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자율형 무인 수상정 및 잠수정의 군집 주행을 위한 제어기 설계

이재용
삼성중공업

Controller Design to Coordinate Autonomous Unmanned Surface and Underwater Vehicles

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KEY WORDS: Unmanned surface vehicle (USV), Unmanned underwater vehicle, Lyapunov stability, Formation control

ABSTRACT: *In this paper, addressed is the control problem of generating a formation for a group of unmanned surface and underwater vehicles. The formation control scheme proposed in this work is based on a fusion of theleader-follower and virtual reference approaches. This scheme gives a formation constraint representation that is independent of the number of vehicles in the formation and the resulting control algorithm is scalable. One of the most important features in controller design is the ability of the controller to globally and exponentially stabilize the formation errors defined by the formation constraints. The proposed controller is based on feedback linearization, and the formation errors are shown to be globally and exponentially stable in the sense of Lyapunov.*

1. 서 론

Since the master plan was first officially released by US Navy in 2007 (US Navy, 2007), Unmanned Surface Vehicles (USVs) have received much attention by underwater robot researchers. At the same time, there have been significant trials to apply multiple number of Unmanned Underwater Vehicles to patrol, surveil, and observe targets or underwater environments (Kemp et. al., 2004). Since multi-vehicle systems can overcome physical limitations of single vehicle capabilities, they are superior to single vehicle systems when they are deployed in cooperative tasks such as tactical oceanography (Bellingham et. al., 1998), harbor protection (Makrinos, 2004), and possibly many other problems.

In order to accomplish multiple tasks, they need to be allocated to each vehicle in the multi-vehicle system. This process is the first step for such a system to fulfill a task. Next, data are exchanged through communication between neighboring vehicles. The final step is to coordinate the vehicles in the system to reach their desired positions. The task allocation problem is also known as the assignment problem. Each task can be assigned to the proper vehicle by linear optimization (Kingston and Schumacher, 2005) or by a distributed algorithm with low ratio bounds (Shehory and Kraus 1998). One of the key issues in information flow is to determine the effect of communication failures and delays between vehicles on formation stability and the effect is determined by using the eigenvalues of the graph Laplacian matrix (Fax and Murray, 2004).

After all tasks are allocated effectively and efficiently, and each vehicle has the required data for group tasks, a control strategy is required for the collaboration of the team of vehicles. The following has been considered in the literature in the control design step: maintaining geometric relationship between vehicles, dynamic constraints, and avoiding inter vehicle collision. In this dissertation we are interested in the formation generating problem.

The formation generating problem is similar to formation keeping except, that formation errors are stabilized globally instead of locally. The key issue in formation generating is the design of a control law for each agent such that all vehicles fall within a preassigned formation. Designing such a control law for each agent requires the reference position for each of the vehicles. Some approaches to deciding the reference position in the literature assume that the reference trajectory for agents is known a-priori rather than computed in real-time. This approach may not be suitable when the shape and the motion of the formation are dependent on the group objectives or tasks.

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Designing reference position in real-time is typically specified roughly by three methods in the literatures: leader-follower (Das et. al., 2002), neighbor reference (Yamaguchi et. al., 2001), and virtual reference (Balch and Arkin, 1998). Designing reference position in real-time is typically specified roughly by three methods in the literatures: leader-follower (Das et. al., 2002), neighbor reference (Yamaguchi et. al., 2001), and virtual reference (Balch and Arkin, 1998).

The main goals of this paper is to develop control laws to specify the motion of multi-vehicle systems to achieve cooperative tasks. The work is aimed at achieving the goals by completing the following objectives. 1) Develop a scalable scheme for generating a formation, 2) Design controllers to stabilize formation errors globally. These features are achieved by introducing virtual vehicles in the proposed scheme. The virtual vehicles not only make the formation constraints meet the desired features but also lend itself to formation guidance.

2. PRELIMINARIES

2.1 Definitions

Formation control has been researched for decades. In the 1990's, formation control was researched via the virtual structure concept. In early 2000's, attention was shifted to distributed algorithms. For example, a group of vehicles come to a common location, which has come to be known as the rendezvous problem (Lin et. al., 2003; Jadbabaie et. al., 2003). However, it is hard to find the definition of a formation. Also, the meaning of a formation is different from one to another. In order to avoid such misconceptions, a definition of a formation is required before further discussion.

Definition 1 (Formation) Let us assume that N rigid body vehicles (vehicles) are assigned to get into a certain formation. Let the mass center of the i^{th}vehicles be denoted by position vector $r_i(t)$. Let $[\ ]_A$ denote the coordinates of a vector in a Frame A. Then, the set $F=\{[r_1(t)]_A,[r_2(t)]_A,...,[r_N(t)]_A\}$ at a given time t is said to be the formation at time t.

By this definition, only positions of vehicles affect the configuration of a formation. Namely, the individual orientations of vehicles in a formation has no effect on the configuration of the formation.

Definition 2 (Rigid Formation) *A* formation *F* is a rigid formation if there exist a positive-definite orthogonal matrix $R(t)$ and a vector $d(t)$ at time t such that $[r_i(t)]_A = R(t)[\boldsymbol{r_i}(t_0)]_A + [\boldsymbol{d}(t)]_A$ for $i=1,2,...,N$ and $t\in[t_0,t_f]$.

Remark 1 If $R(t)$ is a negative definite orthogonal matirx, then the shape of the formation will be flipped.

Remark 2 The distance $d_{i,j}=|r_i(t)-r_j(t)|$ will be maintained for time $t\in[t_0,t_f]$ if the formation is rigid.

Definition 3 (Virtual Structure) A rigid formation may be assumed to be a single virtual rigid body, and this rigid body is called a virtual structure (*VS*).

In general, constructing a certain formation means forming a rigid formation from a non-rigid formation. If all the vehicles are assumed to be rigid bodies in n dimensions, a formation composed of N vehicles has nN degrees of freedom (dof)s. Since a flipped shape may not be allowed in transformation by translation and rotation, the number of dof of a rigid formation is the same as that of a rigid body. Therefore, a rigid formation composed of N vehicles has only $n(n+1)/2$ dofs. In other words, at least nN-$n(n+1)/2$ constraints are required for a rigid formation composed of N vehicles. Let us call these constraints formation constraints.

Definition 4 (Formation Constraints) $Q_j([r_1^*(t)]_I,[r_2^*(t)]_I,...,[r_N^*(t)]_I)=0$ are formation constraints for $j=1,...,k(\geq nN-\frac{n(n+1)}{2})\in\mathrm{R}$, and $Q_j=0$ are sufficient for the formation $F=\{[r_1(t)]_I,[r_2(t)]_I,...,[r_N(t)]_I\}$ to be a rigid formation, where $[r_i^*(t)]_I$ is the desired position of i^{th} vehicle in a rigid formation.

When there is an error between the position of i^{th} vehicle and the desired position of i^{th} vehicle in a formation, we call it a formation error.

2.2 Problem Statement

Suppose N vehicles are assigned to construct a specified rigid formation $F=\{[r_1(t)]_I,[r_2(t)]_I,...,[r_N(t)]_I\}$, and suppose that the motion of each vehicle is governed by the following state equations;

$$\dot{z}_i = f_i(z_i)+g_i(z_i)u_i \qquad (1)$$
$$r = h_i(z_i)$$

where $i=1,2,...N, z_i\in\mathrm{R}^n$ is the state of the i^{th} vehicles, $u_i\in\mathrm{R}^p$ is the control for the i^{th} vehicle, r_i is the position vector of the i^{th} vehicle with respect to an inertial frame. The goal is to design the controller, $u_i(t)$ which satisfies $\lim_{t\to\infty}\sum_i^N\|r_i^*-r_i\|=0$. Each vehicle is assumed to be able to calculate its global position $[r(t)]_I$, is able to determine the desired position in a specified rigid formation $[r^*(t)]_I$ at any time.

2.3 Approach for Scalability

Defining a desired position of each vehicle is the key to constructing a formation. Now we introduce a new approach which is based on a blend of the virtual reference approach and the leader-follower approach. *A* local frame *B* is assumed

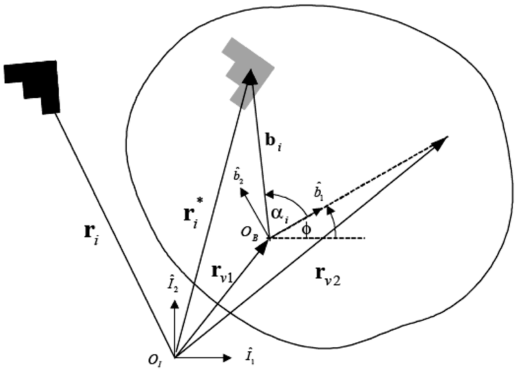

Fig. 1 Configuration of formation in R^2

fixed to the *VS* with its origin O_B. b_i is the position vector of the i^{th} vehicle with respect to O_B. r_i is the position vector of the i^{th} vehicle with respect to an inertial frame *I*, and $\mathbf{r}_i^*$ is the position vector of the desired position of i^{th} vehicle with respect to an inertial frame *I*. Let $[\ ]_B$ denote the coordinates of a vector in the local frame B and $[\ ]_I$ denote the coordinates of a vector in an inertial frame *I*. The position of each of the vehicles can be specified by b_i, which is shown in Fig. 1.

3. ERROR ANALYSIS

Let us start with a definition for a function $\angle$.

Definition 5 (Function $\angle$)

$$\angle\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)=\begin{cases} \arctan(y/x), & \text{for } x>0 \\ \arctan(y/x)+\pi, & \text{for } y\geq 0, x<0 \\ \arctan(y/x)-\pi, & \text{for } y<0, x<0 \\ \pi/2, & \text{for } y>0, x=0 \\ -\pi/2 & \text{for } y<0, x=0 \\ \text{undefined} & \text{for } y=0, x=0 \end{cases}$$

Formation constraints in R^2 can be suggested as following,

$$Q_i=\begin{cases} \|r_i^*-r_{i+1}^*\|^2-d_{i,i+1}^2=0, \text{ for } i=1,\ldots,N-1 \\ \|r_{i-N+1}^*-r_{i-N+3}^*\|^2-d_{i-n+1,i-N+3}^2=0, \text{ for } i=N,\ldots,2N-3 \end{cases} \tag{2}$$

where $d_{a,b}$ is the desired distance between the a^{th} vehicleand the b^{th} vehicle, and the desired position of the i^{th} vehicle is denoted by $\mathbf{r}_i^*$. These 2*N*-3 constraints are not enough to specify one shape of formation although the number of cons

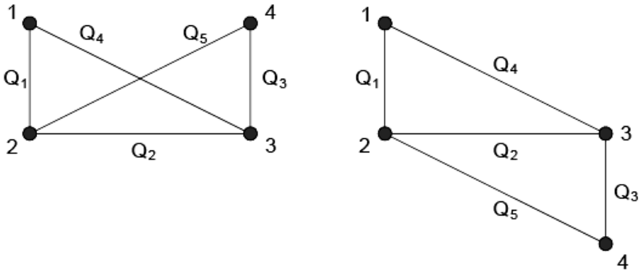

Fig. 2 Non-sufficient formation constraint

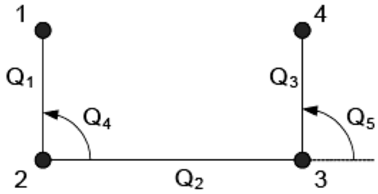

Fig. 3 Sufficient formation constraints

traints satisfies requirements of formationconstraints. Fig. 2 and Fig. 3 shows possible rigid formations with the constraint represented by Eq. (2).

The representation of formation constraints in Eq. (2) can be replaced by the following equation;

$$Q_i=\begin{cases} \|r_i^*-r_{i+1}^*\|^2-d_{i,i+1}^2=0, \text{ for } i=1,\ldots,N-1 \\ \angle(\|r_{i-N+1}^*-r_{i-N+2}^*)_I-\angle(\|r_{i-N+3}^*-r_{i-N+2}^*)_I \\ \quad -\alpha_{i-N+1,i-N+3}=0, \text{ for } i=N,\ldots,2N-3 \end{cases} \tag{3}$$

where $\alpha_{a,a+2}$ is the desired angle between the two vectors, $\mathbf{r}_a^*-\mathbf{r}_{a+1}^*$ and $\mathbf{r}_{a+2}^*-\mathbf{r}_{a+1}^*$. These 2*N*-3 constraints shown in Fig. 3 are sufficient to specify the formation, however, each vehicle is connected to each other in sequence through intermediate vehicles. Namely, the constraint Q_k is not independent of the constraint Q_{k-1}. Consequently, this formation is not scalable. Another idea for realization of formation constraints is given in (Egerstedt and Hu, 2001). Here the formation constraints were defined through a single function. However, this representation depends on the number of vehicles. Therefore, we need to find a new representation of formation constraints which satisfies the following conditions;

- ○ The number of formation constraints should be a multiple of *N* when the formation comprises *N* actual vehicles.
- ○ Each formation constraint should be represented by the configuration of only one vehicle.

By introducing virtual leaders, formation constraints can be made independent of each other, thereby making the formalism for formation generation scalable in the number of vehicles. Another advantage of introducing virtual leaders is that the behavior of the specified rigid formation can be directed by just the behavior of virtual leaders. One can choose new constraints for the formation, which contains two virtual leaders VL_1, VL_2 as follows

$$Q_i=\begin{bmatrix} \|r_i^*-r_{v1}\|-d_i^2 \\ \angle([r_i^*-r_{v1}]_I-\angle([r_{v2}-r_{v1}]_I)-\alpha_i \end{bmatrix}=\begin{bmatrix} q_{i,l} \\ q_{i,\theta} \end{bmatrix}=0, \tag{4}$$

where $\mathbf{r}_{v1}$ and $\mathbf{r}_{v1}$ denote the position vectors of VL_1, VL_2 with respect to the O_I in Fig. 1. respectively. Hence this representation of formation constraints satisfies the two required conditions discussed above. The errors in formation are defined by substituting r_i for $\mathbf{r}_i^*$ in the formation constraints as follows:

$$E_i = \begin{bmatrix} \| r_i - r_{v1} \| - d_i^2 \\ \angle([r_i - r_{v1}]_I - \angle([r_{v2} - r_{v1}]_I) - \alpha_i \end{bmatrix} = \begin{bmatrix} e_{i,l} \\ e_{i,\theta} \end{bmatrix} = 0. \quad (5)$$

4. CONTROLLER DESIGN

Sliding mode control has the feature that it is robust to small disturbances, although the control law suffers from chattering (Khalil, 2002). In (Barth, 2006), the formation error is stabilized asymptotically even when the target changes its moving direction suddenly. However, the algorithm there cannot be expanded to more than two vehicles. Developed is a scheme which is applicable to $N(\geq 1)$ vehicles and which is also scalable.

4.1 2D Dynamic Model of USV

For a USV, a unicycle model shown in Fig. 4 for each vehicle is considered. This is because USVs are supposed to be operated on the surface of the water. However, in most cases, marine vehicles are supposed to be modeled underactuated systems. Although the constraints are different inside unicycle and underactuated UUV system, the control inputs are the same: the forward and yaw speeds (Xiang et. al., 2009). The control law in kinematic level can be exteneded to deal with vehicle dynamics by using backstepping techniques. Therefore, the control design on unicycle can be extended to the early stage of the underactuated system control design.

For a unycycle model, each vehicle has to satisfy the nonholonomic constraint $v_i \cos\theta_i - v_i \sin\theta_i = 0$. Following the representation of Eq. (1), $z_i = [x_i, y_i, \theta_i, v_i]^T$, $f_i(z_i) = [v_i \cos\theta_i, v_i \sin\theta_i, 0, 0]^T$, $g_i(\mathrm{z}_i) = \begin{bmatrix} 0,0,1,0 \\ 0,0,0,1 \end{bmatrix}^T$, $h_i(z_i) = [x_i, y_i]^T$, $\mathrm{u}_i = [\omega_i, u_i]^T$, $\mathrm{y}_i = [x_i, y_i]^T$. Namely, the following form represents the dynamics of the i^{th} vehicle.

$$\begin{cases} \dot{x}_i = v_i \cos\theta_i \\ \dot{y}_i = v_i \sin\theta_i \\ \dot{\theta}_i = \omega_i \\ \dot{v}_i = u_i \end{cases} \quad (6)$$

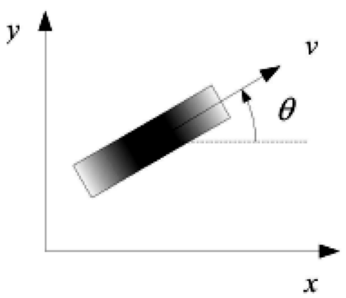

Fig. 4 Unicycle model

If a rigid formation is assigned for N vehicles, the desired position of the i^{th} vehicle in *VS* is specified by α_i and d_i in Eq. (4). can establish a controller which stabilizes these errors using a suitable Lyapunov function. Let us define $s_{i,l}$ and $s_{i,\theta}$ as following;

$$s_{i,l} = \left(\frac{\gamma_{i,l} + \lambda_{i,l}}{\gamma_{i,l}\lambda_{i,l}} + \frac{1 + \gamma_{i,l}\lambda_{i,l}}{(\gamma_{i,l} + \lambda_{i,l})} \right) e_{i,l}^2 + \frac{2}{\gamma_{i,l}\lambda_{i,l}} e_{i,l}\dot{e}_{i,l} + \left(\frac{1 + \gamma_{i,l}\lambda_{i,l}}{\gamma_{i,l}\lambda_{i,l}(\gamma_{i,l} + \lambda_{i,l})} \right) \dot{e}_{i,l}^2, \quad (7)$$

$$s_{i,\theta} = \left(\frac{\gamma_{i,\theta} + \lambda_{i,\theta}}{\gamma_{i,\theta}\lambda_{i,\theta}} + \frac{1 + \gamma_{i,\theta}\lambda_{i,\theta}}{(\gamma_{i,\theta} + \lambda_{i,\theta})} \right) e_{i,\theta}^2 + \frac{2}{\gamma_{i,\theta}\lambda_{i,\theta}} e_{i,\theta}\dot{e}_{i,\theta} + \left(\frac{1 + \gamma_{i,\theta}\lambda_{i,\theta}}{\gamma_{i,\theta}\lambda_{i,\theta}(\gamma_{i,\theta} + \lambda_{i,\theta})} \right) \dot{e}_{i,\theta}^2,$$

where $\lambda_{i,l} > 0$, $\gamma_{i,l} > 0$, $\lambda_{i,\theta} > 0$, and $\gamma_{i,\theta} > 0$. Consider the following Lyapunov function candidate

$$V = \frac{1}{2} \sum_1^N (s_{i,l} + s_{i,\theta}). \quad (8)$$

If u_i satisfies $\ddot{e}_{i,l} = -(\gamma_{i,l} + \lambda_{i,l})\dot{e}_{i,l}^2 - \gamma_{i,l}\lambda_{i,l}e_{i,l}$ and $\ddot{e}_{i,\theta} = -(\gamma_{i,\theta} + \lambda_{i,\theta})\dot{e}_{i,\theta}^2 - \gamma_{i,\theta}\lambda_{i,\theta}e_{i,\theta}$, the derivative of Lyapunov function $\dot{V}$ will be $-\sum_i^N (\dot{e}_{i,l}^2 + e_{i,l}^2 + \dot{e}_{i,\theta}^2 + e_{i,\theta}^2) \leq 0$. Since $(\dot{e}_{i,l}^2, e_{i,l}^2, \dot{e}_{i,\theta}^2, e_{i,\theta}^2) = (0,0,0,0)$ is the largest invariant set in $\{\mathrm{x} \mid \dot{\mathrm{V}}(\mathrm{x}) = 0\}$, the errors will be asymptotically stabilized by the LaSalle's theorem. The control inputs appear in the second derivatives of the errors, and each control law for the actual vehicles can be calculated by solving two linear equations as follows;

$$\ddot{e}_{i,l} = D_i + B_i u_i + C_i \omega_i$$
$$\ddot{e}_{i,\theta} = DD_i + BB_i u_i + CC_i \omega_i$$

where

$$D_i = 2\dot{\tilde{x}}_i^2 + 2\dot{\tilde{y}}_i^2 - 2\tilde{x}_i \ddot{x}_{v1} - 2\tilde{y}_i \ddot{y}_{v1},$$
$$B_i = 2\tilde{x}_i \cos\theta_\mathrm{i} + 2\tilde{\mathrm{y}}_\mathrm{i} \sin\theta_\mathrm{i},$$
$$C_i = -2\tilde{x}_i \dot{y}_i \pm 2\tilde{x}_i y_i,$$

and

$$DD_i = -\frac{\tilde{x}_i \ddot{y}_{v1} - \ddot{x}_{v1}\tilde{y}_i}{\tilde{x}_i^2 + \tilde{y}_i^2} - \frac{\tilde{x}_{v2}\ddot{y}_{v2} - \ddot{x}_{v2}\tilde{y}_{v2}}{\tilde{x}_{v2}^2 + \tilde{y}_{v2}^2} - \frac{(\tilde{x}_i \dot{\tilde{y}}_i - \dot{\tilde{x}}_i \tilde{y}_i)(2\tilde{x}_i \dot{\tilde{x}}_i - \tilde{y}_i \dot{\tilde{y}}_i)}{(\tilde{x}_i^2 + \tilde{y}_i^2)^2} + \frac{(\tilde{x}_{v2}\dot{\tilde{y}}_{v2} - \dot{\tilde{x}}_{v2}\tilde{y}_{v2})(2\tilde{x}_{v2}\dot{\tilde{x}}_{v2} - \tilde{y}_{v2}\dot{\tilde{y}}_{v2})}{(\tilde{x}_{v2}^2 + \tilde{y}_{v2}^2)^2},$$

$$BB_i = \tilde{x}_i \cos\theta_\mathrm{i}$$

$$CC_i = \frac{\tilde{x}_i^2 \dot{x}_i + \tilde{y}_i^2 \dot{y}_i}{\tilde{x}_i^2 + \tilde{y}_i^2},$$ with $\tilde{x}_i = x_i - x_{v1}$, $\tilde{y}_i = y_i - y_{v1}$, $\tilde{x}_{v2} = x_{v2} - x_{v1}$, $\tilde{y}_{v2} = y_{v2} - y_{v1}$.

Therefore, the control law u_i is determined as follows;

$$\begin{bmatrix} C_i & B_i \\ CC_i & BB_i \end{bmatrix} \mathrm{u_i} = \begin{bmatrix} F_i \\ G_i \end{bmatrix} \tag{9}$$

where

$$\begin{aligned} F_i &= -\lambda_{i,l}\dot{e}_{i,l} - \gamma_{i,l}(\dot{e}_{i,l} + \lambda_{i,l}e_{i,l}) - A_i \\ G_i &= -\lambda_{i,\theta}\dot{e}_{i,\theta} - \gamma_{i,\theta}(\dot{e}_{i,\theta} + \lambda_{i,\theta}e_{i,\theta}) - A_i \end{aligned} \tag{10}$$

The paths of the two virtual leaders VL_1, VL_2 coordinates the motion of the VS, and the control law u_i forces the vehicles to remain in formation.

4.2 3D Dynamic Model of UUV

Now, discussed is designing a controller for UUVs which stabilizes the formation errors in R^3. Simple kinematic model for vehicle in R^3 can be represented by Eq. (1) where $\mathrm{z_i} = [x_i, y_i, z_i, \theta_i, \alpha_i, v_i]^T$, $f_i(\mathrm{z}_i) = [v_i\cos\theta_i\cos\alpha_i, v_i\sin\theta_i\cos\alpha_i, v_i\sin\alpha_i, 0,0,0]^T$, $g_i(\mathrm{z}_i) = [g_{i,1}, g_{i,2}, g_{i,3}]$, $g_{i,1} = [0,0,0,1,0,0]^T$, $g_{i,2} = [0,0,0,0,1,0]^T$, $g_{i,3} = [0,0,0,0,0,1]^T$, $h_i = [h_{i,1}, h_{i,2}, h_{i,3}]^T = [x_i - x_i^*, y_i - y_i^*, z_i - z_i^*]^T$, and $\mathrm{u}_i = [w_i, \tau_i, u_i]^T$. Namely, the following form represents the dynamics of each vehicle.

$$\begin{cases} \dot{x}_i = v_i\cos\theta_i\cos\phi_i \\ \dot{y}_i = v_i\sin\theta_i\cos\phi_i \\ \dot{z}_i = v_i\sin\alpha_i \\ \dot{\theta}_i = w_i \\ \dot{\phi}_i = \tau_i \\ \dot{v}_i = u_i \end{cases} \tag{11}$$

Formation Constraints can be defined by $Q_i = [r_i^*]_I - [\mathrm{b_i}] = 0$ and formation errors are represented by $\mathrm{E_i} = [r_i]_I - [r_i^*]_I$. Let us define the error vector for the i^{th} subsystem in the following form;

$$\mathrm{E}_i = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ \bar{z}_i \end{bmatrix} = \begin{bmatrix} x_i - x_i^* \\ y_i - y_i^* \\ z_i - z_i^* \end{bmatrix} \tag{12}$$

Let y_i be E_i, and let us define $\gamma_{i,j}$ to be the relative degree for the j^{th} output of the i^{th} subsystem. We can see that all the relative degrees of outputs are 2 in Eq. (12). Namely, $\gamma_{i,1} = 2$, $\gamma_{i,2} = 2$, and $\gamma_{i,3} = 2$. The total relative degree is then,

$$\gamma_i = \gamma_{i,1} + \gamma_{i,2} + \gamma_{i,3} = 6 = n\,. \tag{13}$$

Therefore no internal dynamics appear in the normal form.

$$\begin{aligned} \dot{\bar{x}}_i &= L_{fi}h_{i,1} + L_{gi,1}h_{i,1}\omega_i + Lg_{i,2}h_{i,1}\tau_i + L_{gi,3}h_{i,1}u_i = v_i\cos\theta_i\cos\phi_i - \dot{x}_i^* \\ \ddot{\bar{x}}_i &= L_{fi}^2 h_{i,1} + L_{gi,1}L_{fi}h_{i,1}\omega_i + L_{gi,2}L_{fi}h_{i,1}\tau_i + L_{gi,3}L_{fi}h_{i,1}u_i \\ &= \ddot{x}_i^* - v_i\sin\theta_i\cos\phi_i\omega_i - v_i\cos\theta_i\sin\phi_i\tau_I + \cos\theta_i\cos\phi_i u_i \end{aligned} \tag{14}$$

$$\begin{aligned} \dot{\bar{y}}_i &= L_{fi}h_{i,2} + L_{gi,1}h_{i,2}\omega_i + L_{gi,2}h_{i,2}\tau_i + L_{gi,3}h_{i,2}u_i = v_i\sin\theta_i\cos\phi_i - \dot{y}_i^* \\ \ddot{\bar{y}}_i &= L_{fi}^2 h_{i,2} + L_{gi,1}L_{fi}h_{i,2}\omega_i + L_{gi,2}L_{fi}h_{i,2}\tau_i + L_{gi,3}L_{fi}h_{i,2}u_i \\ &= -\ddot{y}_i^* + v_i\cos\theta_i\cos\phi_i\omega_i - v_i\sin\theta_i\sin\phi_i\tau_i + \sin\theta_i\cos\phi_i u_i \end{aligned} \tag{15}$$

$$\begin{aligned} \dot{\bar{z}}_i &= L_{fi}h_{i,3} + L_{gi,1}h_{i,3}\omega_i + L_{gi,2}h_{i,3}\tau_i + L_{gi,3}h_{i,3}u_i = -v_1\sin\phi_i - \dot{z}_i^* \\ \ddot{\bar{z}}_i &= L_{fi}^2 h_{i,3} + L_{gi,1}L_{fi}h_{i,3}\omega_i + L_{gi,2}L_{fi}h_{i,3}\tau_i + L_{gi,3}L_{fi}h_{i,3}u_i \\ &= -\ddot{z}_i^* + v_i\cos\phi_i\tau_i - \sin\phi_i u_i \end{aligned} \tag{16}$$

The error dynamics of the i^{th} subsystem can be expressed in the following form;

$$\begin{aligned} \ddot{\mathrm{E}}_i &= -\ddot{\mathrm{r}}_i^* + \mathrm{A}_i(\mathrm{z}_i)\mathrm{u}_i, \\ \ddot{\mathrm{u}}_i &= \mathrm{A}_i(\mathrm{z}_i)^{-1}(\nu_i + \ddot{\mathrm{r}}_i^*), \end{aligned} \tag{17}$$

where

$$\mathrm{A}_i(\mathrm{z}_i) = \begin{bmatrix} L_{gi,1}L_{fi}h_{i,1} & L_{gi,2}L_{fi}h_{i,1} & L_{gi,3}L_{fi}h_{i,1} \\ L_{gi,1}L_{fi}h_{i,2} & L_{gi,2}L_{fi}h_{i,2} & L_{gi,3}L_{fi}h_{i,2} \\ L_{gi,1}L_{fi}h_{i,3} & L_{gi,2}L_{fi}h_{i,3} & L_{gi,3}L_{fi}h_{i,3} \end{bmatrix} = \begin{bmatrix} -\mathrm{v_i}\sin\theta_i\cos\phi_i & -v_i\cos\theta_i\sin\phi_i & \cos\theta_i\cos\phi_i \\ v_i\cos\theta_i\cos\phi_i & -v_i\sin\theta_i\sin\phi_i & \sin\theta_i\cos\phi_i \\ 0 & v_i\cos\phi_i & -\sin\phi_i \end{bmatrix}, \tag{18}$$

and $\ddot{\mathrm{r}}_i^* = [\ddot{x}_i^* \ddot{y}_i^* \ddot{z}_i^*]^T$.

Let us suppose that ν_i satisfies the following form,

$$\begin{aligned} \nu_i &= \mathrm{J}_i\dot{\mathrm{E}}_i + \mathrm{K}_i(\dot{\mathrm{E}}_i - \mathrm{J}_i\mathrm{E}_i) \\ \mathrm{E} &= [\mathrm{E}_1^T, \dot{\mathrm{E}}_1^T, \mathrm{E}_2^T, \dot{\mathrm{E}}_2^T, \ldots, \mathrm{E}_n^T, \dot{\mathrm{E}}_n^T]^T \in \mathrm{R}^{2Nn}. \end{aligned} \tag{19}$$

Lyapunov function candidate can be defined by $V(\mathrm{E}) = \sum_i^N X_i^T \mathrm{P}_i X_i$, where $X_i = [\mathrm{E}_i^T, \dot{\mathrm{E}}_i^T]^T$ and P_i is a symmetric positive definite matrix in $R^{2n\times 2n}$. By Eq. (17) and Eq. (19), X_i should satisfy the following condition:

$$\dot{X}_i = \begin{bmatrix} 0 & \mathrm{I} \\ -\mathrm{K}_i\mathrm{J}_i & \mathrm{J}_i + \mathrm{K}_i \end{bmatrix} X_i = \mathrm{H}_i X_i$$

The derivative of the Lyapunov function is then $\dot{V}(\mathrm{E}) = \sum_i^N X_i^T . (H_i^T \mathrm{P}_i + P_i H_i) X_i$ Suppose $\mathrm{P}_i P_i$ is the solution of $\mathrm{H}_i^T \mathrm{P}_i + \mathrm{P}_i \mathrm{H}_i = -\mathrm{I}$. Then $\dot{V}(\mathrm{E}) = -\sum_i^N X_i^T X_i = -\sum_i^N \|X_i\|^2 = -\|\mathrm{E}\|^2 \le 0$. Since the Lyapunov function satisfies the following inequalities $\min\{\mathrm{eig}(\mathrm{P}_i)\}\|\mathrm{E}\|^2 \le V(\mathrm{E}) \le \max\{\mathrm{eig}(\mathrm{P}_i)\}\|\mathrm{E}\|^2$, $\frac{\partial V}{\partial t} + \frac{\partial V}{\partial \mathrm{E}}\dot{\mathrm{E}} \le -\|\mathrm{E}\|^2$, and the formation errors are stabilized exponentially and globally which follows from theorem 4.10 in [27]. Therefore, the controller u_i is represented as

$$\mathrm{u}_i = \mathrm{A}_i(\mathrm{z}_i)^{-1}\{(\mathrm{J}_i + \mathrm{K}_i)\dot{\mathrm{E}}_i - \mathrm{K}_i\mathrm{J}_i\mathrm{E}_i + \ddot{\mathrm{r}}_i^*\} \tag{20}$$

5. SIMULATION RESULT AND ANALYSIS

Fig. 5 depicts a scenario where six USVs maneuver to construct a hexagonal formation starting from arbitrary initial conditions by employing the controllers ui proposed in Eq. (9). The initial conditions for actual vehicles and the virtual vehicles used in simulation are given by Table 1. The desired hexagonal rigid formation was specified by the values in Table 2. The hexagonal rigid formation was steered by two virtual vehicles VL_1 and VL_2 which were shown by red and blue

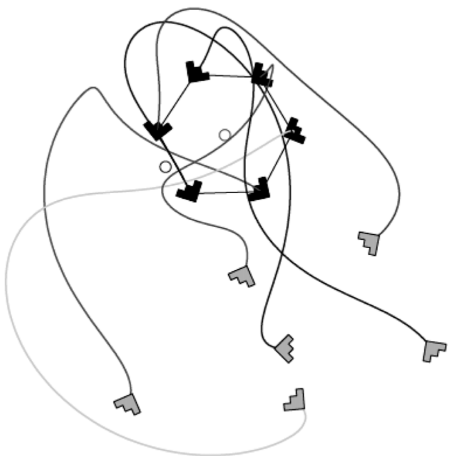

Fig. 5 Forming hexagonal formation. Gray shows initial configuration of vehicles and black shows accomplished formation

Table 1 Initial Conditions

States	i=1	i=2	i=3	i=4	i=5	i=6	$i=u_1$	$i=u_2$
$x_i(m)$	10	-13	12	4	32	23	-10	0
$y_i(m)$	-48	-56	-56	-37	-48	-32	-20	-20
$\theta_i(rad)$	$-\pi$	1.2	-0.7	1.2	2.2	0.6	$\pi/2$	$\pi/2$
$u_i(m/\text{sec})$	10	10	10	10	10	10	1	2

Table 2 Specification of Formation

States	i=1	i=2	i=3	i=4	i=5	i=6
$d_i(m)$	10	10	10	10	10	10
$\alpha_i(rad)$	$\pi/6$	$\pi/2$	$5\pi/6$	$-5\pi/6$	$-\pi/2$	$-\pi/2$

circles in Fig. 5.

Since these two virtual vehicles maintain constant distance d_v, the following equation is always true.

$$\|[\mathrm{r}_{v2}-\mathrm{r}_{v1}]_I\| = d_v \tag{21}$$

In the simulation, models of virtual vehicles were taken to be the same as that of the actual vehicles. However, it is not necessary that the models of virtual vehicles be the same as that of actual vehicles. The first virtual vehicle was located at O_B, and the second virtual vehicle at $[d_v, 0]$ in the frame B. The controls for the first virtual vehicle were chosen to be $u_{v1}=0$, $\omega_{v1}=-0.2\sin(t/10)$, and the control for the second virtual vehicle was chosen to be $u_{v2}=0$. However, these values can be chosen arbitrarily. The angular velocity of the second virtual vehicle was determine d_v from Eq. (21). Also, Fig. 6 shows that all errors are stabilized exponentially by the controllers u_i in Eq. (9). The i^{th} row in Fig. 6 shows the errors between $[\mathrm{r}_i]_I$ and $[\mathrm{r}_i^*]_I$. The convergence rate was determined by ,$\lambda_{i,l}$, $\lambda_{i,\theta}$, $\gamma_{i,l}$ and $\lambda_{i,\theta}$, and these values were chosen to be $\lambda_{i,l}=0.5$, $\lambda_{i,\theta}=0.5$, $\gamma_{i,l}=0.4$ and $\lambda_{i,\theta}=0.4$,

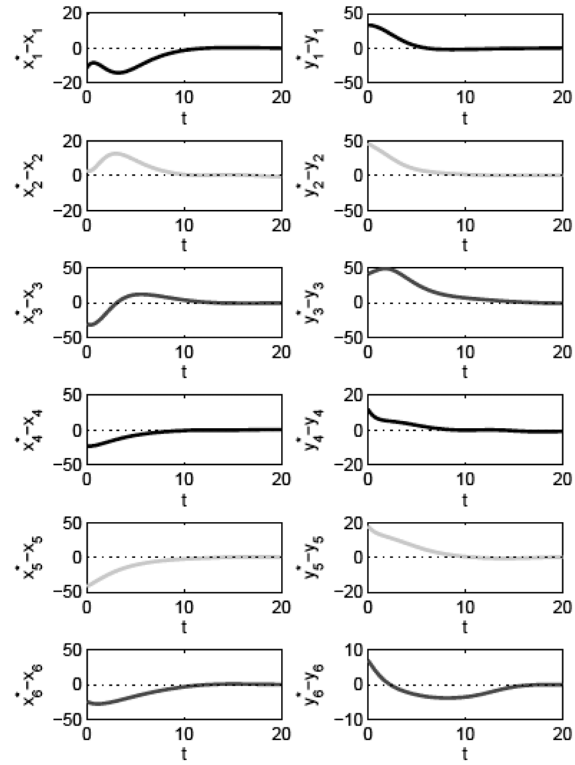

Fig. 6 All errors are stabilized exponentially by the proposed controller

Note that the matrix $P=\begin{bmatrix} C_i & B_i \\ CC_i & BB_i \end{bmatrix}$ can be singular when the i^{th} vehicle coincides with the virtual leaders or when the i^{th} vehicle is stationary. While the former case can be avoided by choosing $\alpha_i \neq 0$, $d_i > 0$, in the latter case, no smooth time-invariant control law can stabilize the error.

6. CONCLUSION

A scalable scheme was proposed for rigid formation construction by a novel representation of formation constraints. This representation is independent of the number of vehicles in a formation and the resulting control algorithm is scalable. The proposed approach is based on a fusion of leader-foll ower and virtual reference approach. The group behavior is directed by specifying the behavior of virtual vehicles in the proposed method. Also desired positions of all the vehicles in a rigid formation can be expressed by the position of virtual vehicles since the virtual structure constructed by virtual vehicles behaves like a rigid body. However, inter-collision avoidance was not considered in the proposed scheme. Collision avoidance may be handled by the potential field approach or the homotopy approach based on homotopy of polynomials. A challenging issue is how to utilize these approaches in the proposed scheme without loss of scalability. The potential field approach and homotopy approach require all the position of vehicles in order that a trajectory of one vehicle is determined. This conflicts with the condition of formation constraints for scalability algorithm. Future research should include a scalable algorithm considering inter-collision avoidance. This work will be valuable when the algorithm is applied to a large number of vehicle systems. Formation errors are exponentially and globally stabilized by the proposed control laws. To represent more realistic model for AUV systems, nonholonomic models will be replaced with underactuated systems, and vehicles dynamics including environmantal forces and disturbance rejection will be considered.

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