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EXISTENCE OF PERIODIC SOLUTION AND PERSISTENCE FOR A DELAYED PREDATOR-PREY SYSTEM WITH DIFFUSION AND IMPULSE†

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Abstract. By using Mawhin continuation theorem and comparison theorem, the existence of periodic solution and persistence for a predator-prey system with diffusion and impulses are investigated in this paper. An example and simulation are given to show the effectiveness of the main results.

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1. Introduction

The dispersion is a ubiquitous phenomenon in natural world. Because of the ecological effects of human activities and industry, more and more habitats have been broken into patches and some of them have been polluted, then species have to diffuse in search for better environment. The importance of dispersal in understanding ecological and evolutionary dynamics of populations is mirrored by large number of mathematical models in the scientific literature, see [1-6] and references cited therein.

For example, Xu and Ma [1] proposed a predator-prey system with diffusion and time delays as follows:

$$\begin{cases} x_1'(t) = x_1(t)(r_1 - a_{11}x_1(t) - a_{12}y(t)) + D_{21}x_2(t) - D_{12}x_1(t), \\ x_2'(t) = -r_2x_2(t) + D_{12}x_1(t) - D_{21}x_2(t), \\ y'(t) = a_{31}x_1(t-\tau)y(t-\tau) - ry(t) - a_{32}y^2(t), \end{cases}$$

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where the parameters are all constants. By using an iteration technique, the permanence and extinction of the model were studied there.

However in real world, the environment always changes periodically. Considering the periodically environmental factors, the parameters in the model should not be assumed to be constant. Hence it is reasonable to study the predator-prey system with periodic coefficients. On the other hand, ecological systems are often perturbed by environmental changes and human activities such as planting and harvesting, etc. These short-time perturbations are often assumed to be in the form of impulses in the modeling process. Impulsive equations provide a natural description of such systems. Consequently, it is necessary to consider ecological system with impulsive effect. Impulsive ecological system attracts more and more attention[2,3,5,7,12,16]. Hence it is necessary and interesting to study dispersed predator-prey system with periodicity of the environment and impulsive perturbations.

Motivated by the above discussion, in this paper, we study the following nonautonomous predator-prey system with time delays due to the gestation of the predator, diffusion and impulsive effects,

$$\begin{cases} \left.\begin{aligned} &x_1'(t) = x_1(t)(r_1(t) - a_{11}(t)x_1(t) - a_{12}(t)y(t)) + D_{21}(t)x_2(t) - D_{12}(t)x_1(t),\\ &x_2'(t) = x_2(t)(r_2(t) - a_{21}(t)x_2(t)) + D_{12}(t)x_1(t) - D_{21}(t)x_2(t),\\ &y'(t) = a_{31}(t)x_1(t-\tau)y(t-\tau) + r(t)y(t) - a_{32}(t)y^2(t), \end{aligned}\right\} t \neq t_k,\\ \Delta x_1(t_k) = x_1(t_k^+) - x_1(t_k^-) = b_{1k}x_1(t_k),\\ \Delta x_2(t_k) = x_2(t_k^+) - x_2(t_k^-) = b_{2k}x_2(t_k),\\ \Delta y(t_k) = y(t_k^+) - y(t_k^-) = b_{3k}y(t_k), \end{cases} \tag{1}$$

with initial conditions

$$x_i(\theta) = \phi_i(\theta) \geq 0,\ y(\theta) = \psi(\theta) \geq 0,\ \theta \in [-\tau, 0],\ \phi_i(0) > 0, \psi(0) > 0, i = 1, 2, \tag{2}$$

where $x_1(t)$ and $y(t)$ represent the population densities of prey species x and predator species y in patch 1, $x_2(t)$ represents the density of prey x in patch 2, predator y is confined to patch 1 while prey x can diffuse between two patches. Parameters $D_{12}(t)$ and $D_{21}(t)$ are the dispersal rates of the prey between two patches, $\tau > 0$ is a constant delay due to the gestation of the predator. We note that the dispersal term $D_{ij}(t)x_i(t) - D_{ji}(t)x_j(t)$ is different from the commonly used dispersal term $D_{ij}(t)(x_j(t) - x_i(t))$ (see [8,9]). Further, we assume that $x_i(t_k^-) = x_i(t_k), y(t_k^-) = y(t_k)$, $x_i(t_k^+), y(t_k^+)$ exist, $0 < t_1 < t_2 < \cdots, \lim_{k\to\infty} t_k = +\infty$. $\phi_i(\theta), \psi(\theta) \in C([-\tau, 0], R_+)$. By symbol $C([-\tau, 0], R_+)$, we mean the Banach space of continuous functions mapping the interval $[-\tau, 0]$ into $R_+ := \{x \geq 0\}, i = 1, 2$.

By using Mawhin continuation theorem and comparison theorem, we aim to investigate the existence of periodic solutions and persistence of system (1).

Throughout this paper, we assume that:

(C_1) $r_1(t), r_2(t), a_{11}(t), a_{12}(t), a_{21}(t), a_{31}(t), a_{32}(t), D_{12}(t), D_{21}(t)$ are all positive periodic continuous functions with period $\omega > 0$, $r(t)$ is continuous and ω-periodic.

(C_2) $-1 < b_{ik} \leq 0$ and there exists a positive integer q such that $t_{q+k} = t_k + \omega, b_{i(k+q)} = b_{ik} + \omega$, if $t_k \neq 0, \omega$, then $[0, \omega] \cap \{t_k\} = \{t_1, t_2, \cdots, t_q\}, i = 1, 2, 3, k \in N$.

For convenience, we introduce the following notation.

(1) $PC(R^+, R) = \{f(t) | f : R^+ \to R, \lim_{s \to t} f(s) = f(t)$ if $t \neq t_k, f(t_k^-) = f(t_k)$, $f(t_k^+)$ exists, $k \in Z^+\}$.

(2) $PC'(R^+, R) = \{f(t) | f : R^+ \to R, f'(t) \in PC(R^+, R)\}$.

(3) $PC_\omega = \{f \in PC(R^+, R) | f(t) = f(t + \omega)\}$,
$PC'_\omega = \{f \in PC'(R^+, R) | f(t) = f(t + \omega)\}$.

(4) For $f \in PC_\omega$, we denote

$$\bar{f} = \frac{1}{\omega} \int_0^\omega f(t) dt, \; f^L = \min_{t \in [0, \omega]} |f(t)|, \; f^M = \max_{t \in [0, \omega]} |f(t)|.$$

This paper is organized as follows. In Section 2, by using Mawhin continuation theorem, the existence of positive periodic solutions of (1) is studied. In Section 3, by constructing suitable functional and employing comparison theorem, the persistence of (1) is investigated. In Section 4, an example and numerical simulation are given to show the validity of the main results. Finally, a brief discussion is given to conclude the paper in Section 5.

2. Existence of periodic solution

In this section, by using Mawhin continuation theorem, we aim to derive conditions ensuring the existence of periodic solution of system (1). Firstly, we introduce Mawhin continuation theorem [10].

Let X, Y be real Banach spaces, $L : DomL \cap X \to Y$ be a Fredholm mapping of index zero. $P : X \to X, Q : Y \to Y$ be continuous projectors such that $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Ker}Q = \mathrm{Im}L$ so that $X = \mathrm{Ker}L \oplus \mathrm{Ker}P, Y = \mathrm{Im}L \oplus \mathrm{Im}Q$. Denote by L_p the restriction of L to $\mathrm{Dom}L \cap \mathrm{Ker}P$ and by $K_p : \mathrm{Im}L \to \mathrm{Dom}L \cap \mathrm{Ker}P$ the inverse to L_p. Let $J : \mathrm{Im}Q \to \mathrm{Ker}L$ be an isomorphism of $\mathrm{Im}Q$ onto $\mathrm{Ker}L$. Then the continuation theorem can be described as follows.

Lemma 1 ([10]). *Let $\Omega \subset X$ be an open bounded set and $N : X \to Y$ be a continuous operator which is L-compact on $\bar{\Omega}$ (i.e., $QN : \bar{\Omega} \to Y$ and $K_p(I - Q)N : \bar{\Omega} \to Y$ are compact). Assume that*
(i) for each $\lambda \in (0, 1), x \in \partial\Omega \cap DomL, Lx \neq \lambda Nx$,
(ii) for each $x \in \partial\Omega \cap DomL, QNx \neq 0$,
(iii) $deg\{JQN, \Omega \cap KerL, 0\} \neq 0$.
Then $Lx = Nx$ has at least one solution in $\bar{\Omega} \cap KerL$.

Theorem 1. *In addition to (C_1) and (C_2) assume,*
(C_3) $r(t) > 0$;
(C_4) $\bar{r}\omega + \sum_{k=1}^q \ln(1 + b_{3k}) > 0, \overline{r_2 - D_{21}}\omega + \sum_{k=1}^q \ln(1 + b_{2k}) > 0$;
(C_5) $\omega\overline{r_1 - D_{12}} + \sum_{k=1}^q \ln(1 + b_{1k}) > a_{12}^M \omega B$;

where $A=\max\left\{\frac{(r_1-D_{12})^M+D_{21}^M}{a_{11}^L},\frac{(r_2-D_{21})^M+D_{12}^M}{a_{21}^L}\right\}, B=\frac{a_{31}^M A+r^M}{a_{32}^L}$. *Then system (1) has at least one positive* $\omega-$*periodic solution.*

Proof. By the fundamental theory of functional differential equations [11], system (1) has a unique solution satisfying the initial conditions (2) and all solutions of (1) remain positive.

Let $x_1(t)=\mathrm{e}^{u_1(t)}, x_2(t)=\mathrm{e}^{u_2(t)}, y(t)=\mathrm{e}^{u_3(t)}$, then (1) can be transformed into

$$\begin{cases} \left.\begin{array}{l} u_1'(t)=r_1(t)-D_{12}(t)-a_{11}(t)\mathrm{e}^{u_1(t)}-a_{12}(t)\mathrm{e}^{u_3(t)}+D_{21}(t)\mathrm{e}^{u_2(t)-u_1(t)},\\ u_2'(t)=r_2(t)-D_{21}(t)-a_{21}(t)\mathrm{e}^{u_2(t)}+D_{12}(t)\mathrm{e}^{u_1(t)-u_2(t)},\\ u_3'(t)=r(t)+a_{31}(t)\mathrm{e}^{u_1(t-\tau)+u_3(t-\tau)-u_3(t)}-a_{32}(t)\mathrm{e}^{u_3(t)}, \end{array}\right\} t\neq t_k,\\ \Delta u_i(t_k)=\ln(1+b_{ik}), i=1,2,3, k=1,2,\cdots. \end{cases} \tag{3}$$

It is easy to see that if (3) has one $\omega-$periodic solution $(u_1(t),u_2(t),u_3(t))^T$, then $(x_1(t),x_2(t),y(t))^T=(\mathrm{e}^{u_1(t)},\mathrm{e}^{u_2(t)},\mathrm{e}^{u_3(t)})^T$ is a positive $\omega-$periodic solution of (1). Therefore, to complete the proof, we need only prove that (3) has one $\omega-$periodic solution. Let $X=\{(u_1(t),u_2(t),u_3(t))^T|u_i(t)\in PC_\omega, i=1,2,3\}$ with $\|(u_1(t),u_2(t),u_3(t))^T\|=\sum\limits_{i=1}^{3}\sup\limits_{t\in[0,\omega]}|u_i(t)|$, $Z=X\times R^{3q}$ with $\|z\|=\|x\|+|v|$ for $z\in Z, x\in X, v\in R^{3q}$, where $|\cdot|$ denotes the Euclidean norm, then $(X,\|\cdot\|)$ and $(Z,\|\cdot\|)$ are both Banach spaces. Let

$$L:\mathrm{Dom}L\cap X\to Z, u\to(u',\Delta u(t_1),\Delta u(t_2),\cdots,\Delta u(t_q)),\ N:X\to Z,$$

$$N\begin{pmatrix}u_1(t)\\u_2(t)\\u_3(t)\end{pmatrix}=\left(\begin{pmatrix}f_1(t)\\f_2(t)\\f_3(t)\end{pmatrix},\begin{pmatrix}\ln(1+b_{11})\\\ln(1+b_{21})\\\ln(1+b_{31})\end{pmatrix},\ \cdots,\begin{pmatrix}\ln(1+b_{1q})\\\ln(1+b_{2q})\\\ln(1+b_{3q})\end{pmatrix}\right)$$

where

$$\begin{aligned} \mathrm{Dom}L&=\{u(t)=(u_1(t),u_2(t),u_3(t))^T\in X|u_i(t)\in PC_\omega', i=1,2,3\},\\ f_1(t)&=r_1(t)-D_{12}(t)-a_{11}(t)\mathrm{e}^{u_1(t)}-a_{12}(t)\mathrm{e}^{u_3(t)}+D_{21}(t)\mathrm{e}^{u_2(t)-u_1(t)},\\ f_2(t)&=r_2(t)-D_{21}(t)-a_{21}(t)\mathrm{e}^{u_2(t)}+D_{12}(t)\mathrm{e}^{u_1(t)-u_2(t)},\\ f_3(t)&=r(t)+a_{31}(t)\mathrm{e}^{u_1(t-\tau)+u_3(t-\tau)-u_3(t)}-a_{32}(t)\mathrm{e}^{u_3(t)}. \end{aligned}$$

Obviously, $\mathrm{Ker}L=\{u(t)=c\in R^3, t\in[0,\omega]\}$, $\mathrm{Im}L=\{z=(f,a_1,a_2,\cdots,a_q)\in Z:\int_0^\omega f(s)ds+\sum_{k=1}^q a_k=0\}$ and $\dim\mathrm{Ker}L=\mathrm{codim}\mathrm{Im}L=3$. Therefore $\mathrm{Im}L$ is closed in Z and L is a Fredholm mapping of index zero. Define

$$\begin{aligned} &P:X\to X, Pu=\frac{1}{\omega}\int_0^\omega u(t)dt,\\ &Q:Z\to Z, Qz=Q(f,a_1,a_2\cdots,a_q)=\left(\frac{1}{\omega}\left(\int_0^\omega f(s)ds+\sum_{k=1}^q a_k\right),0,\cdots,0\right). \end{aligned}$$

It is easy to show that P and Q are continuous projectors satisfying $\mathrm{Im}P = \mathrm{Ker}L, \mathrm{Im}L = \mathrm{Ker}Q + \mathrm{Im}(I-Q)$. By an easy computation, the inverse of $L_p = L|_{\mathrm{Dom}L\cap\mathrm{Ker}P}$ is given by $K_p : \mathrm{Im}L \to \mathrm{Ker}P \cap \mathrm{Dom}L$,

$$K_p(z) = \int_0^t f(s)ds + \sum_{t_k<t} a_k - \frac{1}{\omega}\int_0^\omega \int_0^t f(s)dsdt - \sum_{k=1}^q a_k.$$

Therefore,

$$QNu = \left(\frac{1}{\omega}\begin{pmatrix} \int_0^\omega f_1(t)dt + \sum_{k=1}^q \ln(1+b_{1k}) \\ \int_0^\omega f_2(t)dt + \sum_{k=1}^q \ln(1+b_{2k}) \\ \int_0^\omega f_3(t)dt + \sum_{k=1}^q \ln(1+b_{3k}) \end{pmatrix}, \quad 0, \quad 0, \quad \cdots, \quad 0\right),$$

$$\begin{aligned} K_p(I-Q)Nu = &\begin{pmatrix} \int_0^t f_1(s)ds + \sum_{t_k<t} \ln(1+b_{1k}) \\ \int_0^t f_2(s)ds + \sum_{t_k<t} \ln(1+b_{2k}) \\ \int_0^t f_3(s)ds + \sum_{t_k<t} \ln(1+b_{3k}) \end{pmatrix} \\ &+ (\frac{1}{2} - \frac{t}{\omega})\begin{pmatrix} \int_0^\omega f_1(s)ds + \sum_{k=1}^q \ln(1+b_{1k}) \\ \int_0^\omega f_2(s)ds + \sum_{k=1}^q \ln(1+b_{2k}) \\ \int_0^\omega f_3(s)ds + \sum_{k=1}^q \ln(1+b_{3k}) \end{pmatrix} \\ &- \begin{pmatrix} \frac{1}{\omega}\int_0^\omega \int_0^t f_1(s)dsdt + \sum_{k=1}^q \ln(1+b_{1k}) \\ \frac{1}{\omega}\int_0^\omega \int_0^t f_2(s)dsdt + \sum_{k=1}^q \ln(1+b_{2k}) \\ \frac{1}{\omega}\int_0^\omega \int_0^t f_3(s)dsdt + \sum_{k=1}^q \ln(1+b_{3k}) \end{pmatrix}. \end{aligned}$$

Clearly, QN and $K_p(I-Q)N$ are continuous. Using Ascoli-Arzela lemma of [12], it is not difficult to show that $QN(\bar{\Omega})$ and $K_p(I-Q)N(\bar{\Omega})$ are relatively compact for any open bounded set $\Omega \subset X$. Hence N is $L-$compact on $\bar{\Omega}$ for any open bounded set $\Omega \subset X$.

Now we are in the position to search an appropriate open, bounded subset Ω for application of Lemma 1. Corresponding to equation $Lu = \lambda Nu, \lambda \in (0,1)$, we have

$$\left\{\begin{aligned} &\left.\begin{aligned} &u_1'(t) = \lambda(r_1(t) - D_{12}(t) - a_{11}(t)\mathrm{e}^{u_1(t)} - a_{12}(t)\mathrm{e}^{u_3(t)} \\ &\qquad\quad + D_{21}(t)\mathrm{e}^{u_2(t)-u_1(t)}), \\ &u_2'(t) = \lambda\left(r_2(t) - D_{21}(t) - a_{21}(t)\mathrm{e}^{u_2(t)} + D_{12}(t)\mathrm{e}^{u_1(t)-u_2(t)}\right), \\ &u_3'(t) = \lambda\left(r(t) + a_{31}(t)\mathrm{e}^{u_1(t-\tau)+u_3(t-\tau)-u_3(t)} - a_{32}(t)\mathrm{e}^{u_3(t)}\right), \end{aligned}\right\} t \neq t_k, \\ &\Delta u_i(t_k) = \lambda \ln(1+b_{ik}), i = 1,2,3, k = 1,2,\cdots. \end{aligned}\right. \tag{4}$$

Since $u_i(t)$ is $\omega-$periodic for $i = 1,2,3$, we only need to prove the result in the interval $[0,\omega]$. Suppose that $u(t) \in X$ is a solution of (4) for some $\lambda \in (0,1)$, integrating (4) over the interval $[0,\omega]$ leads to

$$\begin{aligned}&\int_0^\omega a_{11}(t)\mathrm{e}^{u_1(t)}dt+\int_0^\omega a_{12}(t)\mathrm{e}^{u_3(t)}dt\\&=\sum_{k=1}^q\ln(1+b_{1k})+\int_0^\omega (r_1(t)-D_{12}(t))dt+\int_0^\omega D_{21}(t)\mathrm{e}^{u_2(t)-u_1(t)}dt,\end{aligned}\tag{5}$$

$$\int_0^\omega a_{21}(t)e^{u_2(t)}dt=\sum_{k=1}^q\ln(1+b_{2k})+\int_0^\omega (r_2(t)-D_{21}(t))dt+\int_0^\omega D_{12}(t)\mathrm{e}^{u_1(t)-u_2(t)}dt,\tag{6}$$

$$\int_0^\omega a_{32}(t)\mathrm{e}^{u_3(t)}dt=\int_0^\omega r(t)dt+\int_0^\omega a_{31}(t)\mathrm{e}^{u_1(t-\tau)+u_3(t-\tau)-u_3(t)}dt+\sum_{k=1}^q\ln(1+b_{3k}).\tag{7}$$

From (4)- (7), we have

$$\begin{cases}\int_0^\omega |u_1'(t)|dt\le 2\int_0^\omega a_{11}(t)\mathrm{e}^{u_1(t)}dt+2\int_0^\omega a_{12}(t)\mathrm{e}^{u_3(t)}dt+2\int_0^\omega D_{12}(t)dt\\ \qquad\qquad -\sum_{k=1}^q\ln(1+b_{1k})\\ \int_0^\omega |u_2'(t)|dt\le 2\int_0^\omega D_{21}(t)dt+2\int_0^\omega a_{21}(t)\mathrm{e}^{u_2(t)}dt-\sum_{k=1}^q\ln(1+b_{2k})\\ \int_0^\omega |u_3'(t)|dt\le 2\int_0^\omega a_{32}(t)\mathrm{e}^{u_3(t)}-\sum_{k=1}^q\ln(1+b_{3k}).\end{cases}\tag{8}$$

Multiplying the first equation of (4) by $\mathrm{e}^{u_1(t)}$ and integrating it over $[0,\omega]$, we have

$$\begin{aligned}&-\sum_{k=1}^q\left[(1+b_{1k})^\lambda-1\right]\mathrm{e}^{u_1(t_k)}+\int_0^\omega a_{11}(t)\mathrm{e}^{2u_1(t)}dt\\ \le&(r_1-D_{12})^M\int_0^\omega \mathrm{e}^{u_1(t)}dt+D_{21}^M\int_0^\omega \mathrm{e}^{u_2(t)}dt,\end{aligned}$$

Since $-1<b_{1k}<0$, then

$$a_{11}^L\int_0^\omega \mathrm{e}^{2u_1(t)}dt\le(r_1-D_{12})^M\int_0^\omega \mathrm{e}^{u_1(t)}dt+D_{21}^M\int_0^\omega \mathrm{e}^{u_2(t)}dt.\tag{9}$$

Similarly, Multiplying the second equation of system (4) by $e^{u_2(t)}$ and integrating it over $[0,\omega]$, then

$$a_{21}^L\int_0^\omega \mathrm{e}^{2u_2(t)}dt\le(r_2-D_{21})^M\int_0^\omega \mathrm{e}^{u_2(t)}dt+D_{12}^M\int_0^\omega \mathrm{e}^{u_1(t)}dt.\tag{10}$$

By employing the following inequality

$$\left(\int_0^\omega e^{u_i(t)}dt\right)^2\le\omega\int_0^\omega e^{2u_i(t)}dt,$$

we derive from (9) and (10) that

$$
\begin{aligned}
a_{11}^{L}\left(\int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt\right)^{2} &\leq \omega\left(r_{1}-D_{12}\right)^{M} \int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt+\omega D_{21}^{M} \int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt, \\
a_{21}^{L}\left(\int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt\right)^{2} &\leq \omega\left(r_{2}-D_{21}\right)^{M} \int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt+\omega D_{12}^{M} \int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt.
\end{aligned} \tag{11}
$$

Moreover,
(i) If $\int_0^\omega \mathrm{e}^{u_2(t)}dt \leq \int_0^\omega \mathrm{e}^{u_1(t)}dt$, then it follows from (11) that

$$
a_{11}^{L}\left(\int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt\right)^{2} \leq \omega\left(r_{1}-D_{12}\right)^{M} \int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt+\omega D_{21}^{M} \int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt,
$$

which implies

$$
\int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt \leq \int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt \leq \frac{\omega\left(r_{1}-D_{12}\right)^{M}+\omega D_{21}^{M}}{a_{11}^{L}}. \tag{12}
$$

(ii) If $\int_0^\omega \mathrm{e}^{u_1(t)}dt \leq \int_0^\omega \mathrm{e}^{u_2(t)}dt$, from (11) again, we have

$$
a_{21}^{L}\left(\int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt\right)^{2} \leq \omega\left(r_{2}-D_{21}\right)^{M} \int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt+\omega D_{12}^{M} \int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt,
$$

which implies

$$
\int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} dt \leq \int_{0}^{\omega} \mathrm{e}^{u_{2}(t)} dt \leq \frac{\omega\left(r_{2}-D_{21}\right)^{M}+\omega D_{12}^{M}}{a_{21}^{L}}. \tag{13}
$$

Let $A=\max\left\{\frac{(r_1-D_{12})^M+D_{21}^M}{a_{11}^L}, \frac{(r_2-D_{21})^M+D_{12}^M}{a_{21}^L}\right\}$, then (12) and (13) implies that

$$
\int_{0}^{\omega} \mathrm{e}^{u_{i}(t)} dt \leq \omega A, \ i=1,2. \tag{14}
$$

Since $u(t) \in X$, there exist $\xi_i, \eta_i \in [0,\omega]$ such that $u_i(\xi_i) = \min_{t\in[0,\omega]} u_i(t)$, $u_i(\eta_i) = \max_{t\in[0,\omega]} u_i(t)$, then

$$
u_{i}(\xi_{i}) \leq \ln A, \ i=1,2. \tag{15}
$$

On the other hand, multiplying the third equation of system (4) by $e^{u_3(t)}$ and integrating it over $[0,\omega]$, we have

$$
\int_{0}^{\omega} a_{32}(t) \mathrm{e}^{2 u_{3}(t)} dt \leq a_{31}^{M} \int_{0}^{\omega} \mathrm{e}^{u_{1}(t-\tau)+u_{3}(t-\tau)} dt+\int_{0}^{\omega} r(t) \mathrm{e}^{u_{3}(t)} dt.
$$

By the periodicity of $u(t)$, $\int_0^\omega \mathrm{e}^{u_1(t-\tau)}dt = \int_0^\omega \mathrm{e}^{u_1(t)}dt$ and $\int_0^\omega \mathrm{e}^{u_3(t-\tau)}dt = \int_0^\omega \mathrm{e}^{u_3(t)}dt$. From (9), (14) and Hölder inequality [13],

$$
\int_{0}^{\omega} \mathrm{e}^{u_{1}(t)} \mathrm{e}^{u_{2}(t)} dt \leq\left(\int_{0}^{\omega} \mathrm{e}^{2 u_{1}(t)} dt\right)^{1/2}\left(\int_{0}^{\omega} \mathrm{e}^{2 u_{2}(t)} dt\right)^{1/2},
$$

we have

$$a_{32}^L \int_0^\omega \mathrm{e}^{2u_3(t)}dt \le a_{31}^M \left(\int_0^\omega \mathrm{e}^{2u_1(t-\tau)}dt\right)^{\frac{1}{2}} \left(\int_0^\omega \mathrm{e}^{2u_3(t-\tau)}dt\right)^{\frac{1}{2}} + \int_0^\omega r(t)\mathrm{e}^{u_3(t)}dt$$

$$= a_{31}^M \left(\int_0^\omega \mathrm{e}^{2u_1(t)}dt\right)^{\frac{1}{2}} \left(\int_0^\omega \mathrm{e}^{2u_3(t)}dt\right)^{\frac{1}{2}} + \int_0^\omega r(t)\mathrm{e}^{u_3(t)}dt$$

$$\le a_{31}^M \left(\omega A \frac{(r_1 - D_{12})^M + D_{21}^M}{a_{11}^L}\right)^{\frac{1}{2}} \left(\int_0^\omega \mathrm{e}^{2u_3(t)}dt\right)^{\frac{1}{2}} + r^M \int_0^\omega \mathrm{e}^{u_3(t)}dt$$

$$\le a_{31}^M \omega^{\frac{1}{2}} A \left(\int_0^\omega \mathrm{e}^{2u_3(t)}dt\right)^{\frac{1}{2}} + r^M \left(\omega \int_0^\omega \mathrm{e}^{2u_3(t)}dt\right)^{1/2}.$$

That is,

$$\left(\int_0^\omega \mathrm{e}^{2u_3(t)}dt\right)^{\frac{1}{2}} \le \frac{a_{31}^M \omega^{\frac{1}{2}} A + r^M \omega^{1/2}}{a_{32}^L}.$$

According to $(\int_0^\omega \mathrm{e}^{u_3(t)}dt)^2 \le \omega \int_0^\omega e^{2u_3(t)}dt$ again, we have

$$\left(\int_0^\omega \mathrm{e}^{u_3(t)}dt\right)^2 \le \omega^2 \left(\frac{a_{31}^M A + r^M}{a_{32}^L}\right)^2,$$

then

$$\int_0^\omega \mathrm{e}^{u_3(t)}dt \le \omega \frac{a_{31}^M A + r^M}{a_{32}^L} := \omega B, \tag{16}$$

i.e.,

$$u_3(\xi_3) \le \ln B. \tag{17}$$

By (8), (14) and (16), we have

$$\begin{aligned} &\int_0^\omega |u_1'(t)|dt \le 2a_{11}^M \omega A + 2a_{12}^M \omega B - \sum_{k=1}^q \ln(1 + b_{1k}) + 2\bar{D}_{12}\omega := d_1, \\ &\int_0^\omega |u_2'(t)|dt \le 2a_{21}^M \omega A - \sum_{k=1}^q \ln(1 + b_{2k}) + 2\bar{D}_{21}\omega := d_2, \\ &\int_0^\omega |u_3'(t)|dt \le 2a_{32}^M \omega B - \sum_{k=1}^q \ln(1 + b_{3k}) := d_3. \end{aligned} \tag{18}$$

Therefore, we derive from (15) and (18) that

$$u_1(t) = \begin{cases} u_1(\xi_1) + \int_{\xi_1}^t u_1'(s)ds + \sum_{\xi_1 < t_k < t} \ln(1 + b_{1k}), t \in (\xi_1, \omega]; \\ u_1(\xi_1) + \int_{\xi_1}^t u_1'(s)ds - \sum_{t \le t_k \le \xi_1^-} \ln(1 + b_{1k}), t \in (0, \xi_1]; \end{cases}$$

$$\le u_1(\xi_1) + \int_0^\omega |u_1'(t)|dt - \sum_{k=1}^q \ln(1 + b_{1k}), \tag{19}$$

$$\le \ln A + d_1 - \sum_{k=1}^q \ln(1 + b_{1k}).$$

Similarly,

$$u_2(t) \leq \ln A + d_2 - \sum_{k=1}^{q} \ln(1+b_{2k}),\ u_3(t) \leq \ln B + d_3 - \sum_{k=1}^{q} \ln(1+b_{3k}). \quad (20)$$

On the other hand, it follows from (7) that $\int_0^\omega a_{32}(t)e^{u_3(t)}dt \geq \int_0^\omega r(t)dt + \sum_{k=1}^{q} \ln(1+b_{3k})$, which leads to $a_{32}^M \int_0^\omega e^{u_3(t)}dt \geq \omega\bar{r} + \sum_{k=1}^{q} \ln(1+b_{3k})$, hence,

$$u_3(\eta_3) > \ln \frac{\bar{r}\omega + \sum_{k=1}^{q} \ln(1+b_{3k})}{\omega a_{32}^M} := G_1. \quad (21)$$

From (18) and (21), we have

$$u_3(t) \geq u_3(\eta_3) - \int_0^\omega |u_3'(t)|dt + \sum_{k=1}^{q} \ln(1+b_{3k}) \geq G_1 - d_3 + \sum_{k=1}^{q} \ln(1+b_{3k}). \quad (22)$$

Therefore, by (20) and (22), the following inequality hold.

$$\max_{t\in[0,\omega]} |u_3(t)| < \max\{|\ln B| + d_3 - \sum_{k=1}^{q} \ln(1+b_{3k}), |G_1| + d_3 - \sum_{k=1}^{q} \ln(1+b_{3k})\} := R_3.$$

In view of (6), then $\int_0^\omega a_{21}(t)e^{u_2(t)}dt \geq \omega\overline{r_2 - D_{21}} + \sum_{k=1}^{q} \ln(1+b_{2k})$. Thus,

$$\int_0^\omega e^{u_2(t)}dt \geq \frac{\omega\overline{r_2 - D_{21}} + \sum_{k=1}^{q} \ln(1+b_{2k})}{a_{21}^M}$$

and

$$u_2(\eta_2) \geq \ln \frac{\omega\overline{r_2 - D_{21}} + \sum_{k=1}^{q} \ln(1+b_{2k})}{a_{21}^M \omega} := G_2.$$

Hence

$$u_2(t) \geq u_2(\eta_2) - \int_0^\omega |u_2'(t)| + \sum_{k=1}^{q} \ln(1+b_{2k}) \geq G_2 - d_2 + \sum_{k=1}^{q} \ln(1+b_{2k}). \quad (23)$$

From (20) and (23), then

$$\max_{t\in[0,\omega]} |u_2(t)| < \max\{|\ln A| + d_2 - \sum_{k=1}^{q} \ln(1+b_{2k}), |G_2| + d_2 - \sum_{k=1}^{q} \ln(1+b_{2k})\} := R_2.$$

From (5) and (16), we have $\int_0^\omega a_{11}(t)e^{u_1(t)}dt \geq \omega\overline{r_1 - D_{12}} + \sum_{k=1}^{q} \ln(1+b_{1k}) - \int_0^\omega a_{12}(t)e^{u_3(t)}dt \geq \omega\overline{r_1 - D_{12}} + \sum_{k=1}^{q} \ln(1+b_{1k}) - a_{12}^M \omega B$. Therefore

$$\int_0^\omega e^{u_1(t)}dt \geq \frac{\omega\overline{r_1 - D_{12}} + \sum_{k=1}^{q} \ln(1+b_{1k}) - a_{12}^M \omega B}{a_{11}^M}$$

and

$$u_1(\eta_1) \geq \ln \frac{\omega\overline{r_1 - D_{12}} + \sum_{k=1}^{q} \ln(1+b_{1k}) - a_{12}^M \omega B}{\omega a_{11}^M} := G_3 \quad (24)$$

Take $R_1 = \max\{|\ln A| + d_1 - \sum_{k=1}^{q} \ln(1+b_{1k}), |G_3| + d_1 - \sum_{k=1}^{q} \ln(1+b_{1k})\}$, then from (19) and (24), $\max_{t\in[0,\omega]} |u_1(t)| < R_1$ holds. Obviously, $R_i (i = 1, 2, 3)$

is independent of λ. Similar Proof to Theorem 2.1 [14], we can find a sufficiently large constant $M > 0$, if $\Omega = \{u(t) \in X : \|u\| < M\}$ for each $u \in KerL \cap \partial\Omega$, we can derive that

$$QNu \neq 0 \text{ and } \deg\{JQN, \Omega \cap \mathrm{Ker}L, 0\} = -1 \neq 0.$$

By now, we have proved that Ω satisfies all the requirements in Lemma 1. Hence (3) has at least one $\omega-$periodic solution. Accordingly, system (1) has at least one $\omega-$periodic solution. The proof is complete. □

3. Persistence

In this section, the boundedness and uniform permanence of system (1) are discussed. Firstly some preliminaries are introduced as follows.

Definition 1. System (1) is uniformly persistent if there exists a compact region $D \subset \mathrm{Int}R^3_+$ such that every solution $x(t) = (x_1(t), x_2(t), y(t))^T$ of (1) eventually enters and remains in region D.

Lemma 2 ([6]). *If $a > 0, b > 0$, $x'(t) \geq (\leq) x(t)(b - ax(t))$ when $t \geq 0$ and $x(0) > 0$, then we have*

$$x(t) \geq (\leq) \frac{b}{a}\left[1 + (\frac{bx^{-1}(0)}{a} - 1)e^{-bt}\right]^{-1}.$$

Lemma 3 ([15]). *Considering the following equation*

$$x'(t) = ax(t-\tau) - bx(t) - cx^2(t),$$

where $a, b, c, \tau > 0, x(t) > 0$ for $t \in [-\tau, 0]$, we have
(i) if $a > b$, then $\lim_{t\to\infty} x(t) = \frac{a-b}{c}$;
(ii) if $a < b$, then $\lim_{t\to\infty} x(t) = 0$.

In this section, we always assume that

(C_6) $r(t) < 0$.

Under hypotheses $(C_1) - (C_2)$ and (C_6), we consider the non-impulsive delay differential equation

$$\begin{cases} \dot{z}_1(t) = z_1(t)(r_1(t) - A_{11}(t)z_1(t) - A_{12}(t)z_3(t)) + D_{21}(t)z_2(t) \\ \qquad \prod_{0<t_k<t}(1 + b_{2k})(1 + b_{1k})^{-1} - D_{12}(t)z_1(t), \\ \dot{z}_2(t) = z_2(t)(r_2(t) - A_{21}(t)z_2(t)) + D_{12}(t)z_1(t) \prod_{0<t_k<t}(1 + b_{1k})(1 + b_{2k})^{-1} \\ \qquad - D_{21}(t)z_2(t), \\ \dot{z}_3(t) = A_{31}(t)z_1(t-\tau)z_3(t-\tau) + r(t)z_3(t) - A_{32}(t)z_3^2(t). \end{cases} \tag{25}$$

with initial conditions

$$z_i(\theta) = \phi_i(\theta) \geq 0,\ \theta \in [-\tau, 0];\ \phi_i(0) > 0, i = 1, 2, 3. \tag{26}$$

where

$$A_{11}(t) = a_{11}(t) \prod_{0<t_k<t} (1 + b_{1k}), \qquad A_{12}(t) = a_{12}(t) \prod_{0<t_k<t} (1 + b_{3k}),$$
$$A_{21}(t) = a_{21}(t) \prod_{0<t_k<t} (1 + b_{2k}), \qquad A_{31}(t) = a_{31}(t) \prod_{0<t_k<t} (1 + b_{1k}),$$
$$A_{32}(t) = a_{32}(t) \prod_{0<t_k<t} (1 + b_{3k}).$$

For system (1) and (25), similar proof to Theorem 1 [16], the following lemma holds, which plays key role in the proof of the main results.

Lemma 4. *Assume that* $(C_1) - (C_2)$ *hold. Then*
(i) If $z(t) = (z_1(t), z_2(t), z_3(t)^T$ *is a solution of (25) on* $[-\tau, \infty)$*, then* $x_i(t) = \prod_{0<t_k<t}(1 + b_{ik})z_i(t)(i = 1, 2), y(t) = \prod_{0<t_k<t}(1 + b_{3k})z_3(t)$ *is a solution of (1) on* $[-\tau, \infty)$*.*
(ii) If $(x_1(t), x_2(t), y(t)^T$ *is a solution of (1) on* $[-\tau, \infty)$*, then* $z_i(t) = \prod_{0<t_k<t}(1+ b_{ik})^{-1}x_i(t)(i = 1, 2), z_3(t) = \prod_{0<t_k<t}(1 + b_{3k})^{-1}y(t)$ *is a solution of (25) on* $[-\tau, \infty)$*.*

Lemma 5. *Let* $z(t) = (z_1(t), z_2(t), z_3(t))^T$ *be any solution of (25), then there exists a constant* $T > 0$ *such that*

$$0 < z_i(t) \le M_i, i = 1, 2, 3. \tag{27}$$

where $M_1 = M_2 := \max\left\{\frac{r_1^M + \tilde{D}_{21}^M}{A_{11}^L} + \varepsilon, \frac{r_2^M + \tilde{D}_{12}^M}{A_{21}^L} + \varepsilon\right\}$, $M_3 = \frac{A_{31}^M M_1}{A_{32}^L} + \varepsilon, \varepsilon > 0$ *is an arbitrary small positive constant,* $\tilde{D}_{21}(t) = D_{21}(t)\prod_{0<t_k<t}(1 + b_{2k})(1 + b_{1k})^{-1}$, $\tilde{D}_{12}(t) = D_{12}(t)\prod_{0<t_k<t}(1 + b_{1k})(1 + b_{2k})^{-1}$.

Proof. Define $V(t) = \max\{z_1(t), z_2(t)\}$. Calculating the upper right derivative of V along the positive solution of (25), we have the following possibilities.
(i) If $z_1(t) > z_2(t)$ or $z_1(t) = z_2(t)$ and $z_1'(t) > z_2'(t)$, then

$$\begin{aligned} D^+V(t) = z_1'(t) &= z_1(t)(r_1(t) - A_{11}(t)z_1(t) - A_{12}(t)z_3(t)) \\ &\quad + D_{21}(t)z_2(t) \prod_{0<t_k<t} (1 + b_{2k})(1 + b_{1k})^{-1} - D_{12}(t)z_1(t) \\ &\le z_1(t)(r_1^M - A_{11}^L z_1(t)) + \tilde{D}_{21}^M z_2(t) \\ &\le z_1(t)(r_1^M + \tilde{D}_{21}^M - A_{11}^L z_1(t)); \end{aligned}$$

It follows from Lemma 2 that, for arbitrary small positive constant ε, there exists $T_1 > 0$ such that

$$V(t) \le \frac{r_1^M + \tilde{D}_{21}^M}{A_{11}^L} + \varepsilon, \ t > T_1.$$

(ii) If $z_2(t) > z_1(t)$ or $z_1(t) = z_2(t)$ and $z_2'(t) > z_1'(t)$, similarly we can derive

$$D^+V(t) = z_2'(t) \le z_2(t)(r_2^M + \tilde{D}_{12}^M - A_{21}^L z_2(t)).$$

By Lemma 2, for arbitrary small positive constant ε, there exists $T_2 > 0$ such that

$$V(t) \leq \frac{r_2^M + \tilde{D}_{12}^M}{A_{21}^L} + \varepsilon,\ t > T_2.$$

Set

$$M_1 = M_2 := \max\left\{\frac{r_1^M + \tilde{D}_{21}^M}{A_{11}^L} + \varepsilon, \frac{r_2^M + \tilde{D}_{12}^M}{A_{21}^L} + \varepsilon\right\}$$

$T_3 = \max\{T_1, T_2\}$, then for any $t > T_3$, one has $V(t) = \max\{z_1(t), z_2(t)\} \leq M_i, i = 1, 2$.

On the other hand, from the third equation of system (25), for $t > T_3$, $z_3'(t) \leq A_{31}^M M_1 z_3(t-\tau) - A_{32}^L z_3^2(t)$. Consider the auxiliary equation $z_3'(t) = A_{31}^M M_1 z_3(t-\tau) - A_{32}^L z_3^2(t)$, by Lemma 3, then $\lim_{t\to\infty} z_3(t) = \frac{A_{31}^M M_1}{A_{32}^L}$. Therefore,

$$z_3(t) \leq \frac{A_{31}^M M_1}{A_{32}^L} + \varepsilon := M_3.$$

Take $T = T_3$, then the conclusion of Lemma 5 follows immediately. The proof is complete. □

Theorem 2. *Suppose that* $(C_1), (C_2)$ *and* (C_6) *hold. Further,*

(C_7) $r_1^L - A_{12}^M M_3 - D_{12}^M > 0,\ r_2^L - D_{21}^M > 0,\ A_{31}^L m_1 - r^M > 0.$

Then system (1) is uniformly persistent, i.e., there exist $T^* > T$ *and* $m_i > 0$ *such that* $m_i \leq z_i(t) \leq M_i$ *for* $t > T^*, i = 1, 2, 3,$ *where* M_i *is defined by (27) and*

$$m_1 = m_2 = \min\left\{\frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{2A_{11}^M}, \frac{r_2^L - D_{21}^M}{2A_{21}^M}\right\}, m_3 = \frac{A_{31}^L m_1 - r^M}{2A_{32}^M}.$$

Proof. By Lemma 5, we only need to prove that there exist $T^* > T$ and $m_i > 0$ such that $z_i(t) \geq m_i$ for $t > T^*, i = 1, 2, 3$.

Define $V(t) = \min\{z_1(t), z_2(t)\}$. Calculating the lower right derivative of V along the positive solution of system (25), we have the following possibilities.
(i) If $z_1(t) < z_2(t)$ or $z_1(t) = z_2(t)$ and $z_1'(t) \leq z_2'(t)$, then for any $t > T$, we have

$$\begin{aligned} D_+V(t) = z_1'(t) &= z_1(t)(r_1(t) - A_{11}(t)z_1(t) - A_{12}(t)z_3(t)) + D_{21}(t)z_2(t) \\ &\prod_{0<t_k<t}(1 + b_{2k})(1 + b_{1k})^{-1} - D_{12}z_1(t)) \\ &\geq z_1(t)(r_1^L - A_{11}^M z_1(t) - A_{12}^M M_3 - D_{12}^M). \end{aligned}$$

By Lemma 2, there exists $T_1 > T > 0$ such that $z_1(t) \geq \frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{A_{11}^M} - \varepsilon$ for any $t > T_1$. It implies that there exists a positive integer N such that

$$z_1(t) \geq \frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{A_{11}^M} - N\varepsilon,\ t > T_1.$$

Noting that ε is an arbitrary small positive number, we can choose ε small enough such that

$$N\varepsilon \leq \frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{2A_{11}^M},$$

then

$$z_1(t) \geq \frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{2A_{11}^M}, \ t > T_1.$$

(ii) If $z_1(t) > z_2(t)$ or $z_1(t) = z_2(t)$ and $z_1'(t) > z_2'(t)$, similarly we can derive

$$D_+V(t) = z_2'(t) \geq z_2(t)(r_2^L - D_{21}^M - A_{21}^M z_2(t)).$$

From Lemma 2 again, there exists $T_2 > 0$, for any $t > T_2$, $z_2(t) \geq \frac{r_2^L - D_{21}^M}{A_{21}^M} - \varepsilon$. Similarly we have

$$z_2(t) \geq \frac{r_2^L - D_{21}^M}{2A_{21}^M} \text{ for any } t > T_2.$$

Let $T_3 = \max\{T_1, T_2\}, m_1 = m_2 := \min\left\{\frac{r_1^L - A_{12}^M M_3 - D_{12}^M}{2A_{11}^M}, \frac{r_2^L - D_{21}^M}{2A_{21}^M}\right\}$, then we have

$$V(t) = \min\{z_1(t), z_2(t)\} \geq m_i, \ t > T_3, i = 1, 2.$$

Further, combining the third equation of system (25), for $t > T_3 + \tau$, we have

$$z_3'(t) \geq A_{31}^L m_1 z_3(t-\tau) - r^M z_3(t) - A_{32}^M z_3^2(t).$$

By Lemma 3, we can derive from the auxiliary equation $z_3'(t) = A_{31}^L m_1 z_3(t - \tau) - r^M z_3(t) - A_{32}^M z_3^2(t)$ that

$$\limsup_{t\to\infty} z_3(t) = \frac{A_{31}^L m_1 - r^M}{A_{32}^M}.$$

By comparison theorem again,

$$z_3(t) \geq \frac{A_{31}^L m_1 - r^M}{A_{32}^M} - \varepsilon \ \text{ and } z_3(t) \geq \frac{A_{31}^L m_1 - r^M}{2A_{32}^M}.$$

Let $m_3 = \frac{A_{31}^L m_1 - r^M}{2A_{32}^M}$ and $T^* = T_3 + \tau$, then Theorem 2 follows immediately. This completes the proof. □

4. Example and simulations

In this section, an example and simulation are given to show the validity of the main results.

Example. Consider the following Lotka-Volterra model with prey dispersal and impulses.

$$
\begin{cases}
\left.\begin{aligned}
x_1'(t) &= x_1(t)(9+\cos t - 5x_1(t) - 2y(t)) + (2-\sin t)x_2(t) \\
&\quad - (3+\cos t)x_1(t), \\
x_2'(t) &= x_2(t)(5-\sin t - 8x_2(t)) + (3+\cos t)x_1(t) \\
&\quad - (2-\sin t)x_2(t), \\
y'(t) &= (2+\sin t)x_1(t-0.15)y(t-0.15) + y(t) - 10y^2(t),
\end{aligned}\right\} t \neq t_k, \\
\Delta x_1(t_k) = -1/2x_1(t_k), \\
\Delta x_2(t_k) = -1/3x_2(t_k), \\
\Delta y(t_k) = -1/4y(t_k),
\end{cases}
\tag{28}
$$

Let $t_{k+2} = t_k + 2\pi$ and $[0, 2\pi] \cap \{t_k\} = \{t_1, t_2\}$. It is easy to show that $(C_1) - (C_5)$ hold. Theorem 1 implies system (28) has at least one $2\pi-$periodic solution. By Matlab, numerical simulation indicates the existence and stability of the periodic solution(see Fig 1).

5. Conclusion

In this paper, considering the combined effects from real world, a class of nonautonomous predator-prey system with diffusion, time delays and impulsive effects is studied. By using Mawhin continuation theorem, conditions ensuring the existence of positive periodic solution for system (1) are established. Then by employing inequality analysis techniques and comparison theorem, the persistence of (1) is investigated. Theorem 1 implies that the intrinsic growth rates of the prey and predator are high, the dispersal rates, the capturing rate of the predator and the impulses are low, then system (1) has at least one positive periodic solution. Theorem 2 shows that the intrinsic growth rates of prey in two patches are high and the death rate of predator is low, then system (1) is persistent. It is reasonable and useful for the coexistence of predator and prey so as to keep ecological balance. Finally, an example and its numerical simulation by Matlab are given. Simulation further indicates that the main results are valid.

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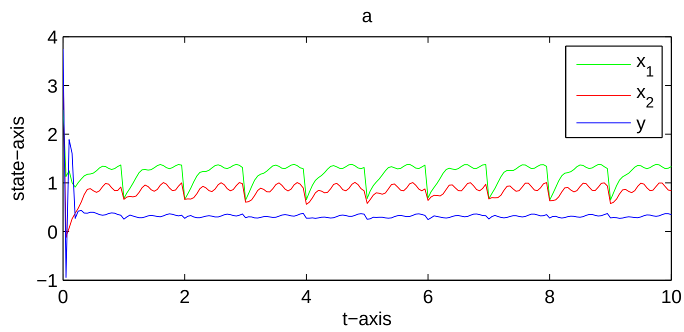
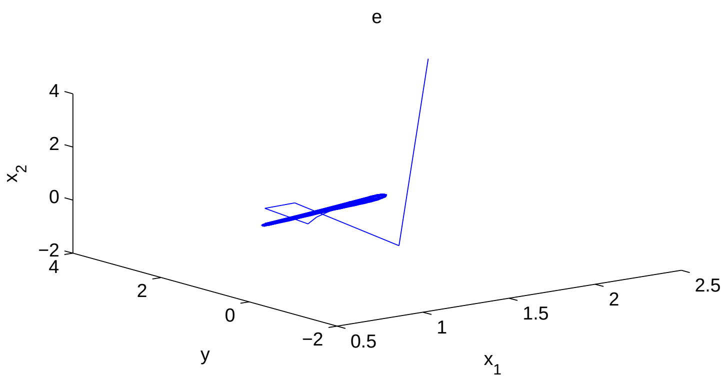


(a) time series of $(x_1(t), x_2(t), y(t))$.



(b) portrait of $(x_1(t), x_2(t), y(t))$.

FIGURE 1. Dynamics of (28).

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