

# Improved Model of the Iron Loss for the Permanent Magnet Synchronous Motors

Ikram Junaid \*, Khan Nasrullah \*, and Byung-il Kwon \*\*

**Abstract** –This paper presents an improved iron loss model, for the computation of the no load iron loss in the stator core of the in-wheel permanent magnet synchronous motors (PMSM), for the cases of with and without stator skew. 2-D analytical model is used for the computation of tooth and yoke flux densities of the in-wheel PMSM. The no load iron loss computed by the improved iron loss model, for the cases of with and without skew is compared with the finite element method (FEM) and the results show good consistency.

**Keywords:** Iron Loss, Permanent magnet motors, Finite element method, Stator skewing, 2-D analytical model, Slot pitch

## 1. Introduction

A permanent magnet (PM) machine has many advantages over other machines like high efficiency and smaller size. Accurate prediction of the iron loss is required for the higher efficiency and design optimization [1]. The Computation of iron losses in the permanent magnet synchronous motor (PMSM) can be achieved by using finite element method (FEM) and by using analytical methods. However FEM is more accurate but it takes long computation time. Selmon et al. [1] calculated the iron loss by using theoretically derived formulas and by using the correction factors. The correction factors which were used in [1], calculated by FEM takes long computation time. Waseem Roshan [2] proposed the iron loss modeling by removing the correction factors. However both [1, 2] had calculated the iron loss by using the flux densities computed by the FEM. Both [1, 2] do not consider the effect of stator skew on the iron loss. Also both [1, 2] do not consider the effect of minor hysteresis loops on the iron loss. Deng et al. [3] calculated the iron loss for the cases of with and without stator skew. The eddy current loss and excess eddy current loss in tooth calculated by [3] considered rise time of flux density equal to one tooth width for the transversal of magnet. But the loss calculated by

considering tooth width effect is almost twice the loss calculated by FEM [4].

This paper presents an improved iron loss model for the calculation of iron loss. The improved iron loss model can calculate skew and non skew model iron loss by considering minor hysteresis loops. In addition, to obtain the tooth eddy current loss and excess eddy current loss, slot pitch effect is also considered.

The improved iron loss model is based on [3], [6] for the computation of minor hysteresis loop loss effect on iron loss. The improved iron loss model calculates iron loss in tooth for the skew and non skew model by considering rise time of flux density as one slot pitch for the transversal of magnet. The iron loss computed by improved model is compared with the 2-D FEM results for the case of no skew and with 3D-FEM in the case of stator skew.

## 2. Conventional Iron Loss Models

In general there are three components of iron loss, which occurs in a magnetic material. These loss components are the hysteresis loss, eddy current loss and excess eddy current loss. The hysteresis loss density, eddy current loss density and excess eddy current loss density are generally expressed as follows:

$$P_h = K_h B^\beta \omega_s \quad (1)$$

$$P_e = K_e B_m^2 \omega_s^2 = 2K_e \left( \frac{\Delta B}{\Delta t} \right)^2 = \frac{d^2}{3\pi\rho} \frac{1}{T} \int_0^T \left( \frac{\Delta B}{\Delta t} \right)^2 \Delta t \quad (2)$$

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$$P_{exc} = K_{exc} B_m^{1.5} \omega_s^{1.5} = 2^{\frac{3}{4}} K_{exc} \left( \frac{\Delta B}{\Delta t} \right)^{\frac{3}{2}} \quad (3)$$

$$= \sqrt{\frac{A\alpha n_0}{\rho}} 2^{\frac{3}{4}} \frac{1}{T} \int_0^T \left( \frac{\Delta B}{\Delta t} \right)^2 \Delta t$$

where  $P_h$ ,  $P_e$  and  $P_{exc}$  are the hysteresis loss density, eddy current loss density and excess eddy current loss density,  $K_h$  is the hysteresis loss constant,  $B$  is flux density,  $\beta$  is the Steinmetz constant and  $\omega_s$  is the angular frequency,  $K_e$  is the eddy current loss density and  $B_m$  is the maximum value of the flux density,  $\rho$  is the resistivity of the material,  $d$  is the lamination thickness and  $T$  is time period,  $\alpha$  is a numerical constant,  $A$  is the area of cross section and  $n_0$  is statistical distribution of coercive field.

Deng evaluated the iron losses in a surface mounted permanent magnet in-wheel PMSM using a similar procedure derived by Slemon and Liu [3]. In addition to their approach Deng model considered the excess losses and also the effect of skew on iron losses. The model also considered the harmonics effect on hysteresis losses. The expression developed by Deng is as follows:

$$P_t = K_{ch} K_h B_m^\alpha f + \frac{K_e}{2\pi^2} \left( \frac{\Delta B}{\Delta t} \right)_{rms}^2 + \frac{K_{exc}}{(2\pi^2)^{0.75}} \left( \frac{\Delta B}{\Delta t} \right)_{rms}^{1.5} \quad (4)$$

This expression was evaluated further by putting the values of  $\frac{\Delta B}{\Delta t}$ , for case of no skew and with skew, respectively as follows:

$$P_t = \left\{ K_{ch} K_h B_m^\alpha f + \frac{4f^2 B_{tm}^2 K_e}{\alpha_u \pi} + \left( \frac{4K_{exc}}{\alpha_u \pi} \right)^{\frac{3}{4}} f^{1.5} B_{tm}^{1.5} \right\} w_t + \left\{ K_{ch} K_h B_m^\alpha f + \frac{8f^2 B_{ym}^2 K_e}{\beta_m \pi} + \left( \frac{8K_{exc}}{\alpha_u \pi} \right)^{\frac{3}{4}} f^{1.5} B_{ym}^{1.5} \right\} w_y \quad (5)$$

$$P_t = \left\{ K_{ch} K_h B_m^\alpha f + \frac{4f^2 B_{tm}^2 K_e}{(\alpha_u + \sigma_t) \pi} + \left( \frac{4K_{exc}}{(\alpha_u + \sigma_t) \pi} \right)^{\frac{3}{4}} f^{1.5} B_{tm}^{1.5} \right\} w_t + \left\{ K_{ch} K_h B_m^\alpha f + \frac{8f^2 B_{ym}^2 K_e}{\beta_m \pi} + \left( \frac{8K_{exc}}{\alpha_u \pi} \right)^{\frac{3}{4}} f^{1.5} B_{ym}^{1.5} \right\} w_y \quad (6)$$

where  $f$  is the frequency,  $\alpha$  is the constant determined by the manufacturer provided loss data,  $K_{exc}$  is the excess eddy current loss constant,  $\alpha_u$  is the effective arc in electrical radian,  $\beta_m$  is pole width in electrical radian,  $\sigma_t$  is the skew angle in electrical radian,  $w_t$  is the total tooth weight,  $w_y$  is the total yoke weight,  $B_m$  is the maximum value of flux density in teeth and  $B_{ym}$  is the maximum flux density in yoke.

The above model considered that the flux density is not pure trapezoidal in the core of the in-wheel PMSM but it has round corners at the edges. To consider this effect the above model has taken effective tooth width. The loss calculated by this model when tooth was skewed considered the effect of tooth width.

Waseem Roshan presented a model to calculate the no load core losses in the PM motor [2]. In his study he considered the effect of excess eddy current loss as substitute for the correction factors which were used in the model developed by [1] for the computation of iron loss. This model also considered the rise time of tooth flux density as one slot pitch for the transversal of magnet. Also this model expressed the eddy current loss constant and excess eddy current loss constant, in the form of resistivity and lamination thickness. The expressions proposed by Waseem Roshan are as follows:

$$P_h = K_h B_{th}^\beta \omega_s + K_h B_{cy}^\beta \omega_s \quad (7)$$

$$P_e = \frac{d^2}{3\rho\pi^3} m q B_{th}^2 \omega_s^2 + \frac{d^2}{3\rho\alpha'\pi^3} B_{cy}^2 \omega_s^2 + \frac{cd^2}{6\rho\pi^3} q B_{cy}^2 \omega_s^2 \quad (8)$$

$$P_{ex} = \sqrt{\frac{A\alpha n_0}{\rho}} \frac{\sqrt{mq}}{(2\pi)^{\frac{3}{2}}} B_{th}^{\frac{3}{2}} \omega_s^{\frac{3}{2}} + \sqrt{\frac{8A\alpha n_0}{\rho\alpha'}} \frac{d^2}{3\rho\alpha'\pi^3} B_{cy}^{\frac{3}{2}} \omega_s^{\frac{3}{2}} + D \sqrt{\frac{A\alpha n_0}{\rho}} B_{cy}^{\frac{3}{2}} \omega_s^{\frac{3}{2}} \quad (9)$$

where  $B_{th}$  is the tooth flux density,  $B_{cy}$  is the circumferential component of yoke flux density,  $\rho$  is the resistivity of the material,  $d$  is the lamination thickness,  $A$  is the area of cross section of lamination,  $\alpha$  is the numerical constant,  $n_0$  is the statistical distribution of coercive field,  $m$  is the no of phases,  $q$  is the no of slot per pole per phase,  $\alpha'$  is the magnet coverage,  $C$  is constant of integration for eddy current loss and  $D$  is the constant of integration for excess eddy current loss.

The model did not consider the effect of minor hysteresis loops loss also the above model cannot predict the effect of stator skew on the iron loss.

### 3. Improved Iron Loss Model

This paper proposes an improved model for the prediction of iron loss of the in-wheel PMSM. The improved iron loss model does not contain any correction factor, as were used by [1] for the calculation of eddy current loss. The improved iron loss model is based on [2] for the calculation of the eddy current loss and the excess eddy current loss in the in-wheel PMSM, by modifying its derivations. The improved iron loss model calculates the hysteresis loss, by considering the effect of minor hysteresis loops on the hysteresis loss, due to non sinusoidal flux density, by using [3]. The improved iron loss model also calculates the iron loss for the case of skewed stator tooth, by modifying [3].

In the improved iron loss model all main iron loss components, i.e. hysteresis loss, eddy current loss and excess eddy current loss will be considered for the calculation of the iron loss.

For the improved iron loss model the flux density will be decomposed into two orthogonal components: radial and circumferential components, to evaluate hysteresis, eddy current and excess eddy current loss densities in the tooth and yoke.

For a  $p$ -pole machine, the time period,  $T$ , related to the machine rotating at a speed,  $\omega_{mech}$ , can be expressed as follows:

$$T = \frac{1}{f} = \frac{2\pi}{\omega_s} = \frac{2\pi}{\frac{p\omega_{mech}}{2}} = \frac{4\pi}{p\omega_{mech}} \quad (10)$$

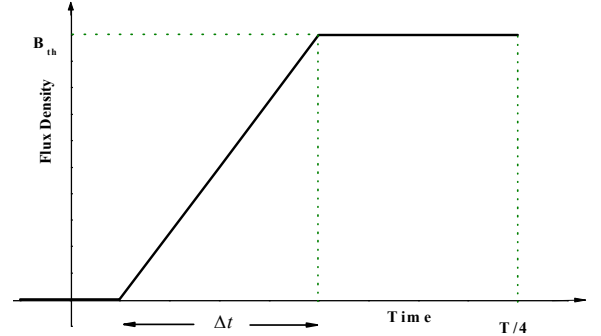
where  $p$  is the number of poles and  $\omega_{mech}$  is the mechanical speed in rad/sec.

#### 3.1 Tooth Loss Model

In general there are two components of flux density: radial component and the circumferential component. The radial component of flux density is the predominant component of flux density in tooth and circumferential component of flux density is negligible in tooth of the in-wheel PMSM [1], [2]. Therefore only the radial component of flux density will be considered here in the improved iron loss. As the study of this paper will also show, it is adequate to consider only radial component in order to obtain good agreement with FEM results.

**Eddy Current Loss:** The tooth flux density waveform studied by [1], [2] will be used in the improved iron loss model. Generally this waveform is piece wise linear. In

particular the waveform for the tooth flux density is trapezoidal as shown in Fig. 1.



**Fig. 1.** Tooth flux density waveform

For an  $m$ -phase machine, with “ $q$ ” slots per pole phase, the time required for the magnet to transverse one slot pitch is as follows:

$$\Delta t = \frac{T}{2mq} \quad (11)$$

For the waveform shown in Fig. 1, the time rate of change of flux density is given by as follows:

$$\frac{dB}{dt} = \frac{B_{th}}{\Delta t} \quad (12)$$

where  $B_{th}$  is the maximum value of radial component tooth flux density and  $\Delta t$  is the time to transverse one slot pitch by the magnet.

This change in flux density occurs four times in a given time period time  $T$ . Therefore, the average eddy current loss density in tooth by using (12) in (2) can be expressed as follows:

$$P_{ct} = \frac{d^2}{3\pi\rho} \left( \frac{B_{th}}{\Delta t} \right)^2 \left( \frac{4\Delta t}{T} \right) \quad (13)$$

Using (11) in (13) yields

$$P_{ct} = \frac{2d^2}{3\pi\rho^3} mq B_{th}^2 \omega_s^2 \quad (14)$$

**Excess Eddy Current Loss:** The change in flux density for excess eddy current loss will be as given (12). This change in flux density occurs four times in one time period. Therefore using (12) in (3) excess eddy current can be expressed as follows:

$$P_{\text{ext}} = \left( \frac{4\Delta t}{T} \right) \sqrt{\frac{A\alpha n_0}{\rho}} \left( \frac{B_{th}}{\Delta t} \right)^{\frac{3}{2}} \quad (15)$$

Using (11) in (15) yields

$$P_{\text{ext}} = 4 \sqrt{\frac{A\alpha n_0}{\rho}} \frac{\sqrt{2mq}}{(2\pi)^{\frac{3}{2}}} B_{th}^{\frac{3}{2}} \omega_s^{\frac{3}{2}} \quad (16)$$

The main differences between the formulas for eddy current loss and excess eddy current loss are: 1) both the frequency and flux density exponents are 2 for eddy current loss and 1.5 for excess eddy current loss and 2) eddy current loss is linearly dependent on slot per pole phase,  $q$ , while excess eddy current loss is square root dependent on slot per pole per phase. Also note the difference between the model proposed by the Waseem Roshan and improved iron loss model are: 1) in improved iron loss model for eddy current loss a factor of 2 is multiplied and 2) for excess eddy current loss a factor of 4 is multiplied with the formula derived for eddy current loss and excess eddy current loss by Waseem Roshan.

In the improved model circumferential component of tooth eddy current loss has been ignored. As the results of the improved model will show that by ignoring this component we can still get good agreement with FEM results. This component of flux density is negligible at the centre of tooth and has considerable value at the shoes and tooth surface.

### 3.2 Yoke Loss Model Due to $B_{cy}$

The flux density waveforms studied by [1], [2] will be used for the yoke eddy current and excess eddy current loss modeling in the improved iron loss model. Generally this waveform is piece wise linear. In particular the waveform of the circumferential component of flux density of the yoke is trapezoidal as shown in Fig. 2.

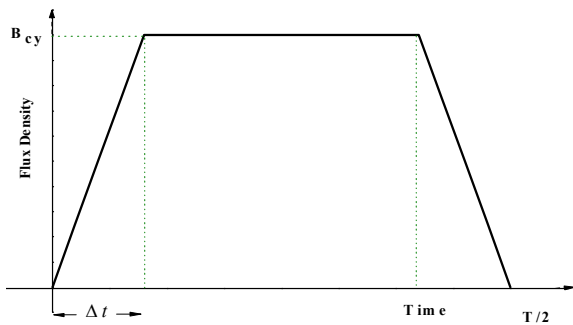


Fig. 2. Yoke flux density circumferential component

The waveform of the circumferential component of flux density is trapezoidal. The flux density is nearly evenly distributed over the thickness of the yoke and the rise time of flux from negative peak to positive peak is about the time required for one point of the yoke to transverse one magnet width.

The magnet coverage  $\alpha'$  for width  $W_m$  of the magnet can be expressed as follows:

$$P_{\text{ext}} = 4 \sqrt{\frac{A\alpha n_0}{\rho}} \frac{\sqrt{2mq}}{(2\pi)^{\frac{3}{2}}} B_{th}^{\frac{3}{2}} \omega_s^{\frac{3}{2}} \quad (17)$$

$$\alpha' = \frac{\omega_m}{\frac{2\pi r}{p}} \quad (18)$$

Where  $r$  is the outer radius of rotor.

The time required for the magnet of width  $W_m$  to pass a point in the stator yoke is as follows:

$$\Delta t = \frac{w_m}{r\omega_{mech}} = \frac{w_m}{r \frac{2\omega_s}{p}} = \frac{\alpha'\pi}{\omega_s} \quad (19)$$

During the time interval  $\Delta t$ , the circumferential component of the yoke flux density changes from  $-B_{cy}$  to  $B_{cy}$ . This change in flux density occurs twice in one time period.

Therefore the change of flux density over the time interval  $\Delta t$  can be written as follows:

$$\frac{dB}{dt} = \frac{2B_{cy}}{\Delta t} = \frac{2\omega_s B_{cy}}{\alpha'\pi} \quad (20)$$

Where  $B_{cy}$  is the maximum value of the circumferential component of the yoke flux density and  $\Delta t$  is the time for the magnet to pass over point of the stator yoke.

Eddy Current Loss: Therefore by using (2) and (20), the eddy current loss in yoke due to circumferential component of flux density can be written as follows:

$$P_{\text{eyc}} = \frac{d^2}{3\pi\rho} \left( \frac{B_{cy}}{\Delta t} \right)^2 \left( \frac{2\Delta t}{T} \right) \quad (21)$$

By using (19) in (21), the eddy current loss in yoke due to circumferential component of flux density can be written as follows:

$$P_{\text{eyc}} = \frac{4d^2}{3\rho\alpha'} B_{\text{cy}}^2 \omega_s^2 \quad (22)$$

Excess Eddy Current Loss: The change in flux density for excess eddy current loss will be the same of that of eddy current loss as given by (20). Therefore by using (20) in (3) excess eddy current loss density can be expressed as follows:

$$P_{\text{exyc}} = \sqrt{\frac{A\alpha n_0}{\rho}} \left( \frac{2B_{\text{cy}}}{\Delta t} \right)^{\frac{3}{2}} \left( \frac{2\Delta t}{T} \right) \quad (23)$$

By using (19) and (21) in (23), we can write the excess eddy current loss in the yoke due to circumferential component of flux density as follows:

$$P_{\text{exyc}} = \sqrt{\frac{8A\alpha n_0}{\rho\alpha'}} \frac{B_{\text{th}}^{\frac{3}{2}} \omega_s^{\frac{3}{2}}}{(\pi)^{\frac{3}{2}}} \quad (24)$$

The difference between the yoke loss due to circumferential component obtained by improved iron loss model and by Waseem Roshan are: 1) the formula of the eddy current loss in yoke due to circumferential component has been multiplied by a factor of 4 and 2) the excess eddy current for the yoke due to circumferential component has been divided by factor of  $\pi^{1.5}$ .

### 3.3 Yoke Loss Model Due to $B_{ry}$

The variation of the radial component of flux density in yoke is considered to be the same as those of the variation of radial component of tooth flux density as shown by Fig.3.

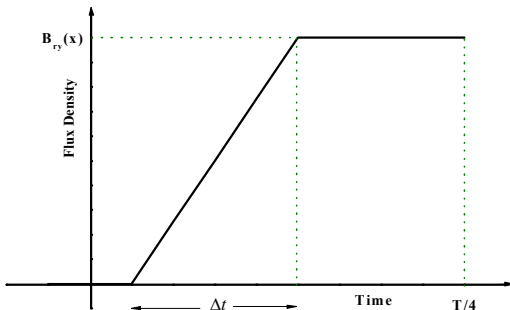


Fig. 3. Yoke flux density radial component

The radial component of yoke flux density has similar waveform to that of radial component of tooth flux density waveform but with different plateau at each layer of flux

density as given by [1], [2]. The plateau has its maximum value near the tooth and dramatically goes to zero near the surface of the yoke and the maximum value of flux density depends strongly on the position along radial direction. This variation was studied by Waseem Roshan and expressed it as a quadric fit, as shown be Fig. 4, and it can be expressed as follows [2]:

$$B_{ry}(x) = (ax + bx^2) B_{\text{cy}} \quad (25)$$

where  $B_{\text{cy}}$  is the maximum value of the circumferential component of the flux density in the yoke,  $B_{ry}$  is the maximum value of the radial component of flux density at the outer edge of yoke and  $a$ ,  $b$  are the coefficient of the quadratic fit.

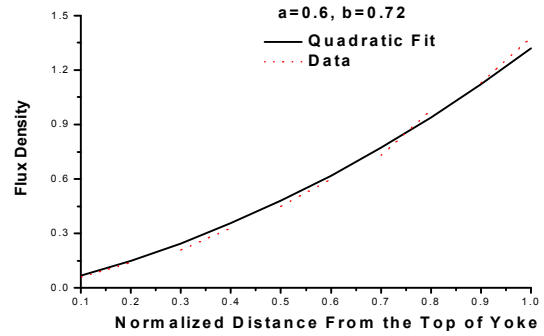


Fig. 4. Quadratic Fit of the Yoke Flux Density

The variable  $x$  is normalized with the width of yoke. This fit is also employed in the improved iron loss model to calculate the average eddy current loss and excess eddy current loss densities in the yoke due to radial component of flux density in the yoke.

For the waveform as shown in Fig. 3, the radial component of yoke flux density changes from zero to maximum value, in time required by the magnet to transverse one slot pitch. If there are  $p$  poles then the distance travelled by the magnet in one slot pitch is given by as follows:

$$\Delta x = \frac{2\pi r}{mpq} \quad (26)$$

Therefore, the time required to transverse one slot pitch is as follows:

$$\Delta t = \frac{\Delta x}{r\omega_{\text{mech}}} = \frac{\pi}{mq\omega_s} \quad (27)$$

During this time the flux density changes from zero to its plateau. Therefore the rate of the change of flux density is given by as follows:

$$\frac{dB}{dt} = \frac{B_{ry}}{\Delta t} = \frac{qm\omega_s B_{ry}(x)}{\pi} \quad (28)$$

Where  $\Delta t$  is the time to transverse one slot pitch by the magnet.

This change in flux density occurs four times in one time period.

Eddy Current Loss: Using (2), the eddy current loss at a point  $x$  in yoke due to radial component of flux density can be written as follows:

$$P_{\text{eyc}}(x) = \frac{d^2}{3\pi\rho} \left( \frac{B_{ry}}{\Delta t} \right)^2 \left( \frac{4\Delta t}{T} \right) \quad (29)$$

By integrating over  $x$  and by using (27) and (28) in (29) eddy current losses in yoke due to circumferential component of flux density can be written as follows:

$$P_{\text{eyr}} = C \frac{d^2 m q}{6\rho\pi^3} B_{cy}^2 \omega_s^2 \quad (30)$$

Where  $C$  is a constant, calculated by integrating over  $x$  with limit from  $0$  to  $l$  and its value is  $0.445$ .

Excess Eddy Current Loss: The excess eddy current loss at a point  $x$  in the yoke due to radial component of flux density can be obtained by using (27) and (28) in (3):

$$P_{\text{exyr}}(x) = \frac{1}{2} \sqrt{\frac{A \alpha n_0 q}{\rho}} \frac{B_{ry}^{\frac{3}{2}} \omega_s^{\frac{3}{2}}}{(\pi)^{\frac{3}{2}}} \quad (31)$$

By integrating over  $x$  the average excess eddy current loss density in the yoke due to radial component of yoke flux density is as follows:

$$P_{\text{exyr}} = D \sqrt{\frac{A \alpha n_0 q}{\rho}} \frac{B_{ry}^{\frac{3}{2}} \omega_s^{\frac{3}{2}}}{(\pi)^{\frac{3}{2}}} \quad (32)$$

Where  $D$  is a constant, calculated by integrating over  $x$  with limits from  $0$  to  $l$  and its value is  $0.225$ .

The difference between the yoke loss due to radial component of flux density obtained by the improved iron loss model and by reference [2]: 1) the formula of eddy

current loss in yoke due to radial component is multiplied by "m", 2) the formula of excess eddy current for the yoke due to circumferential component is divided by factor of  $\pi^{1.5}$ .

### 3.4 Skewing Effect

For a tooth skewed by an angle  $\sigma$ , the rate of change of flux density is decreased in comparison with those shown in Fig. 1, for the case of without skewing.

Let  $\Delta t_1$  is the time to transverse one slot pitch when there is now skew, and then the time for the change in flux density can be expressed as follows:

$$\Delta t_1 = \frac{T}{2mq} \quad (33)$$

If the tooth is skewed by an angle  $\sigma$ , then the time for the change in flux density can be written as follows:

$$\Delta t = \Delta t_1 \times \sigma + \Delta t_1 \quad (34)$$

Where  $\sigma$  is the skew angle and  $\Delta t_1$  is the time for a magnet to transverse one slot pitch.

Substituting the value of  $\Delta t_1$  in above equation it can be rewritten as follows:

$$\Delta t = \frac{T}{2mq} (\sigma + 1) \quad (35)$$

Now the eddy current loss in the tooth due to radial component of flux density can be obtained by putting the  $\Delta t$  in (13), can be written as follows:

$$P_{\text{et}} = \frac{2d^2}{3\rho\pi^3} m q \frac{B_{th}^2 \omega_s^2}{(\sigma + 1)} \quad (36)$$

Similarly the excess the eddy current loss by the radial component of tooth flux density when tooth is skewed can be obtained by putting the value of  $\Delta t$  in equation (15) can be written as follows:

$$P_{\text{ext}} = \left( \frac{4\Delta t}{T} \right) \sqrt{\frac{A \alpha n_0}{\rho}} \left( \frac{B_{th}}{\Delta t} \right)^{\frac{3}{2}} = \sqrt{\frac{A \alpha n_0}{\rho}} \frac{\sqrt{2mq}}{(2\pi)^{\frac{3}{2}} (\sigma + 1)} \frac{4}{B_{th}^{\frac{3}{2}} \omega_s^{\frac{3}{2}}} \quad (37)$$

The difference between the iron loss calculated when skewed by improved iron loss model and reference [3] is that: 1) the improved iron loss model considers the rise time

of flux density in tooth for the time to transverse one slot pitch whereas the reference [3] calculated for the time to transverse one tooth width and 2) improved iron loss model use resistivity as a parameter and reference [3] use  $K_e$  and  $K_{ex}$  as parameter, respectively for the eddy current and excess eddy current loss.

### 3.5 Hysteresis Loss Model

The hysteresis loss density formula that is used in the improved iron loss model is as follows:

$$P_h = K_h K_{ch} B^\beta \omega_s \quad (38)$$

In the improved iron loss model, the hysteresis loss density in tooth and yoke is calculated by considering the effect of minor hysteresis loops. The total hysteresis loss density is expressed as follows:

$$P_h = P_{ht} + P_{hy} \quad (39)$$

Where the hysteresis loss density in tooth is expressed as follows:

$$P_{ht} = K_h K_{cht} B_{ht}^\beta \omega_s \quad (40)$$

Where the hysteresis loss density in yoke is expressed as follows:

$$P_{hy} = K_h K_{chy} B_{hy}^\beta \omega_s \quad (41)$$

### 3.6 Total Iron Loss Model

The total iron loss is obtained by summing the eddy current losses, excess eddy current losses and hysteresis losses in the tooth and yoke as follows:

$$P_t = (P_{et} + P_{ext} + P_{ht}) V_t + (P_{eyc} + P_{exyc} + P_{eyr} + P_{hy} + P_{exyr}) V_y \quad (42)$$

Where  $V_t$  is the volume of the tooth and  $V_y$  is the volume of the yoke of the In-wheel PMSM.

The eddy current loss density in tooth  $P_{et}$ , excess eddy current loss density in tooth  $P_{ext}$  and hysteresis loss density in tooth  $P_{ht}$  are given by (25), (27) and (40) respectively. The eddy current loss density in yoke  $P_{eyc}$ , excess eddy current loss density in yoke  $P_{exyc}$  and hysteresis loss density in yoke  $P_{hy}$  are given by (22), (24) and (41) respectively.

The eddy current loss density in yoke  $P_{eyr}$ , excess eddy current loss density in yoke  $P_{exyr}$  are given by (30) and (32) respectively.

## 4. Model Verification and Discussion

### 4.1 Analysis Results

Figure 5 shows the in-wheel PMSM motor model that is used in this paper. The various parameters of the PMSM are as shown in Table 1.

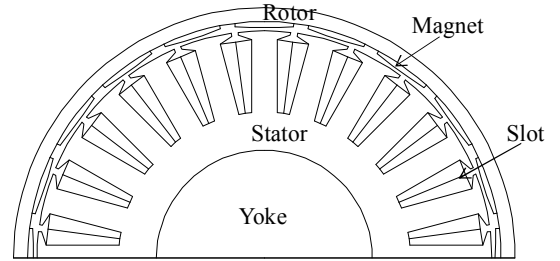


Fig. 5. In-Wheel PM Motor Model.

Table 1. Parameters of the In-wheel PMSM

Name (mm)	Value	Name (mm)	Value
Stator Outer radius	78.0	Inner radius of stator	37.0
Thickness of magnet	2.03	Yoke width	4.8
Air gap length	1.16	Slot opening	2.4309
Rotor Outer radius	86.0	Inner radius of rotor	81.2

The values of the other parameters taken for improved iron loss model are  $K_h=40$ ,  $\beta=1.8$ ,  $\rho=10^{-7}$ ,  $K_{chy}=1.35$ ,  $K_{cht}=1.0$ ,  $\alpha n_o=0.372$ ,  $\alpha'=0.777$ ,  $A=0.0000044$ , no of slots=24, no of poles=20 and  $d=0.00021$ .

Figure 6 shows the no load air gap flux density obtained by using [7].

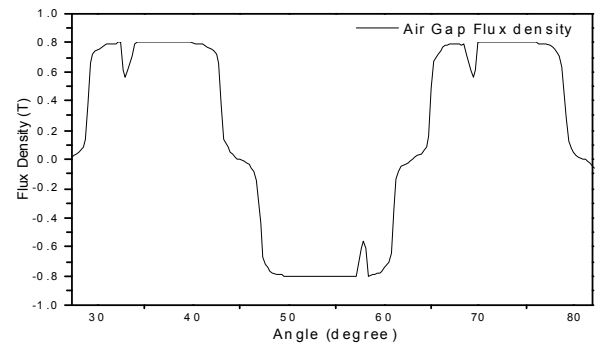


Fig. 6. Air Gap Flux Density

Table 2. Shows the comparison of the improved iron loss model results for the case of no skew with the 2-D FEM and the results shows good consistency.

**Table 2.** Iron Loss Comparison of Non Skewed Model

Speed(RPM)	300	350	400	450	500
By 2D-FEM(W)	11.94	14.12	16.98	20.07	23.25
By improved Model(W)	11.74	14.25	16.88	19.63	22.49

Table 3. shows the comparison of the improved iron loss model results for the skew case with the 3-D FEM and results shows good consistency.

**Table 3.** Iron Loss Comparison of Skewed Model

Speed(RPM)	300	350	400	450	500
By improved model (W)	6.61	8.12	9.68	11.53	13.14
By 3D-FEM (W)	6.48	8.18	9.94	11.84	13.86

## 5. Conclusions

An improved model for the iron loss computation was presented in this paper. The improved iron loss model included the effect of minor hysteresis loops loss on the iron loss for the case and without stator skew. The circumferential component of tooth flux density was neglected for the calculation of tooth loss for both skew and non skew case. The good agreement of the improved loss model with the FEM indicates that the effect of including this component must be small. The results of the iron loss calculated by improved iron loss model in stator teeth and yoke were compared with the FEM results for several operating speeds and results shows good consistency (within 5%).

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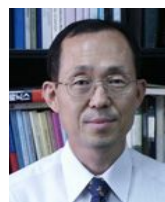
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