Communications of the Korean Statistical Society
2012, Vol. 19, No. 3, 395–403

DOI: http://dx.doi.org/10.5351/CKSS.2012.19.3.395

Process Improvement in Feedback Adjustment

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Abstract

Process adjustment, also called engineering process control(EPC), is applied to maintain process output close to a target value by manipulating controllable variables, but special causes may still make the process deviate from the target and result in significant costs. Thus, it is important to detect and mediate deviations as early as possible. We propose a one-step detection method, the *moving search block*(MSB), with which the time and type of a special cause can be identified in short periods. A modified control rule that can entertain the effects of the special cause is proposed. A numerical example is presented to evaluate the performance of the proposed scheme.

Keywords: Responsive system, special causes, outliers, moving search block.

1. Introduction

We consider a *responsive feedback control system*(RFCS) for which all the effects of a change in a controllable variable will be realized on the output in one time interval. We assume that the underlying process follows a non-stationary process, focusing on ARIMA(1, 1, 1) model. Here, all parameters in the model are assumed known, which is usual in process adjustment. It is well known that the minimum mean squared error(MMSE) control rule minimizes output variation. However, some special causes, such as sudden changes in environmental conditions or a mistake by the operator, may still cause the process to deviate from the target additionally. Therefore, it is important to detect the occurrence of a special cause and entertain its effects as early as possible.

Special causes of different kinds (types) lead to different patterns of effects on adjusted outputs, and identifying the type could greatly narrow the set of possible root sources to be investigated. When its source is identified but cannot be eliminated (*e.g.*, some chemical processes), MMSE control rules can be modified, using the pattern of effects on outputs, so that the effects also can be properly adjusted. Many articles employed types of outliers in time series as models of special causes. In time series, types and procedures for outlier problems were researched by Tsay (1986) and Chen and Liu (1993).

It is to be noted that the problem of detecting special causes in EPC is essentially equivalent to the problem of monitoring autocorrelated processes. Many researches have dealt with the monitoring problem in the autocorrelated processes. Jiang and Tsui (2002) and Tsung and Tsui (2003) investigated the mean-shifts patterns in residuals when a level shift(LS) type outlier occurred in a stationary process, in which a Shewhart type scheme is considered. Meanwhile, CUSUM and EWMA schemes were considered by Vander Wiel *et al.* (1992), Vander Wiel (1996), and Wardell *et al.* (1994). On the other hand, Tsung *et al.* (1999) and Tsung and Apley (2002) considered a multivariate chart to

This research was supported by Inha University.

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investigate effects on the fitted values and residuals jointly. All these works mainly focused only on the performance of SPC for the LS outlier. Note that no other type is considered and effectiveness for early detection has not been seriously investigated.

It is well-known that an outlier can be detected accurately when it occurs at the middle of observed series (Chen and Liu, 1993). Meanwhile, the early detection in process adjustment means detecting a special cause occurred at or near the end of series. If existing methods are used, sufficient accuracy for early detection is in doubt. Therefore, we propose a one-step detection scheme, the MSB scheme, searching only latest few observations, which can detect special causes in short periods.

This article is organized as follows. A general framework for the RFCS is firstly introduced and mean-shifts patterns of the MMSE-controlled process are derived for two representative types of special causes. In Section 3, a detection scheme based on MSB is proposed and the performance of the proposed scheme is evaluated. In Section 4, we introduce a modified control rule that can be used when a special cause is detected but root sources of the cause cannot be eliminated. In Section 5, a numerical example is presented to illustrate the issues discussed. We conclude with some discussion.

2. Special Cause Problems in Process Adjustment

2.1. RFCS and MMSE control rule

We consider a responsive feedback control system,

$$U_t = Y_t + Z_t = gX_{t-1} + Z_t, \tag{2.1}$$

where X_t is a controllable *input variable*, Z_t is the *underlying process* which is the amount of deviation from the target when no control action is applied, Y_t is the amount of compensation on the output at time t when the input variable is set as X_{t-1} at $t-1$, and U_t is *the adjusted (or controlled) output process*. In (2.1), $Y_t = gX_{t-1}$ where g is the steady state gain. Note that a change in X_t will be realized on the output in one time interval.

In this paper, we assume that Z_t is represented by ARIMA(1, 1, 1) model,

$$(1-\phi B)(1-B)Z_t = (1-\theta B)a_t,$$

where B is the back shift operator such that $B^k Z_t = Z_{t-k}$, and $\phi \neq \theta$ (no common factor) with $|\phi| < 1$ and $|\theta| < 1$ (*i.e.*, stationary and invertible), and a_t is a white noise series with variance σ^2.

In addition, we assume that the series Z_t starts at a fixed time point t_0 with fixed initial values and innovations. This process is of particular importance in process adjustment, because if it is left unadjusted there is no guarantee that it will return to the target value in finite time periods.

The MMSE forecast of Z_t computed at time $t-1$ can be expressed as

$$Z_{t-1}(1) = \pi_1 Z_{t-1} + \pi_2 Z_{t-2} + \pi_3 Z_{t-3} + \cdots, \tag{2.2}$$

where, with $\lambda = 1-\theta$ and $\delta = \theta - \phi$, $\pi_1 = \phi + \lambda$ and $\pi_k = \delta\lambda\theta^{k-2}$ for $k \geq 2$. It is well known that the MMSE control rule at time $t-1$ is

$$X_{t-1} = -\frac{Z_{t-1}(1)}{g}, \tag{2.3}$$

and from which the adjusted output becomes MMSE forecast error, that is a white noise, as

$$U_t = e_{t-1}(1) = a_t. \tag{2.4}$$

Table 1: Mean-Shifts in Adjusted Outputs at $t(>T)$: $\lambda = 1-\theta$ and $\delta = \theta - \phi$

time	Types of special cause	
	AO	LS
$t = T$	ω	ω
$t = T+1$	$-\omega(\phi+\lambda)$	$\omega\delta$
$t = T+k$	$-\omega\delta\lambda\theta^{k-2}$	$\omega\delta\theta^{k-1}$

2.2. Effects of a special cause on adjusted outputs

Process adjustment employs a system of statistical forecasting and thus special causes may lead to carry-over-effects on X_t as well as U_t. Hu and Roan (1996) and Tsung and Tsui (2003) investigated mean-shift patterns in U_t for stationary Z_t. In this study, we consider two types of special causes: additive outlier(AO) and level shift(LS) which are spot and persistent mean changes, respectively. Atienza *et al.* (1998) presented such cases of special causes in real situations. We investigate their effects on U_t.

A contaminated underlying process, when a special cause occurs, can be modeled by

$$N_t = f(t) + Z_t = \omega\xi(B)I_t(T) + Z_t, \tag{2.5}$$

where T, ω, and $\xi(B) = 1 + \xi_1 B + \xi_2 B^2 + \cdots$ are the occurrence time, the size of level change, and the type of a special cause, respectively. In (2.5), $f(t)$ represents the effect of a special cause on Z_t, and $I_t(T) = 1$ if $t = T$ and 0 otherwise signifies the pulse indicator at time T. Two types of special causes can be specified as $\xi(B) = 1$ for AO type and $\xi(B) = 1/(1-B)$ for LS type (Tsay, 1986; Chen and Liu, 1993).

If a special cause occurred at time T, N_t instead of Z_t will be observed and applied to (2.3). Since $\xi(B)I_t(T) = \xi_{t-T}$, the forecast computed at $t-1$ can be expressed in terms of $Z_{t-1}(1)$ as

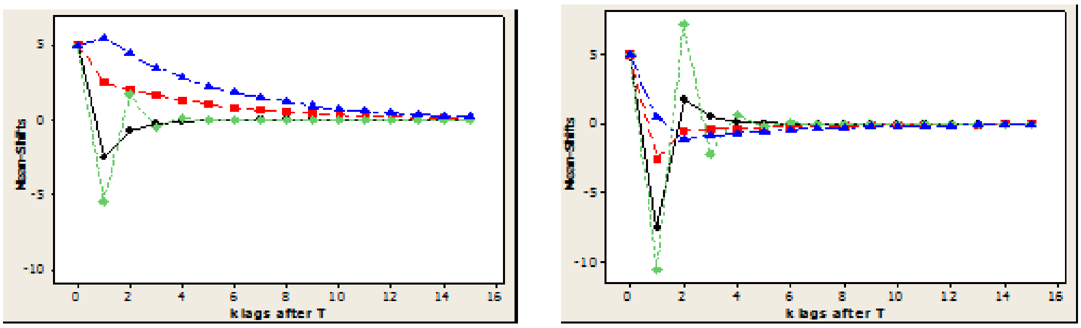
$$\widehat{Z}_{t-1}(1) = Z_{t-1}(1) + \omega \sum_{i=1}^{t-T} \pi_i \xi_{t-T-i},$$

where $\xi_0 = 1$ and $\xi_j = 0$ for $j < 0$. From now on, the hat '$\widehat{\ }$' above any character signifies the use of contaminated series N_t instead of Z_t when computing such statistics. The input variable given in (2.3) will then be manipulated by $\widehat{X}_{t-1} = -\widehat{Z}_{t-1}(1)/g$, and adjusted outputs can be written as

$$\widehat{U}_t = N_t + g\widehat{X}_{t-1} = U_t + \omega\left(\xi_{t-T} - \sum_{i=1}^{t-T} \pi_i \xi_{t-T-i}\right). \tag{2.6}$$

In (2.6), the second term is the mean-shifts in adjusted outputs, denoted by $\mu_t = E[\widehat{U}_t]$. Specifically, the mean shift in adjusted output at each time point can be summarized as given in Table 1.

It is evident that, different models and cause types may produce distinct patterns of mean-shifts, which will vanish eventually for all types since $\theta^k \to 0$ as k increases. Consider four ARIMA$(1,1,1)$ models: $(\phi, \theta) = (0.8, 0.3), (0.3, 0.8), (0.8, -0.3)$, and $(-0.3, 0.8)$. It is noted that, from Figure 1, the effects of AO type can be greater than that of LS type, as shown in models with parameters $(\phi, \theta) = (0.8, 0.3)$ and $(0.8, -0.3)$. From Table 1, mean-shifts are $-\omega\delta\lambda\theta^{k-2}$ for AO type and $\omega\delta\theta^{k-1}$ for LS type, and thus the effects of AO can be greater than that of LS when $|\lambda| = |1-\theta| > |\theta|$, *i.e.*, $|\theta| < 0.5$. Thus, though AO type may not be the one greatly concerned for SPC, it can deteriorate the performance of process adjustment more than LS.



(a) LS Type Cause

(b) AO Type Cause

Figure 1: *Mean-Shift levels in adjusted Outputs due to (a) LS Type and (b) AO Type Cause:* $(\phi, \theta) = (0.8, 0.3)$*, solid;* $(0.3, 0.8)$*, dash;* $(0.8, -0.3)$*, dot;* $(-0.3, 0.8)$*,dash 1-dot*

3. A Scheme for Early Detection

3.1. The moving search block

It is well known that the Equation (2.6) can be expressed as a simple linear regression model, and thus the detection statistic for testing occurrence of a special cause of type $\xi(B)$ at time T can be written as

$$\lambda_\xi(T) = \frac{\tau_\xi(T)\widehat{\omega}_\xi(T)}{\sigma} = \sum_{t=1}^{n} \frac{b_t(T)\widehat{U}_t}{\tau_\xi(T)\sigma}, \tag{3.1}$$

where $\widehat{\omega}_\xi(T) = \sum_{t=1}^{n} b_t(T)\widehat{U}_t/\tau_\xi^2(T)$ with $b_t(T) = \xi_{t-T} - \sum_{i=1}^{t-T} \pi_i \xi_{t-T-i}$ and $\tau_\xi^2(T) = \sum_{t=1}^{n} b_t^2(T)$. Here, $\sigma_a/\tau_\xi(T)$ is the standard deviation of $\widehat{\omega}_\xi(T)$ and, when no outlier is present, The $\lambda_\xi(T)$ in (3.1) follows a standard normal distribution. In practice, the time and type are determined as follows (Tsay, 1986):

(1) Compute the maximum of detection statistics over all times ($1 \leq T \leq n$) and types (AO, LS),

(2) Decide the time and type if the maximum exceeds a pre-specified criterion (C).

Early detection in process adjustment is conceptually equivalent to outlier detection when it occurred near the end of series. Chen and Liu (1993) and Wright *et al.* (2001) claimed that an outlier may be detected accurately at two time periods after the occurrence. In their studies, they employed a relatively small detection criterion ($C = 2.0$) compared with the more customary $C = 3.5$. It is to be noted that the criterion $C = 2.0$ would be useful for early detection, but it would lead to a high probability of type I error, *i.e.*, wrong detection when no outlier is present.

We propose a modified detection scheme, the *MSB scheme*, in which only time periods in the MSB will be searched as possible occurrence times. The size of MSB will be denoted by m and the last period of the MSB, n, is the time of process adjustment, called *control origin*. Thus, if the MSB scheme can detect the time and type at small k (*i.e.*, number of periods after occurrence), where $n = T + k$, the goal of early detection is accomplished.

3.2. Design of MSB

Key factors in MSB scheme are the size of a block (m) and a decision criterion (C). In order to determine C, we generated outlier-free series of size 100 for 342 ARIMA(1, 1, 1) models, where

Table 2: Probability of False Signal: Mean(Standard Deviation)

m	C		
	2.0	2.25	2.5
4	.058(.004)	.036(.004)	.019(.004)
6	.053(.005)	.035(.004)	.019(.004)
8	.058(.005)	.034(.004)	.019(.005)
10	.052(.05)	.033(.004)	.019(.05)

parameters ϕ and θ range from -0.9 to $+0.9$ with increment of 0.1, respectively. For each model, 2,000 series are generated. In this analysis, the cases of $\phi = \theta$ are excluded because those models have a common factor.

The estimate of ω in (3.1) for time point t as possible occurrence time, computed at control origin $n = t + k$, can be specifically written as,

$$\widehat{\omega}_{AO}(t) = \frac{\left(\widehat{U}_t - \pi_1 \widehat{U}_{t+1} - \cdots - \pi_k \widehat{U}_n\right)}{\tau_{AO}},$$
$$\widehat{\omega}_{LS}(t) = \frac{\left(\widehat{U}_t + (1-\pi_1)\widehat{U}_{t+1} + \cdots + (1-\pi_1-\cdots-\pi_k)\widehat{U}_n\right)}{\tau_{LS}},$$

where $\tau_{AO}^2 = 1 + \pi_1^2 + \cdots + \pi_k^2$ and $\tau_{LS}^2 = 1 + (1-\pi_1)^2 + \cdots + (1-\pi_1-\cdots-\pi_k)^2$. Note that both $\widehat{\omega}$ statistics depend only on observations after the time points t (*i.e.*, $\widehat{U}_j$, $j \geq t$.) Therefore, we do not need to search all time points, rather enough to search within the range of MSB with a smaller criterion.

As possible criteria for a MSB scheme, we focused on the three critical values suggested by Chen and Liu (1993), C= 2.0, 2.25, and 2.50. The probabilities of false signals for outlier-free series are computed for k=2, 4, 6, and 10, respectively, and given in Table 2. We note that overall probabilities of false signals are slightly over 0.05 for $C = 2.0$, about 0.035 for $C = 2.25$, and about 0.02 for $C = 2.50$ with small standard deviations. Though $C = 2.0$ was suggested by Chen and Liu (1993), we choose $C = 2.25$ in this study.

It is well known that when an outlier occurred at the middle of a series, called *middle case*, the standard procedures such as Tsay (1986) detect the outlier accurately. Therefore, in this simulation study to detect an outlier that occurs at or near the end of a series, we determine a proper MSB size that provides almost the same performance as that of middle case. To determine the detection power for the middle case, contaminated series by AO and LS are generated with $T = 50$ and $\omega = 4.0\sigma$ for each model, respectively, and the detection procedure has been applied in the last period, $n = 100$. The detection accuracy is computed with the criterion $C = 2.25$.

In determining the proper size of MSB, m, we generated contaminated series by imposing the effects of AO and LS at time $T = 90$ with $\omega = 4\sigma$ to pure series, respectively. The starting time was fixed at $t = 88$ so that two time points without outlier effect are included in the MSB, therefore, the last time point in MSB is $m - 3$, denoting k, periods after occurrence. We now focus on k instead of m for early detection problem. For each model (ϕ, θ), the detection procedure is provided below the Equation (3.1) is applied to observations of time points in MSB for $k = m - 3 = 2, 3, \ldots$ until the performance is close enough to that of middle case. The simulation study is summarized as follows:

Step 1: for each type (AO, LS), determine the smallest $k\,(\geq 2)$ satisfying detection power at $k \geq$ detection power at the middle case $-\epsilon$

Step 2: determine the proper k as the maximum of k's (of both types) of Step 1.

		+								phi			-							
		.9	.8	.7	.6	.5	.4	.3	.2	.1	0	.1	.2	.3	.4	.5	.6	.7	.8	.9
+	.9	X	2	2	2	3	2	2	2	2	2	2	2	2	2	6	3	3	3	3
	.8	2	X	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3
	.7	2	2	X	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3
	.6	2	2	2	X	2	2	2	2	2	2	2	2	2	2	2	2	2	3	2
	.5	2	2	2	2	X	2	2	2	2	2	2	2	2	2	2	2	2	2	3
	.4	2	2	2	2	2	X	2	2	2	2	2	2	2	2	2	2	2	2	2
	.3	2	2	2	2	2	2	X	2	2	2	2	2	2	2	2	2	2	2	2
	.2	2	2	2	2	2	2	2	X	2	2	2	2	2	2	2	2	2	2	2
theta	.1	2	2	2	2	2	2	2	2	X	2	2	2	2	2	2	2	2	2	2
	0	2	2	2	2	2	2	2	2	2	X	2	2	2	2	2	2	2	2	2
	.1	2	2	2	2	2	2	2	2	2	2	X	2	2	2	2	2	2	2	2
-	.2	2	2	2	2	5	2	2	2	2	2	2	X	2	2	2	2	2	2	2
	.3	2	2	2	2	5	5	3	2	2	2	2	2	X	2	2	2	2	2	2
	.4	2	2	2	2	5	6	5	3	2	2	2	2	2	X	2	2	2	2	2
	.5	3	3	3	3	5	6	6	5	2	2	2	2	2	2	X	2	2	2	2
	.6	4	3	3	3	5	5	5	5	5	3	2	2	2	2	2	X	2	2	2
	.7	4	3	4	4	5	5	5	5	5	5	2	2	3	2	2	2	X	2	2
	.8	5	4	5	5	5	6	6	6	5	6	6	5	5	2	2	2	2	X	2
	.9	5	5	5	5	6	6	6	5	6	6	6	5	5	3	3	2	2	2	X

Figure 2: *Proper k to each model for early detection*

In this study, $\epsilon = 0.02$ is used as a criterion for closeness in detection accuracy for $k = 2, 3, \ldots, 5$, while $\epsilon = 0.04$ is considered for $k = 6$ to simplify the procedure. The results are summarized in Figure 2. Note that at two periods after occurrence ($k = 2$), a special cause can be detected accurately for the majority of the models as claimed by Chen and Liu (1993) and Wright *et al.* (2001), but larger k may be needed for models with large $|\theta|$. That is, for some model, a special cause can be detected at 3-6 periods after occurrence. Note that the size of MSB is $m = k + 3$.

4. Process Adjustment when Special Causes are Detected

For some chemical processes, the source of detected cause cannot be eliminated. We introduce a method for adjusting the effects of a special cause, which can be applied to such situations. Assume that a special cause is detected correctly at control origin n, then the contaminated process at $t = n + 1$ can be written as

$$N_t = Z_t + \omega\xi(B)I_t(T) = Z_t + \omega\xi_{t-T}.$$

Pretending that Z_t for $t \geq T$ are observable, a control rule (input) set at time $n = t-1$ as

$$X_{t-1} = -\frac{Z_{t-1}(1) + \omega\xi_{t-T}}{g} \tag{4.1}$$

will adjust the effects of the cause at t, $\omega\xi_{t-T}$, and thus produce the adjusted output as shown in (4.2), which is the MMSE control procedure applied to a contaminated series, where

$$U_t = N_t + gX_{t-1} = e_{t-1}(1). \tag{4.2}$$

In practice, for $t \geq T$, Z_t are unobservable because they are contaminated by a special cause. If detected time and type are denoted by $(\widehat{T}, \widehat{\xi}(B))$, Z_t can be estimated by

$$\widetilde{Z}_t = \widehat{U}_t - g\widehat{X}_{t-1} - \widehat{\omega}(\widehat{T})\widehat{\xi}_{t-T}, \tag{4.3}$$

where $\widehat{\omega}(\widehat{T})$ is the estimated magnitude of the cause computed at the control origin n. Here, ' $\sim$ ' above any character will signify that it is computed using $\widetilde{Z}$ instead of Z. Substituting $\widetilde{Z}_t$ in place of $Z_t(1)$ in Equation (4.1) for periods $t \geq T$, the input variable at $t(\geq n)$ can be manipulated by $\widetilde{X}_t = -(\widetilde{Z}_t(1) + \widehat{\omega}\widehat{\xi}_{t+1-\widehat{T}})/g$. By applying $\widetilde{X}_t$ to the Equation (4.2), where $\widetilde{e}_t(1) = Z_{t+1} - \widetilde{Z}_t(1)$, adjusted outputs becomes $\widetilde{U}_{t+1} = \widetilde{e}_t(1) + (\omega\xi_{t+1-T} - \widehat{\omega}\widehat{\xi}_{t+1-\widehat{T}})$. Note that, if the detected time and type are correct, $\widehat{\omega}$ is the only source of additional variation having $\mathrm{var}(\widehat{\omega}_\xi) = \sigma^2/\tau_\xi^2$.

5. An Illustrative Example

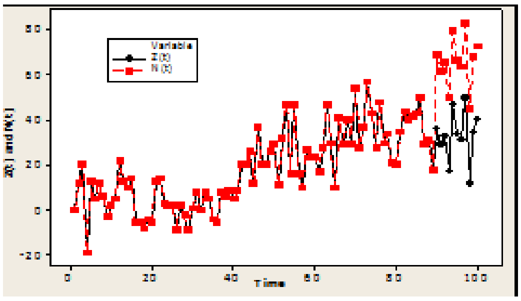
We apply the proposed MSB scheme to the data of thickness measurements of metallic film, which has been analyzed by Box and Luceno (1997). The data set of size 100 is considered as an underlying process Z_t, which is the deviation of the characteristic from its target value 80 when no adjustment is made. They analyzed that Z follows IMA(1, 1) model with $\theta = 0.8$ and $\sigma = 11.0$, as given in (5.1). In their analysis, deposition rate X is a controllable variable, and a unit change in X will produce $g = 1.2$ units of change in Z_t taking full effect in one time interval.

$$Z_t = Z_{t-1} + a_t - 0.8a_{t-1}, \quad \sigma^2 = 11.0^2. \tag{5.1}$$

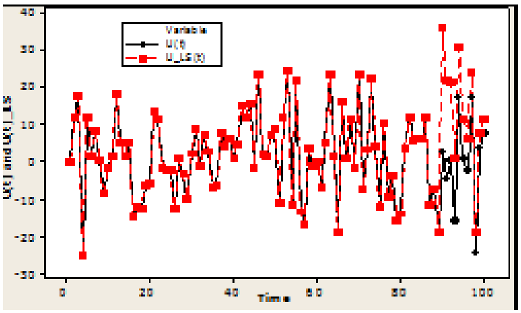
To evaluate the performance of the MSB scheme, level changes(LS) of size $3\sigma = 33.0$ are added to the underlying process Z_t for $90 \leq t \leq 100$ to get a contaminated series N_t. The pure and contaminated time series are plotted in Figure 3(a). If control rule (2.3) is applied to N_t, the variability of adjusted output series is affected by LS, as shown in Figure 3(b). Compared with (solid line) produced from Z_t, $\widehat{U}_t$(dash line) affected by an LS shows larger deviations for $90 \leq t \leq 100$ for relatively long periods, though the effects decreases as time passes.

In order to apply MSB scheme, $k = 6$ (*i.e.*, $m = 9$) is selected from Figure 2 for model with $(\phi, \theta) = (0.0, 0.8)$. The MSB scheme is applied to the series N_t and results for detection at each control origin for $n = 88, 89, \ldots, 100$ are summarized in Table 3. At $t = 90$, the values of two statistics are the same and exceeds $C = 2.25$, and AO type is identified though actually unidentifiable. At $t = 91$ and thereafter, a special cause is detected as $\widehat{T} = 90$ and $\widehat{\xi}(B) = \mathrm{LS}$. We conclude that a cause of LS type occurred at $T = 90$.

Using the results of MSB scheme, the modified adjustment method can be applied to mediate the cause effects. Since a special cause with $T = 90$ and $\widehat{\xi}(B) = \mathrm{LS}$ is detected, ω_{LS} is estimated as $\widehat{\omega}_{LS}(90) = 32.50$. Then, pure underlying series for $90 \leq t \leq 100$ are estimated by $\widetilde{Z}_t$ using the Equation (4.3). Applying $\widetilde{Z}_t$, the adjusted outputs $\widetilde{U}_t$ are obtained recursively. The root mean squares error(RMSE) is obtained as 33.57 for series $\widehat{U}_t$ and 12.68 for series $\widetilde{U}_t$. Modified adjustment scheme leads to about 62% reduction in RMSE.



(a) LS Type Cause



(b) AO Type Cause

Figure 3: *Time series of underlying and adjusted output processes: (a) Z_t(solid line) and N_t(dash line), (b) U_t(solid line) and $\widehat{U}_t$(dash line)*

Table 3: Results from MSB Scheme: (a) Detected Time and Type, (b) Computed Detection Statistics at Each Control Origin

	n	88	89	90	91	92	93	94
(a)	Time	86	87	90	90	90	90	90
	Type	NO	NO	AO	LS	LS	LS	LS
(b)	LS	1.322	1.841	3.278	3.801	4.273	4.059	4.652
	AO	1.361	1.698	3.278	2.825	2.808	2.805	3.016

6. Summary and Concluding Remarks

We considered a responsive feedback control system, in which the underlying process follows a non-stationary ARIMA(1, 1, 1) process with known parameters. It is well known that MMSE control rules minimize output process variation but a special cause may results in large deviations on adjusted outputs, possibly for long periods. We employed two types of outliers, AO and LS, as models of special causes and showed that the effects of AO type may exceed that of LS type.

We introduced a modified scheme that detects a special cause in short periods, namely MSB scheme, with sufficient accuracy. The proper size of MSB, for each model, is determined by a simulation study, with which special causes can be detected as efficiently as a standard procedure which has a suspicious Type I error probability. In addition to that, we proposed a modified control rule that can be applied to the case when a special cause is detected but its sources cannot be eliminated. The data of thickness measurements of metallic film has been analyzed by the proposed scheme. A reduction of 62% in RMSE has been achieved, showing the effectiveness of the proposed scheme.

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Received February 10, 2012; Revised March 27, 2012; Accepted March 28, 2012