fuzzy r-M α-continuous and fuzzy r-$M(M^*)$ α-open mappings, and investigate characterization for such mappings.

2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* [5] of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{1} - A$. All other notations are standard notations of fuzzy set theory.

An *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha \text{ if } y = x \\ 0 \text{ if } y \neq x. \end{cases}$$

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A *smooth topology* [3] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

(1) $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
(2) $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
(3) $\mathcal{T}(\cup A_i) \geq \wedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space.*

Let X be a nonempty set and $r \in (0, 1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* [4] if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{0}$ and $\tilde{1}$.

Then the pair $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* [4] (simply r-FMS). Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A (simply, A^c) is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure of A, denoted by $mC(A, r)$, is defined as

$$mC(A, r) = \cap\{B \in I^X : B^c \in \mathcal{M}_r \text{ and } A \subseteq B\}.$$

The fuzzy r-minimal interior of A, denoted by $mI(A, r)$, is defined as

$$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A\}.$$

Theorem 2.1 ([4])**.** *Let $(X, \mathcal{M})$ be an r-FMS and $A, B \in I^X$.*

(1) *$mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.*

(2) *$A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.*

(3) *If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.*

(4) *$mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.*

(5) *$mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.*

(6) *$\tilde{1} - mC(A, r) = mI(\tilde{1} - A, r)$ and $\tilde{1} - mI(A, r) = mC(\tilde{1} - A, r)$.*

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal semiopen set* [2] in X if

$$A \subseteq mC(mI(A, r), r).$$

A fuzzy set A is called a *fuzzy r-minimal semiclosed* set if the complement of A is fuzzy r-minimal semiopen.

Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then $f : X \to Y$ is said to be *fuzzy r-M continuous* function if for every $A \in \mathcal{N}_r$, $f^{-1}(A)$ is in $\mathcal{M}_r$.

3. Fuzzy r-minimal α-open sets

Definition 3.1. Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal α-open set* in X if

$$A \subseteq mI(mC(mI(A, r), r), r).$$

A fuzzy set A is called a *fuzzy r-minimal α-closed* set if the complement of A is fuzzy r-minimal α-open.

Remark 3.2. The following implications are obtained but the converses are not true in general.

fuzzy r-minimal open $\Rightarrow$ fuzzy r-minimal α-open $\Rightarrow$ fuzzy r-minimal semiopen.

Example 3.3. Let $X = I = [0, 1]$ and let A and B be fuzzy sets defined as follows

$$A(x) = \begin{cases} -x + \frac{1}{2}, & \text{if } 0 \leq x \leq \frac{1}{4}, \\ \frac{1}{3}(x-1) + \frac{1}{2}, & \text{if } \frac{1}{4} \leq x \leq 1; \end{cases}$$

$$B(x) = \frac{1}{4}, \quad \text{if } 0 \leq x \leq 1.$$

Let us consider a fuzzy minimal structure

$$\mathcal{M}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, A, \\ 0, & \text{otherwise.} \end{cases}$$

Then the fuzzy set B is a fuzzy $\frac{2}{3}$-minimal α-open set but not fuzzy $\frac{2}{3}$-minimal open. The $\tilde{1} - A$ is a fuzzy $\frac{2}{3}$-minimal semiopen set but not fuzzy $\frac{2}{3}$-minimal α-open.

Lemma 3.4. *Let $(X, \mathcal{M})$ be an r-FMS. Then a fuzzy set A is fuzzy r-minimal α-closed if and only if $mC(mI(mC(A, r), r), r) \subseteq A$.*

Theorem 3.5. *Let $(X, \mathcal{M})$ be an r-FMS. Then any union of fuzzy r-minimal α-open sets is fuzzy r-minimal α-open.*

Proof. Let A_i be a fuzzy r-minimal α-open set for $i \in J$. Then

$$A_i \subseteq mI(mC(mI(A_i, r), r), r) \subseteq mI(mC(mI(\cup A_i, r), r), r).$$

So $\cup A_i \subseteq mI(mC(mI(\cup A_i, r), r), r)$ and hence $\cup A_i$ is fuzzy r-minimal α-open. □

As shown in the next example, the intersection of two fuzzy r-minimal α-open sets may not be fuzzy r-minimal α-open.

Example 3.6. Let $X = I = [0, 1]$ and let A and B be fuzzy sets defined as follows

$$A(x) = -\frac{3}{4}(x-1), \quad \text{if } x \in I;$$

$$B(x) = \frac{1}{2}x, \text{ if } x \in I.$$

Let us consider a fuzzy minimal structure

$$\mathcal{N}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, A, B, \\ 0, & \text{otherwise.} \end{cases}$$

Then A and B are fuzzy $\frac{2}{3}$-minimal α-open sets but $A \cap B$ is not fuzzy $\frac{2}{3}$-minimal α-open.

Definition 3.7. Let $(X, \mathcal{M})$ be an r-FMS. For $A \in I^X$, $m\alpha C(A, r)$ and $m\alpha I(A, r)$, respectively, are defined as the following:

$$m\alpha C(A, r) = \cap\{F \in I^X : A \subseteq F,\ F \text{ is fuzzy } r\text{-minimal } \alpha\text{-closed}\};$$
$$m\alpha I(A, r) = \cup\{U \in I^X : U \subseteq A,\ U \text{ is fuzzy } r\text{-minimal } \alpha\text{-open }\}.$$

Theorem 3.8. *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then*

(1) $m\alpha I(A, r) \subseteq A$.
(2) *If $A \subseteq B$, then $m\alpha I(A, r) \subseteq m\alpha I(B, r)$.*
(3) *A is r-minimal α-open if and only if $m\alpha I(A, r) = A$.*
(4) $m\alpha I(m\alpha I(A, r), r) = m\alpha I(A, r)$.
(5) $m\alpha C(\tilde{1} - A, r) = \tilde{1} - m\alpha I(A, r)$ *and* $m\alpha I(\tilde{1} - A, r) = \tilde{1} - m\alpha C(A, r)$.

Proof. (1), (2), (3) and (4) are obvious.

(5) For $A \in I^X$,

$$\begin{aligned} \tilde{1} - m\alpha I(A, r) &= \tilde{1} - \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\} \\ &= \cap\{\tilde{1} - U : U \subseteq A, U \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\} \\ &= \cap\{\tilde{1} - U : \tilde{1} - A \subseteq \tilde{1} - U, U \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\} \\ &= m\alpha C(\tilde{1} - A, r). \end{aligned}$$

Similarly, it is proved that $m\alpha I(\tilde{1} - A, r) = \tilde{1} - m\alpha C(A, r)$. □

Theorem 3.9. *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then*

(1) $A \subseteq m\alpha C(A, r)$.
(2) *If $A \subseteq B$, then $m\alpha C(A, r) \subseteq m\alpha C(B, r)$.*
(3) *F is r-minimal α-closed if and only if $m\alpha C(F, r) = F$.*
(4) $m\alpha C(m\alpha C(A, r), r) = m\alpha C(A, r)$.

Proof. It is similar to the proof of Theorem 3.8. □

Lemma 3.10. *Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then $x_\alpha \in m\alpha C(A, r)$ if and only if $A \cap V \neq \tilde{0}$ for every r-minimal α-open set V containing x_α.*

Proof. If there is a fuzzy r-minimal β-open set V containing x_α such that $A \cap V = \tilde{0}$, then $\tilde{1} - V$ is a fuzzy r-minimal α-closed set such that $A \subseteq \tilde{1} - V$, $x_\alpha \notin \tilde{1} - V$. From this fact, $x_\alpha \notin m\alpha C(A, r)$.

The converse is easily proved. □

4. Fuzzy r-M α-continuity and fuzzy r-M α-open mappings

Definition 4.1. Let $(X,\mathcal{M})$ and $(Y,\mathcal{N})$ be r-FMS's. Then a mapping $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ is said to be *fuzzy r-M α-continuous* if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal α-open set U containing x_α such that $f(U) \subseteq V$.

Let $(X,\mathcal{M})$ and $(Y,\mathcal{N})$ be r-FMS's. Then a mapping $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ is said to be *fuzzy r-M semicontinuous* [2] if for each point x_α and each fuzzy r-minimal open set V containing $f(x_\alpha)$, there exists a fuzzy r-minimal semiopen set U containing x_α such that $f(U) \subseteq V$.

Remark 4.2. It is obvious that every fuzzy r-M α-continuous mapping is fuzzy r-M semicontinuous but the converse may not be true as shown in the next example.

fuzzy r-M continuous $\Rightarrow$ fuzzy r-M α-continuous $\Rightarrow$ fuzzy r-M semicontinuous.

Example 4.3. Let $X = I = [0,1]$ and let A, B and C be fuzzy sets defined as follows

$$A(x) = \begin{cases} x + \frac{1}{2}, & \text{if } 0 \le x \le \frac{1}{4}, \\ -\frac{1}{3}(x-1) + \frac{1}{2}, & \text{if } \frac{1}{4} \le x \le 1; \end{cases}$$

$$B(x) = \frac{1}{4}(x+3), \ \text{if } x \in I;$$

$$C(x) = -\frac{1}{4}(x-1), \ \text{if } x \in I.$$

Let us consider fuzzy minimal structures $\mathcal{L}, \mathcal{M}, \mathcal{N}$ as the following:

$$\mathcal{L}(\mu) = \begin{cases} \frac{2}{3}, & \text{if} \mu = \tilde{0}, \tilde{1}, A, \\ 0, & \text{otherwise}; \end{cases}$$

$$\mathcal{M}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, B, \\ 0, & \text{otherwise}; \end{cases}$$

$$\mathcal{N}(\mu) = \begin{cases} \frac{2}{3}, & \text{if } \mu = \tilde{0}, \tilde{1}, C, \\ 0, & \text{otherwise}. \end{cases}$$

Then:

(1) The identity function $f : (X,\mathcal{L}) \to (X,\mathcal{M})$ is fuzzy r-M α-continuous but not fuzzy r-M continuous.

(2) the identity function $g : (X,\mathcal{N}) \to (X,\mathcal{L})$ is fuzzy r-M semicontinuous but not fuzzy r-M α-continuous.

Theorem 4.4. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following statements are equivalent:*

(1) *f is fuzzy r-M α-continuous.*

(2) *$f^{-1}(V)$ is a fuzzy r-minimal α-open set for each fuzzy r-minimal open set V in Y.*

(3) *$f^{-1}(B)$ is a fuzzy r-minimal α-closed set for each fuzzy r-minimal closed set B in Y.*

(4) *$f(m\alpha C(A, r)) \subseteq mC(f(A), r)$ for $A \in I^X$.*

(5) *$m\alpha C(f^{-1}(B), r) \subseteq f^{-1}(mC(B, r))$ for $B \in I^Y$.*

(6) *$f^{-1}(mI(B, r)) \subseteq m\alpha I(f^{-1}(B), r)$ for $B \in I^Y$.*

Proof. (1) $\Rightarrow$ (2) Let V be any fuzzy r-minimal open set in Y and $x_\alpha \in f^{-1}(V)$. By hypothesis, there exists a fuzzy r-minimal α-open set U containing x_α such that $f(U) \subseteq V$. This implies that $\cup U = f^{-1}(V)$ and hence $f^{-1}(V)$ is fuzzy r-minimal α-open.

(2) $\Rightarrow$ (3) Obvious.

(3) $\Rightarrow$ (4) For $A \in I^X$,

$$\begin{aligned} & f^{-1}(mC(f(A), r)) \\ &= f^{-1}(\cap\{F \in I^Y : f(A) \subseteq F \text{ and } F \text{ is fuzzy } r\text{-minimal closed}\}) \\ &= \cap\{f^{-1}(F) \in I^X : A \subseteq f^{-1}(F) \text{ and } f^{-1}(F) \text{ is fuzzy } r\text{-minimal } \alpha\text{-closed}\} \\ &\supseteq \cap\{K \in I^X : A \subseteq K \text{ and } K \text{ is fuzzy } r\text{-minimal } \alpha\text{-closed}\} \\ &= m\alpha C(A, r). \end{aligned}$$

Hence $f(m\alpha C(A, r)) \subseteq mC(f(A), r)$.

(4) $\Rightarrow$ (5) For $B \in I^Y$,

$$f(m\alpha C(f^{-1}(B), r)) \subseteq mC(f(f^{-1}(B)), r) \subseteq mC(B, r).$$

So $m\alpha C(f^{-1}(B), r) \subseteq f^{-1}(mC(B, r))$.

(5) $\Rightarrow$ (6) For $B \subseteq Y$,

$$\begin{aligned} f^{-1}(mI(B, r)) &= f^{-1}(\tilde{1} - mC(\tilde{1} - B, r)) \\ &= \tilde{1} - f^{-1}(mC(\tilde{1} - B, r)) \\ &\subseteq \tilde{1} - m\alpha C(f^{-1}(\tilde{1} - B), r) \\ &= m\alpha I(f^{-1}(B), r). \end{aligned}$$

Therefore, $f^{-1}(mI(B, r)) \subseteq m\alpha I(f^{-1}(B), r)$.

(6) $\Rightarrow$ (1) Let V be any fuzzy r-minimal open set containing $f(x_\alpha)$ for a fuzzy point x_α. By hypothesis, $x_\alpha \in f^{-1}(V) = f^{-1}(mI(V, r)) \subseteq m\alpha I(f^{-1}(V), r)$. Since $x_\alpha \in m\alpha I(f^{-1}(V), r)$, there exists a fuzzy r-minimal α-open set U containing x_α such that $U \subseteq f^{-1}(V)$. This implies $f^{-1}(V)$ is fuzzy r-minimal α-open. Hence f is fuzzy r-M α-continuous. □

Definition 4.5. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then

(1) f is said to be *fuzzy r-M α-open* if for fuzzy r-minimal open set A in X, $f(A)$ is fuzzy r-minimal α-open in Y;

(2) f is said to be *fuzzy r-M α-closed* if for fuzzy r-minimal closed set A in X, $f(A)$ is fuzzy r-minimal α-closed in Y;

Theorem 4.6. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following are equivalent*:

(1) *f is fuzzy r-M α-open.*

(2) *$f(mI(A, r)) \subseteq m\alpha I(f(A), r)$ for $A \in I^X$.*

(3) *$mI(f^{-1}(B), r) \subseteq f^{-1}(m\alpha I(B, r))$ for $B \in I^Y$.*

Proof. (1) $\Rightarrow$ (2) For $A \in I^X$,

$$\begin{aligned} f(mI(A, r)) &= f(\cup\{B \in I^X : B \subseteq A, B \text{ is fuzzy } r\text{-minimal open}\}) \\ &= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\} \\ &\subseteq \cup\{U \in I^X : U \subseteq f(A), U \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\} \\ &= m\alpha I(f(A), r). \end{aligned}$$

Hence $f(mI(A, r)) \subseteq m\alpha I(f(A), r)$.

(2) $\Rightarrow$ (3) For $B \in I^Y$, from (3) it follows that

$$f(mI(f^{-1}(B), r)) \subseteq m\alpha I(f(f^{-1}(B)), r) \subseteq m\alpha I(B, r).$$

Hence we get (3).

(3) $\Rightarrow$ (2) Obvious.

(2) $\Rightarrow$ (1) Let A be a fuzzy r-minimal open set in X. Then $A = mI(A, r)$. By (2), $f(A) = m\alpha I(f(A), r)$ and it implies $f(A)$ is fuzzy r-minimal α-open. □

Theorem 4.7. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following are equivalent*:

(1) *f is fuzzy r-M α-closed.*

(2) *$mC(f(A), r) \subseteq (f(mC(A, r)))$ for $A \in I^X$.*

(3) *$f^{-1}(mC(B, r)) \subseteq mC(f^{-1}(B), r)$ for $B \in I^Y$.*

Proof. It is similar to the proof of Theorem 4.6. □

Definition 4.8. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then

(1) f is said to be *fuzzy r-M^* α-open* if for every fuzzy r-minimal α-open set A in X, $f(A)$ is fuzzy r-minimal open in Y;

(2) f is said to be *fuzzy r-M^* α-closed* if for every fuzzy r-minimal α-closed set A in X, $f(A)$ is fuzzy r-minimal closed in Y.

Theorem 4.9. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a mapping on r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$.*

(1) *f is fuzzy r-M^* α-open.*

(2) *$f(m\alpha I(A, r)) \subseteq mI(f(A), r)$ for $A \in I^X$.*

(3) *$m\alpha I(f^{-1}(B), r) \subseteq f^{-1}(mI(B, r))$ for $B \in I^Y$.*

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $A \in I^X$,

$$\begin{aligned} f(m\alpha I(A,r)) &= f(\cup\{B \in I^X : B \subseteq A, B \text{ is fuzzy } r\text{-minimal } \alpha\text{-open}\}) \\ &= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B) \text{ is fuzzy } r\text{-minimal open}\} \\ &\subseteq \cup\{U \in I^Y : U \subseteq f(A), U \text{ is fuzzy } r\text{-minimal open}\} \\ &= mI(f(A),r). \end{aligned}$$

Hence $f(m\alpha I(A,r)) \subseteq mI(f(A),r)$.

(2) $\Rightarrow$ (3) For $B \in I^Y$, from (3),

$$f(m\alpha I(f^{-1}(B),r)) \subseteq mI(f(f^{-1}(B)),r) \subseteq mI(B,r).$$

(3) $\Rightarrow$ (2) Obvious. □

Similarly, we have the following theorem:

Theorem 4.10. *Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$.*
(1) *f is fuzzy r-M^* α-closed.*
(2) *$mC(f(A),r) \subseteq (f(m\alpha C(A,r)))$ for $A \in I^X$.*
(3) *$f^{-1}(mC(B,r)) \subseteq m\alpha C(f^{-1}(B),r)$ for $B \in I^Y$.*
Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy r-minimal structure $\mathcal{M}_r$ is said to have the property $(\mathcal{U})$ [4] if for $A_i \in \mathcal{M}_r$ $(i \in J)$,

$$\mathcal{M}_r(\cup A_i) \geq \wedge \mathcal{M}_r(A_i).$$

Theorem 4.11 ([4])**.** *Let $(X,\mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then for $A \in I^X$, $mI(A,r) = A$ if and only if A is fuzzy r-minimal open.*

Obviously the following corollaries are obtained:

Corollary 4.12. *Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$. If $(Y,\mathcal{N})$ has the property $(\mathcal{U})$, then the following are equivalent:*
(1) *f is fuzzy r-M^* α-open.*
(2) *$f(m\alpha I(A,r)) \subseteq mI(f(A),r)$ for $A \in I^X$.*
(3) *$m\alpha I(f^{-1}(B),r) \subseteq f^{-1}(mI(B,r))$ for $B \in I^Y$.*

Corollary 4.13. *Let $f : (X,\mathcal{M}) \to (Y,\mathcal{N})$ be a mapping on r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$. If $(Y,\mathcal{N})$ has the property $(\mathcal{U})$, then the following are equivalent*:
(1) *f is fuzzy r-M^* α-closed.*
(2) *$mC(f(A),r) \subseteq (f(m\alpha C(A,r)))$ for $A \in I^X$.*
(3) *$f^{-1}(mC(B,r)) \subseteq m\alpha C(f^{-1}(B),r)$ for $B \in I^Y$.*

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