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LOCALLY SYMMETRIC HALF LIGHTLIKE SUBMANIFOLDS IN AN INDEFINITE KENMOTSU MANIFOLD

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Abstract. In this paper, we study locally symmetric half lightlike submanifolds M of an indefinite Kenmotsu manifold $\bar{M}$ subject to the conditions: (1) The transversal vector bundle $tr(TM)$ is parallel with respect to the connection $\bar{\nabla}$ of $\bar{M}$ and (2) M is irrotational.

1. Introduction

The class of lightlike submanifolds of codimension 2 is compose entirely of two classes by virtue of the rank of its radical distribution, named by half lightlike and coisotropic submanifolds [2]. Half lightlike submanifold is a special case of r-lightlike submanifold such that $r = 1$ and its geometry is more general form than that of coisotrophic submanifold. Much of the works on half lightlike submanifolds will be immediately generalized in a formal way to general r-lightlike submanifolds of arbitrary codimension and arbitrary rank.

In 1971, K. Kenmotsu proved the following result [6]: If a Kenmotsu manifold is locally symmetric, then it is of constant negative curvature -1.

The objective of this paper is study of the half lightlike version of above Kenmotsu's result. We prove the following results:

Let M be a half lightlike submanifold of an indefinite Kenmotsu manifold $\bar{M}$ equipped with an almost contact metric structure $(J, \zeta, \theta, \bar{g})$.

- The structure 1-form θ is closed, i.e., $d\theta = 0$, on TM.
- If M is a locally symmetric space and the transversal vector bundle $tr(TM)$ of M is parallel with respect to the connection $\bar{\nabla}$ of $\bar{M}$, then the Ricci type tensor $R^{(0,2)}$ is a symmetric Ricci tensor on M.
- If M is an irrotational and locally symmetric space and $tr(TM)$ is parallel with respect to the connection $\bar{\nabla}$ of $\bar{M}$, then M is a space of constant curvature 0 and the vector field ζ of $\bar{M}$ is normal to M.

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2. Half lightlike submanifolds

An odd dimensional semi-Riemannian manifold $\bar{M}$ is said to be an *indefinite Kenmotsu manifold* [4, 6, 8] if there exists a structure $(J,\, \zeta,\, \theta,\, \bar{g})$, where J is a $(1,1)$-type tensor field, ζ is a vector field which called the characteristic vector field, θ is a 1-form and $\bar{g}$ is the semi-Riemannian metric on $\bar{M}$ such that

$$J^2X = -X + \theta(X)\zeta, \quad J\zeta = 0, \quad \theta \circ J = 0, \quad \theta(\zeta) = 1, \tag{2.1}$$
$$\theta(X) = \bar{g}(\zeta, X), \quad \bar{g}(JX, JY) = \bar{g}(X,Y) - \theta(X)\theta(Y),$$

$$\bar{\nabla}_X\zeta = -X + \theta(X)\zeta, \tag{2.2}$$

$$(\bar{\nabla}_X J)Y = -\bar{g}(JX,Y)\zeta + \theta(Y)JX, \tag{2.3}$$

for any vector fields X, Y on $\bar{M}$, where $\bar{\nabla}$ is the Levi-Civita connection of $\bar{M}$.

A submanifold M of a semi-Riemannian manifold $\bar{M}$ of codimension 2 is called a *half lightlike submanifold* if the radical distribution $Rad(TM) = TM \cap TM^{\perp}$ of M is a vector subbundle of the tangent bundle TM and the normal bundle $TM^{\perp}$ of rank 1. Then there exist complementary non-degenerate distributions $S(TM)$ and $S(TM^{\perp})$ of $Rad(TM)$ in TM and $TM^{\perp}$ respectively, called the *screen* and *co-screen distribution* on M, such that

$$TM = Rad(TM) \oplus_{orth} S(TM), \; TM^{\perp} = Rad(TM) \oplus_{orth} S(TM^{\perp}), \tag{2.4}$$

where the symbol $\oplus_{orth}$ denotes the orthogonal direct sum. We denote such a half lightlike submanifold by $M = (M, g, S(TM))$. Denote by $F(M)$ the algebra of smooth functions on M and by $\Gamma(E)$ the $F(M)$ module of smooth sections of a vector bundle E over M. Let ξ be a section on $Rad(TM)$. Choose $L \in \Gamma(S(TM^{\perp}))$ as a unit vector field with $\bar{g}(L,L) = \pm 1$. In this paper we may assume that $\bar{g}(L,L) = 1$ without loss of generality. Consider the orthogonal complementary distribution $S(TM)^{\perp}$ to $S(TM)$ in $T\bar{M}$. Certainly ξ and L belong to $\Gamma(S(TM)^{\perp})$. Then we have the following decomposition

$$S(TM)^{\perp} = S(TM^{\perp}) \oplus_{orth} S(TM^{\perp})^{\perp},$$

where $S(TM^{\perp})^{\perp}$ is the orthogonal complementary to $S(TM^{\perp})$ in $S(TM)^{\perp}$. For any null section ξ of $Rad(TM)$ on a coordinate neighborhood $\mathcal{U} \subset M$, there exists a uniquely defined null vector field $N \in \Gamma(ltr(TM))$ satisfying

$$\bar{g}(\xi, N) = 1, \; \bar{g}(N,N) = \bar{g}(N,X) = \bar{g}(N,L) = 0, \; \forall\, X \in \Gamma(S(TM)). \tag{2.5}$$

We call N, $ltr(TM)$ and $tr(TM) = S(TM^{\perp}) \oplus_{orth} ltr(TM)$ the *lightlike transversal vector field, lightlike transversal vector bundle* and *transversal vector bundle* of M with respect to $S(TM)$ respectively. Thus $T\bar{M}$ is decomposed as follows:

$$\begin{aligned} T\bar{M} &= TM \oplus tr(TM) = \{Rad(TM) \oplus tr(TM)\} \oplus_{orth} S(TM) \\ &= \{Rad(TM) \oplus ltr(TM)\} \oplus_{orth} S(TM) \oplus_{orth} S(TM^{\perp}). \end{aligned} \tag{2.6}$$

Let P be the projection morphism of TM on $S(TM)$ with respect to the first equation of the decompositions (2.4) [denote $(2.4)_1$]. Then the local Gauss and Weingarten formulas are given by

$$\bar{\nabla}_X Y = \nabla_X Y + B(X,Y)N + D(X,Y)L, \tag{2.7}$$

$$\bar{\nabla}_X N = -A_N X + \tau(X)N + \rho(X)L, \tag{2.8}$$

$$\bar{\nabla}_X L = -A_L X + \phi(X)N, \tag{2.9}$$

$$\nabla_X PY = \nabla^*_X PY + C(X,PY)\xi, \tag{2.10}$$

$$\nabla_X \xi = -A^*_\xi X - \tau(X)\xi, \ \forall\, X,\, Y \in \Gamma(TM), \tag{2.11}$$

where ∇ and ∇^* are induced linear connections on TM and $S(TM)$ respectively, B and D are called the *local second fundamental forms* of M, C is called the *local second fundamental form* on $S(TM)$. A_N, A^*_ξ and A_L are linear operators on TM and τ, ρ and ϕ are 1-forms on TM. Since $\bar{\nabla}$ is torsion-free, ∇ is also torsion-free, and B and D are symmetric. From the facts $B(X,Y) = \bar{g}(\bar{\nabla}_X Y, \xi)$ and $D(X,Y) = \bar{g}(\bar{\nabla}_X Y, L)$ for all $X, Y \in \Gamma(TM)$, we know that B and D are independent of the choice of a screen distribution and satisfy

$$B(X,\xi) = 0, \quad D(X,\xi) = -\phi(X), \ \forall\, X \in \Gamma(TM). \tag{2.12}$$

Replacing Y by ξ to (2.7) and using (2.11) and (2.12), we have

$$\bar{\nabla}_X \xi = -A^*_\xi X - \tau(X)\xi - \phi(X)L, \ \forall\, X \in \Gamma(TM). \tag{2.13}$$

The induced connection ∇ of M is not metric and satisfies

$$(\nabla_X g)(Y,Z) = B(X,Y)\,\eta(Z) + B(X,Z)\,\eta(Y), \tag{2.14}$$

for all $X, Y, Z \in \Gamma(TM)$, where η is a 1-form on TM such that

$$\eta(X) = \bar{g}(X,N), \ \forall\, X \in \Gamma(TM). \tag{2.15}$$

But the connection ∇^* on $S(TM)$ is metric. The above three local second fundamental forms are related to their shape operators by

$$B(X,Y) = g(A^*_\xi X, Y), \qquad \bar{g}(A^*_\xi X, N) = 0, \tag{2.16}$$

$$C(X,PY) = g(A_N X, PY), \qquad \bar{g}(A_N X, N) = 0, \tag{2.17}$$

$$D(X,PY) = g(A_L X, PY), \qquad \bar{g}(A_L X, N) = \rho(X), \tag{2.18}$$

$$D(X,Y) = g(A_L X, Y) - \phi(X)\eta(Y), \ \forall\, X,\, Y \in \Gamma(TM). \tag{2.19}$$

Denote by $\bar{R}$ and R the curvature tensors of the connections $\bar{\nabla}$ and ∇ respectively. Using the local Gauss-Weingarten formulas (2.4) $\sim$ (2.6) and (2.19), we have the Gauss-Codazzi equations for M, for all $X, Y, Z \in \Gamma(TM)$:

$$\begin{aligned} \bar{R}(X,Y)Z &\\ = R(X,Y)Z &+ B(X,Z)A_N Y - B(Y,Z)A_N X \\ &+ D(X,Z)A_L Y - D(Y,Z)A_L X \\ &+ \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) + \tau(X)B(Y,Z) - \tau(Y)B(X,Z) \\ &\quad + \phi(X)D(Y,Z) - \phi(Y)D(X,Z)\}N, \\ &+ \{(\nabla_X D)(Y,Z) - (\nabla_Y D)(X,Z) + \rho(X)B(Y,Z) - \rho(Y)B(X,Z)\}L, \end{aligned} \tag{2.20}$$

(2.21) $$\begin{aligned}\bar{R}(X,Y)N &= -\nabla_X(A_N Y) + \nabla_Y(A_N X) + A_N[X,Y] \\ &+ \tau(X)A_N Y - \tau(Y)A_N X + \rho(X)A_L Y - \rho(Y)A_L X \\ &+ \{B(Y,A_N X) - B(X,A_N Y) + 2d\tau(X,Y) + \phi(X)\rho(Y) - \phi(Y)\rho(X)\}N \\ &+ \{D(Y,A_N X) - D(X,A_N Y) + 2d\rho(X,Y) + \rho(X)\tau(Y) - \rho(Y)\tau(X)\}L,\end{aligned}$$

(2.22) $$\begin{aligned}\bar{R}(X,Y) &= -\nabla_X(A_L Y) + \nabla_Y(A_L X) + A_L[X,Y] + \phi(X)A_N Y - \phi(Y)A_N X \\ &+ \{B(Y,A_L X) - B(X,A_L Y) + 2d\phi(X,Y) + \tau(X)\phi(Y) \\ &-\tau(Y)\phi(X)\}N.\end{aligned}$$

Using (2.10) and (2.11), for all $X,\, Y \in \Gamma(TM)$, we obtain

(2.23) $$\begin{aligned}R(X,Y)\xi &= -\nabla^*_X(A^*_\xi Y) + \nabla^*_Y(A^*_\xi X) + A^*_\xi[X,Y] - \tau(X)A^*_\xi Y \\ &+ \tau(Y)A^*_\xi X + \{C(Y,A^*_\xi X) - C(X,A^*_\xi Y) - 2d\tau(X,Y)\}\xi.\end{aligned}$$

The induced Ricci type tensor $R^{(0,\,2)}$ of M is defined by

$$R^{(0,\,2)}(X,Y) = trace\{Z \to R(Z,X)Y\},\ \forall\, X,\, Y \in \Gamma(TM).$$

In general, $R^{(0,\,2)}$ is not symmetric [1, 2]. A tensor field $R^{(0,\,2)}$ of M is called its *induced Ricci tensor* of M if it is symmetric. It is well-known that $R^{(0,\,2)}$ is symmetric if and only if the 1-form τ is closed, i.e., $d\tau = 0$ on any $\mathcal{U} \subset M$ [2].

3. Main results

Theorem 3.1. *Let M be a half lightlike submanifold of an indefinite Kenmotsu manifold $\bar{M}$. Then the 1-form θ is closed, i.e., $d\theta = 0$, on TM.*

Proof. From the decomposition (2.6) of $T\bar{M}$, ζ is decomposed by

(3.1) $$\zeta = W + mN + nL,$$

where W is a smooth vector field on M and $m = \theta(\xi)$ and $n = \theta(L)$ are smooth functions. Substituting (3.1) in (2.2) and using (2.8) and (2.9), we have

(3.2) $$\nabla_X W = -X + \theta(X)W + mA_N X + nA_L X,$$

(3.3) $$Xm + m\tau(X) + n\phi(X) + B(X,\, W) = m\theta(X),$$

(3.4) $$Xn + m\rho(X) + D(X,\, W) = n\theta(X),\ \forall\, X \in \Gamma(TM).$$

Substituting (3.3) and (3.4) into the following two equations

$$[X,Y]m = X(Ym) - Y(Xm),\ [X,Y]n = X(Yn) - Y(Xn),\ \forall\, X,\, Y \in \Gamma(TM),$$

and using (2.20), (2.21), (2.22), (3.1), (3.3), (3.4), we have respectively

(3.5) $$2m\, d\theta(X,Y) = \bar{g}(\bar{R}(X,Y)\zeta,\, \xi),\ 2n\, d\theta(X,Y) = \bar{g}(\bar{R}(X,Y)\zeta,\, L).$$

Substituting (3.2) into $R(X,Y)W = \nabla_X\nabla_Y W - \nabla_Y\nabla_X W - \nabla_{[X,\,Y]}W$ and using (2.20)∼(2.22), (3.2)∼(3.5) and the fact that ∇ is torsion-free, we have

$$\bar{R}(X,Y)\zeta = \theta(X)Y - \theta(Y)X + 2d\theta(X,Y)\zeta, \ \forall\, X,\, Y \in \Gamma(TM). \tag{3.6}$$

Taking the scalar product with ζ to (3.6) and using the fact $\bar{g}(\bar{R}(X,Y)\zeta,\,\zeta) = 0$ and (2.1), we show that $d\theta = 0$ on TM. Thus we have our assertion. □

Theorem 3.2. *Let M be a locally symmetric half lightlike submanifold of an indefinite Kenmotsu manifold $\bar{M}$. If the transversal vector bundle $tr(TM)$ is parallel with respect to the connection $\bar{\nabla}$ on $\bar{M}$, then the tensor field $R^{(0,\,2)}$ is a symmetric Ricci tensor on M and the curvature tensor R of M is given by*

$$R(X,Y)Z = n\phi(Z)\{\eta(X)Y - \eta(Y)X\}, \quad \forall\, X,\, Y,\, Z \in \Gamma(TM).$$

Proof. As $tr(TM)$ is parallel with respect to $\bar{\nabla}$ and $tr(TM) = Span\{N,\, L\}$, by (2.8) and (2.9) we have $A_{N} = A_{L} = 0$. Due to (2.17) and (2.18), we also have $C = \rho = 0$. Replacing X by ξ to $(2.18)_1$ and using $(2.12)_2$, we show that $\phi(PX) = 0$, i.e., $\phi = 0$ on $S(TM)$. Substituting (3.1) into $\bar{R}(X,Y)\zeta$ and using (2.20), (2.21) and (2.22), for all $X,\, Y \in \Gamma(TM)$, we have

$$\bar{R}(X,Y)\zeta = R(X,Y)W + \bar{g}(\bar{R}(X,Y)\zeta,\, \xi)N + \bar{g}(\bar{R}(X,Y)\zeta,\, L)L.$$

From this equation, (3.5) and (3.6) with $d\theta = 0$, we obtain

$$R(X,Y)W = \theta(X)Y - \theta(Y)X, \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.7}$$

Applying $\bar{\nabla}_X$ to $\theta(Y) = g(Y,\zeta)$ and using (2.2) and (2.6), we have

$$(\nabla_X\theta)(Y) = lB(X,Y) + nD(X,Y) - g(X,Y) + \theta(X)\theta(Y), \tag{3.8}$$

for all $X,\, Y \in \Gamma(TM)$, where we set $l = \bar{g}(\zeta, N)$. Applying ∇_Z to (3.7) and using M is locally symmetric, we have

$$R(X,Y)\nabla_Z W = (\nabla_Z\theta)(X)Y - (\nabla_Z\theta)(Y)X, \ \forall\, X,\, Y,\, Z \in \Gamma(TM).$$

Substituting (3.2) and (3.8) in this equation and using (3.7), we obtain

$$\begin{aligned} R(X,Y)Z = &\{g(X,Z) - lB(X,Z) - nD(X,Z)\}Y \\ &- \{g(Y,Z) - lB(Y,Z) - nD(Y,Z)\}X \end{aligned} \tag{3.9}$$

for all $X,\, Y,\, Z \in \Gamma(TM)$. Replacing Z by ξ to (3.9) and using (2.12), we have

$$R(X,Y)\xi = n\{\phi(X)Y - \phi(Y)X\}, \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.10}$$

Taking the scalar product with N to (2.23) and (3.10) respectively and then, comparing this two results, we have

$$2d\tau(X,Y) = n\{\phi(Y)\eta(X) - \phi(X)\eta(Y)\}, \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.11}$$

Due to (2.19), we have $D(X,Y) = -\phi(X)\eta(Y)$ for all $X,\, Y \in \Gamma(TM)$. As D is symmetric, we get $\phi(X)\eta(Y) - \phi(Y)\eta(X) = 0$. From this result and (3.11), we have $d\tau = 0$ on TM. Thus $R^{(0,2)}$ is a symmetric Ricci tensor on M. From (2.20), (2.21), (3.9) and the facts $d\tau = 0$ and $D(X,Y) = -\phi(X)\eta(Y)$, we have

$$\begin{aligned} 0 &= \bar{g}(\bar{R}(X,Y)N,Z) = -\bar{g}(\bar{R}(X,Y)Z,N) = -\bar{g}(R(X,Y)Z,N) \\ &= \{g(Y,Z) - lB(Y,Z)\}\eta(X) - \{g(X,Z) - lB(X,Z)\}\eta(Y) \end{aligned} \tag{3.12}$$

for all $X, Y, Z \in \Gamma(TM)$. Replacing Y by ξ to (3.12) and using (2.12), we get

$$g(X,Y) = lB(X,Y), \quad \forall\, X, Y \in \Gamma(TM). \tag{3.13}$$

Substituting (3.13) into (3.9) and using $D(X,Z) = -\phi(Z)\eta(X)$, we have

$$R(X,Y)Z = n\phi(Z)\{\eta(X)Y - \eta(Y)X\}, \quad \forall\, X, Y, Z \in \Gamma(TM). \tag{3.14}$$ □

Definition 1. A half lightlike submanifold M is said to be *irrotational* [3, 7] if $\bar{\nabla}_X\xi \in \Gamma(TM)$ for any $X \in \Gamma(TM)$. From (2.7) and (2.12), we show that this definition is equivalent to $D(X,\xi) = 0 = \phi(X)$ for all $X \in \Gamma(TM)$.

Theorem 3.3. *Let M be a locally symmetric irrotational half lightlike submanifold of an indefinite Kenmotsu manifold $\bar{M}$ such that $tr(TM)$ is parallel with respect to $\bar{\nabla}$. Then M is a space of constant curvature 0 and the vector field ζ of $\bar{M}$ is normal to M.*

Proof. If M is irrotational, we have $\phi = 0$ on TM. Thus we have $R(X,Y)Z = 0$ for all $X, Y, Z \in \Gamma(TM)$ due to (3.14). Therefore M is a space of constant curvature 0. From (2.19), we have $D = 0$ on TM. Replacing Z by W to (3.9) and then, comparing this result with (3.7) and using the facts $D = 0$ and $\theta(X) = g(X,W) + m\eta(X)$ on TM, we have

$$\{m\eta(X) + lB(X,W)\}Y = \{m\eta(Y) + lB(Y,W)\}X.$$

Replacing Y by ξ to this and using (2.12) and $X = PX + \eta(X)\xi$, we have

$$mPX = lB(X,W)\xi, \ \forall\, X \in \Gamma(TM).$$

The left term of this equation belongs to $S(TM)$ and the right term belongs to $TM^{\perp}$. This imply $mPX = 0$ and $lB(X,W) = 0$ for all $X \in \Gamma(TM)$. From the first equation of this results we deduce to $m = 0$. Replacing Y by W to (3.13) and using $lB(X,W) = 0$, we have $g(X,W) = 0$ for all $X \in \Gamma(TM)$. This implies $W = l\xi$. Thus the vector field ζ is decomposed by $\zeta = l\xi + nL$. This implies that ζ is normal to M. □

Definition 2. A vector field X on $\bar{M}$ is called a *conformal Killing vector field* [5, 9] if $\bar{\mathcal{L}}_X\bar{g} = -2\delta\bar{g}$, where δ is a non-vanishing smooth function on $\bar{M}$ and $\bar{\mathcal{L}}_X$ denotes the Lie derivative on $\bar{M}$ with respect to X. A distribution $\mathcal{G}$ on $\bar{M}$ is said to be a *conformal Killing distribution* on $\bar{M}$ if each vector field belonging to $\mathcal{G}$ is a conformal Killing vector field.

Note 1 [5]. Using (2.9) and (2.19), for all $X, Y \in \Gamma(TM)$, we have

$$\begin{aligned} &\bar{g}(\bar{\nabla}_X L, Y) = -g(A_L X, Y) + \phi(X)\eta(Y) = -D(X,Y), \\ &(\bar{\mathcal{L}}_L\bar{g})(X,Y) = \bar{g}(\bar{\nabla}_X L, Y) + \bar{g}(X, \bar{\nabla}_Y L) = -2D(X,Y). \end{aligned}$$

Thus $S(TM^{\perp})$ is a conformal Killing distribution if and only if D satisfies

$$D(X,Y) = \delta g(X,Y), \quad \forall\, X,\, Y \in \Gamma(TM). \tag{3.15}$$

Corollary 1. *Let M be a locally symmetric half lightlike submanifold of an indefinite Kenmotsu manifold $\bar{M}$ such that $tr(TM)$ is parallel with respect to $\bar{\nabla}$. If $S(TM^{\perp})$ is a conformal Killing distribution on $\bar{M}$, then M is a space of constant curvature* 0 *and the structure vector field ζ of $\bar{M}$ is normal to M.*

Proof. If $S(TM^{\perp})$ is a conformal Killing distribution on $\bar{M}$, then, from $(2.12)_2$ and (3.15), we have $\phi = 0$ on TM. By Theorem 3.3, we have our assertion. □

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