A Game Theoretic Cross-Layer Design for Resource Allocation in Heterogeneous OFDMA Networks

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Abstract – Quality of Service (QoS) and fairness considerations are undoubtedly essential parameters that need to be considered in the design of next generation scheduling algorithms. This work presents a novel game theoretic cross-layer design that offers optimal allocation of wireless resources to heterogeneous services in Orthogonal Frequency Division Multiple Access (OFDMA) networks. The method is based on the Axioms of the Symmetric Nash Bargaining Solution (S-NBS) concept used in cooperative game theory that provides Pareto optimality and symmetrically fair resource distribution. The proposed strategies are determined via convex optimization based on a new solution methodology and by the transformation of the subcarrier indexes by means of time-sharing. Simulation comparisons to relevant schemes in the literature show that the proposed design can be successfully employed to typify ideal resource allocation for next-generation broadband wireless systems by providing enhanced performance in terms of queuing delay, fairness provisions, QoS support, and power consumption, as well as a comparable total throughput.

Keywords: Cross-layer design, Channel-State-Information (CSI), Multiple access, NBS, OFDMA, QoS, Resource allocation, Wireless networks

1. Introduction

Next generation wireless networks need to provide quick ubiquitous network access to their rapidly expanding faction of mobile users. The core process used to achieve such performances is based in the network mechanisms, which must efficiently allocate the available resources. These mechanisms deliberate and combine low-level system dynamics from the medium access control (MAC) and physical (PHY) layers and are widely known as cross-layer schemes.

Extensive research has been attempted in regards to cross-layer designs [1-6], e.g.the adoption of the OFDMA technique approach. Due to itsorthogonality principle, OFDMA is considered to bethe most effective multiple access method for 3G, 4G, and further generations of broadband wireless networks, such as 3GLTE [1] and WiMAX [2]. A notable observation is that most of the existing cross-layer schemes focus on maximizing the overall system's data rate or minimizing the overall system's power consumption [3-6]. However, by optimizing a system's aggregate resource efficiency without considering individuals might benefit users who have good channel conditions, but starve other users with bad channel conditions. As shown in [7] and [8],this problem can lead to unfair and greedy resource distribution.

In order to resolve this issue, some researchers have used cooperative game theory to propose schemes that provide fair resource allocation relying on tradeoffs between a system's efficiency and proportional fairness patterns [9-17]. Such cooperative game theoretical schedulers are discussedin [10] and [11]. It has beenobserved that, on the one hand,the proposed schemes provide fairness amongst users, even to thosewho are in a deep fade, but on the other hand they dramatically increase the system's power consumption. Therefore, fairness considerations induce a power increase or throughput decrease. To overcome this problem two notable attempts are presented in [12] and [13]. In particular, [12] investigates a cooperative game, where players cooperate to achieve a mutually desirable equity solution. In [13], the resource allocation scheme wasdeveloped usingthe S-NBS concept; it operates by forming coalitions among users. Both [12] and [13] show that in addition tofair resource allocation these methodscan also achieve the maximization of the aggregate throughput at thesame levels as opportunistic schemes, i.e., the greedy Maximal-Rate (M-R) algorithm presented in [14]. Consequently, smart scheduling can reduce or even obliterate the costtothroughput or power consumption stemming from fairness considerations.

Another key issue in proportionally fair schemes is that users frequently suffer from fairness deficiencies due to the users' unequal spatial positioning and unequal delays, as pointed in [15]. In other words, resource allocation might be fair between users running thesame applications, but beunfair tousers having different requirements. Therefore, to avoid such deficiencies, it is evidentthat new designs must consider the heterogeneous users' requirements in-

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stead of the homogeneous. However, this heterogeneous assumption induces several additionalsystem constraints, which commonly cause the cross-layer problem to evolve into a NP-hard problem. For this reason the authors of [16] and [17] utilized utility theory from the economic domain; they designed utility functions to combine the users' requirements with their fairness considerations and throughput oriented processes. Such utility functions offer a tangible metric to quantify the level of satisfaction foreach user when a certain amount of resources have been assigned to that user. More precisely, in [16] the problem of subcarriers and power allocationis considered, as a maximization problem regardingeach user's utility function and show that the obtained solution has the property of proportional fairness if the utility function is logarithmic according tothe achieved throughput. Similar findings are presented in 17], which additionally reports that proportional fairness can be considered as a special case of the S-NBS concept, when the rate is the utility function and the minimum required rate is zero. Consequently, it is prudent to allocate the resources on fairness concepts defined directly in terms of users' utilities rather than the users' throughputs, as seen in [10-15].

In this work we address the aforementioned issues andpropose a novel S-NBS-based cross-layer design for OFDMA networks. The key innovationsof our work are summarized as follows:

- Thisis the first work toapproach cooperative (S-NBS) resource scheduling from the cross-layer perspective considering QoS heterogeneity. Other relevant studies have often considered single-layer architectures with homogeneous QoS considerations [10-17]. Thisimproves the performance in terms of transmission delay by reducing the probability of data packet loss at the MT.
- This is the first work to derive explicit mathematical solutions usingthe S-NBSconcept. To the best of our knowledge, other relevant approaches propose either single-layer or cross-layer allocation patterns based on complex numerical solutions, [12, 13, 15-18]. We overcome this major problem by utilizing a new low-complexity solution methodology, presented in Appendix B.
- We introduce a novel utility optimization objective to express the users' satisfaction in terms of throughput. The utility objective fully complies with the S-NBS axioms, meaning that it incorporates the S-NBS properties instead of considering them as individual optimization constraints, as discussed in [12, 16, 17], and [18]. Such incorporation increases the accuracy of the final strategies and decreases their complexity by avoiding further variable relaxations during the solution process.

The rest of the paper is organized as follows: In Section 2, we discuss the OFDMA system regarding the PHY and MAC layers and the queuing model's characteristics. In Section 3, we employthe NBS bargaining framework from game theory to define the utility function that is used as the optimization objective forthe primary optimization problem. Section 4presents the formulation of the primary optimization problem subject to the cross-layer constraints. We show that through applying a time-sharing relaxation of the subcarrier indexes the constrained cross-layer problem can be transformed to a convex optimization problem over a convex and feasible set. The optimal allocation policies are presented in Section 5, whereasin Section 6we provide details regardingthe implementation process of our solution by means of convergence feasibility and complexity performances. Section 7 discussesthe simulation results and comparisons, andour conclusions are drawn in Section 8.

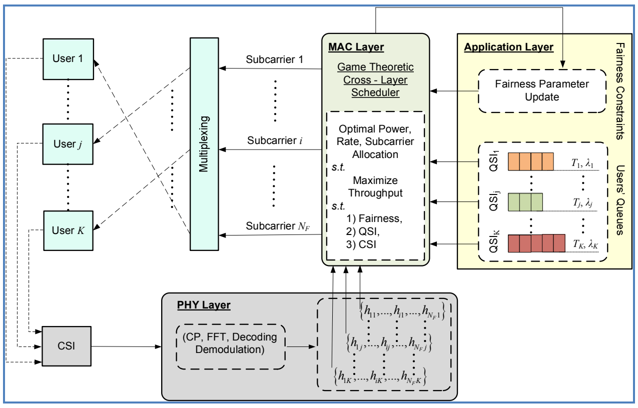

Fig. 1. The cross-Layer modelinan OFDMA system with NBS considerations, heterogeneous QoS, and perfect CSI

2. The OFDMA System Model

Fig. 1 shows the downlink (DL) OFDMA system model with bandwidth BW equally divided to the N_F system subcarriers. Before the scheduling operation is performed, the scheduler collects the Channel State information (CSI) and Queue State Information (QSI) fromall of the K heterogeneoususers. More precisely, in the beginning of each timeslot the scheduler obtains the CSI through the uplink dedicated pilot subcarriers transmitted by the mobile users. The information forthe users' queue dynamics from the higher layers is updated according to an incremental update algorithm, which senses and accordingly modifies the QSI of each user [6]. Based on the CSI and QSI, the Base Station (BS)scheduler can then decide how to distribute the resources according to its allocation policies.The allocation decision isfinally announced to each individual mobile user through separate control channels under the assumption that the CSI is perfectly available.

2.1 The Channel Model

Here we consider DL time-varying transmissions over a quasi-static multi-path slow fading channel. After removing the Cyclic Prefix (CP) and performing Fast Fourier Transforms (FFT) inthe BS, the received Orthogonal Frequency Division Multiplexing (OFDM) symbol of the j-th user, $j = 1,...,K$ on the i-th subcarrier, $i = 1,...,N_F$ is given by:

$$y_{ij} = h_{ij} x_{ij} + z_{ij} , \tag{1}$$

where x_{ij} represents the transmitted OFDM symbol, h_{ij} is the identically independent distributed (i.i.d) actual channel gain, is the complex circularly symmetric Gaussian (CCSG) noise with zero mean, and $\sigma_z^2 = (BW \cdot N_0)/N_F$ variance, i.e. $z_{ij} \sim \mathcal{CN}(0,\sigma_z^2)$, where N_0 denotes the noise power spectral density. Relying on the definition of x_{ij}, we denote the transmitting power allocated from the BS to user j through subcarrier i by $p_{ij} = E\left[\left|x_{ij}\right|^2\right]$ [1], which can be expressed in matrix form by the power allocation policy $P_{N_F \times K} = \left[p_{ij}\right]$, with $E[\cdot]$ to indicate the expectation operator.

In our system it is not allowed for more than one user to occupy the same subcarrier during a timeslot, e.g., for each i if $p_{ij} \neq 0$ then $p_{ij'} = 0$, $\forall j' \neq j$. This admission is expressed by the following subcarrier allocation rule:

$$\sum_{j=1}^{K} s_{ij} = 1 , \ \forall i , \tag{2}$$

where s_{ij} denotes the subcarrier allocation index and is defined as $s_{ij} \in \{0,1\}$ indicating that allocation occurs ($s_{ij} = 1$) when subcarrier i is allocated to user j, otherwise allocation does not occur ($s_{ij} = 0$). In addition, we represent the subcarrier allocation policy in matrix form as $S_{N_F \times K} = \left[s_{ij}\right]$, with individual matrix elements $s_{ij} \in \{0,1\}$.

Finally, to guarantee the feasibility of the transmissions in our system, the average total transmitting power from the BS over all users and subcarriers $E\left[\sum_{j=1}^{K}\sum_{i=1}^{N_F} s_{ij} \cdot p_{ij}\right]$ [1] mustnot exceed the total available power inthe BS, expressed as P_{TOTAL}. This is expressed by the following power allocation rule:

$$E\left[\sum_{j=1}^{K}\sum_{i=1}^{N_F} s_{ij} \cdot p_{ij}\right] \leq P_{TOTAL} \text{ [1]}. \tag{3}$$

[1] The expectation operator $E[\cdot]$ refers to the average power over the random realizations $\left\{\left|x_{ij}\right|^2\right\}$ of the transmitted OFDM symbols $\left\{x_{ij}\right\}$.

2.2 The Physical Layer Model for Multi-User OFDMA Systems

Recalling our assumption that a perfect CSI is available tothe BS, we can straightforwardly consider that the channel is Gaussian. Therefore, during a fading slot the maximization of the mutual information $M(\cdot)$ between the received y_{ij} and transmitted x_{ij} symbol yields the maximum achievable instantaneous capacity c_{ij} for user j on subcarrier i. In other words, according to Shannon's capacity theorem we can define the maximum achievable instantaneous capacity c_{ij} as:

$$c_{ij} = \max_{p(x_{ij})} M\left(x_{ij} : y_{ij} \left| h_{ij}\right.\right) = \log_2\left(1 + p_{ij} \cdot \left|h_{ij}\right|^2\right). \tag{4}$$

To achieve maximum performance, each user's j instantaneous data rate r_{ij} on subcarrier i needs tomatch the maximum instantaneous capacity c_{ij} in, given the channel realizations $\left\{\left|h_{ij}\right|^2\right\}$. In other words, weapply M-ary Quadrature Amplitude Modulation (M-QAM) to offer a finite set $\mathbf{D} = \{0,1,\ldots,D\}$ of the possible transmission data rates to each user j, with D used to express the maximum amount of information that can be transmitted by each subcarrier (in *bits/OFDM symbol*).

By applying M-QAM, theBit-Error Rate (*BER*) of user j on subcarrier i can be expressed as a function of the instantaneous rate r_{ij} and theSignal-to-Noise Ratio (SNR) [19]:

$$BER_{ij} \approx 0.2 \cdot \exp\left(\frac{-1.5 \cdot \Gamma_{ij}}{2^{r_{ij}} - 1}\right), \ BER_{ij} \leq 10^{-3}. \tag{5}$$

The variable Γ_{ij} in (5) represents the SNR and it is given by $\Gamma_{ij} = \left(p_{ij} \cdot \left|h_{ij}\right|^2\right)/\sigma_z^2$ [19]. Consequently, from (5) the maximum achievable instantaneous data rate r_{ij} of user j on subcarrier i can bewritten as:

$$r_{ij} = \log_2\left(1 + \frac{p_{ij} \cdot \left|h_{ij}\right|^2 \cdot \eta_{ij}}{\sigma_z^2}\right), \tag{6}$$

where η_{ij} is a variable used for notational brevity, denoted as $\eta_{ij} = -1.5/\ln\left(5 \cdot BER_{ij}\right)$. Therefore, we express the data rate allocation policy in matrix form as $\mathbf{R}_{N_F \times K} = \left[r_{ij}\right]$, with the individual matrix elements the terms from r_{ij} determined by (6).

In Section 3 we will use the utility theory to express each user's level of satisfactionin terms of r_{ij} relyingon the S-NBS properties. In the following subsectionwe will correlate r_{ij} from the physical layer with each user's traffic parameters from the higher layers to derive the system conditions by means of cross-layer constraints.

2.3 The Queuing Model

To express the traffic parameters of each user from the higher layers, we assume that each user's queue is described by the $M/G/1$ model with a non-selected timeslot [3]. In our queuing system, thedata packets are fixed at *B bits* and arrive to the users' queues by following an independent Poisson arrival process atrate λ_j (in *packets per timeslot*). Let us denote the maximum delay tolerance of each user j as $T_j^{\max}$ (in *timeslots*) in order to represent each user's QoS characteristics with a3-tuple structure $\left[B, \lambda_j, T_j^{\max}\right]$. We can now correlate the data rate r_{ij} from the physical layer withthe traffic rate from the higher layers with the following *Lemma*.

Lemma 1: To guarantee the QoS requirements of each heterogeneoususer j, the cross-layer QoS condition

$$E\left[\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij}\right] \geq q_j\left(B, T_j^{\max}, \lambda_j\right) \quad (bits/sec/Hz)^2, \tag{7}$$

must be satisfied with the equivalent rate at the user's queue to be given by $q_j\left(B, T_j^{\max}, \lambda_j\right) = \frac{B \cdot N_F \cdot \left(\sqrt{T_j^{\max} \cdot \lambda_j \cdot \left(T_j^{\max} \cdot \lambda_j + 2\right)} - \lambda_j \cdot T_j^{\max}\right)}{2 \cdot T_j^{\max} \cdot t_s \cdot BW}$ and t_s to denote the duration of the scheduling timeslot.

Proof: The proof of *Lemma 1* is the same asthe proof for*Lemma 1* in our previous work [3] and has been omitted due to space limitations. ■

Lemma 1 implies that the average scheduled data rate $E\left[\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij}\right]$ should be at least the same as the minimum required traffic arrival rate $q_j\left(B, T_j^{\max}, \lambda_j\right)$ to the queue of each user j. In other words, from *Lemma 1* we derive the cross-layer condition that associates the data rates from the physical and higher layers expressed in *bits/sec/Hz*. In the case where users have no delay requirements, i.e., $T_j^{\max} \to \infty$, then by applying the De L'Hospital's rule condition (7) becomes:

$$E\left[\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij}\right] \geq \frac{B \cdot \lambda_j \cdot N_F}{t_s \cdot BW} \quad (bits/sec/Hz), \tag{7a}$$

meaning that for such users the average data rate should be at least equal to their corresponding queue arrival rate $B\lambda_j$. Finally, the packets at the $M/G/1$ queues and the arriving packets at the PHY layer are multiplexed over time according to the multiplexing process described in [20]. For simplicity, we have omitted the details of the cross-layer multiplexing process, howeverwe still consider it inour simulations.

2.4 The Cross-Layer Model

In this subsection, we discuss how our scheme at the cross-layer realizes its queue and channel dynamics, given the aforementioned channel, physical and queuing models. At the cross-layer, our system's dynamics are characterized by the system state $\left(\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}\right)$, with $\mathbf{H}_{K \times N_F} = \left[\left|h_{ij}\right|^2\right]$ composing the CSI realization matrix and $\mathbf{Q}_{K \times 1} = \left[q_j\right]$ beingthe $K \times 1$ vectorwith its j-th element used to define the number of packets remaining in j-th user's buffer. Therefore, the cross-layer policies differ from the physical layer policies found in Subsections 2.1. and 2.2., sinceat the cross-layer the scheduler is dependentonboth the channel and queuing characteristics. In other words, the cross-layer scheduler determines the subcarrier, power, andrate allocation from the policies $\mathcal{S}_{K \times N_F}\left[\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}\right]$, $\mathcal{P}_{K \times N_F}\left[\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}\right]$, and $\mathcal{R}_{K \times N_F}\left[\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}\right]$, respectively.

3. The Formulation of the Objective Function with S-NBS Considerations

In this section,we express each user j's satisfaction rate by means of throughput relying on cooperative game theory and especially on the S-NBS bargaining concept.

It wasshown in [13] that the S-NBSconcept can provide fair and efficient bandwidth allocation subject to the users' (players') rate requirements. In such bargaining each user j has a utility function f_j and an initial utility u_j^0. The initial utility u_j^0 denotes the minimum rate that the scheduler must provide to the user in order foruser to participate in the S-NBS game[3]. Moreover, each f_j is defined as a subset of R^K termed as Ψ that describes the set of

[2] The expectation operator $E[\cdot]$ refers to the average throughput over random realizations $\left\{\left|h_{ij}\right|^2\right\}$ of the channel gains $\left\{h_{ij}\right\}$ and the queue-state-information (QSI) $q_j\left(B, T_j^{\max}, \lambda_j\right)$.

game strategies for the K users. Assuming that u_j^0 can be achieved for each user that participates in the game, then a $\Psi_0 \in \Psi$ exists so that $\Psi_0 = \{\psi \in \Psi | f(\psi) \geq u^0\}$, where $f(\psi) = (f_1, ..., f_K)$ is the utility vector and $u^0 = (u_1^0, ..., u_K^0)$ is the initial utility vector. Let the set of achievable utilities be denoted by $U = \{f(\psi) | \psi \in \Psi\}$ and the class of sets of utility measures that satisfy the minimum utility bounds u^0 be denoted by $G = [U, u^0 | U \subset R^K]$. Then the S-NBS $S_{NBS} | G \to R^K$ satisfies the following Axioms [21]:

(1) $S_{NBS}(U, u^0)$ is Pareto optimal.

(2) Guarantees the minimum required utility $S_{NBS}(U, u^0) \in U^0$, where $U^0 = \{u \in U | u \geq u^0\}$.

(3) Is independent of irrelevant alternatives: If the feasible set shrinks but the solution outcome remains feasible, then the solution outcome for the smaller feasible set willbe the same point. This can be written as $V \subset U$, $(V, u^0) \in G$ and $S_{NBS}(U, u^0) \in G$ then $S_{NBS}(U, u^0) = S_{NBS}(V, u^0)$. This axiom offers fairness.

(4) Provides symmetry, which means that all of the users have the same priorities. S_{NBS} satisfies symmetry if U is symmetric with respect to subset $J_{Class} \subseteq \{1, ..., j, ..., K\}$ $u \in U$, $j, j' \in J$. Thus if $u_j^0 = u_{j'}^0$ then $S_{NBS}(U, u^0)_j = S_{NBS}(U, u^0)_{j'}$.

Considering thesefour Axioms we present the S-NBS propertyutilizingthe following *Theorem*:

[3] It is important to examine the difference between each user's initial utility u_j^0 and its minimum required traffic arrival rate $q_j(B, T_j^{max}, \lambda_j)$. The initial utility u_j^0 is the data rate required by a user to participate in the S-NBS game. The minimum required traffic arrival rate $q_j(B, T_j^{max}, \lambda_j)$ is the data rate required by a user to satisfy its minimum QoS requirements. In the case where resource starvation occurs, the scheduler may not be able to satisfy all users' QoS requirements ($q_j(B, T_j^{max}, \lambda_j)$) but may be able to provide a rate equal to each user's u_j^0. This means that users may not be totally satisfied but they participate in the S-NBS game to increase system's performance. In [31] we proved that the correlation between these two parameters is given by $q_j(B, T_j^{max}, \lambda_j) - 1 \geq u_j^0$. This correlation has major practical significance as it ensures that the S-NBS game is feasible even if the allocated data rate is less than the incoming traffic arrival rate to each user's queue.

Theorem 1: If the utility function f_j is concave upper-bounded defined on Ψ, which is convex and a subset of R^K, and J is the set of indices of users who are able to achieve a performance strictly superior to their initial performance, then there exists a symmetric Nash bargaining point ψ that verifies $f_j(\psi) \geq u_j^0, j \in J$ and comprises the unique solution of the maximization problem:

$$\max \prod_{j \in J} \left(f_j(\psi) - u_j^0 \right), \quad \psi \in \Psi_0. \tag{8}$$

Proof: The proof of *Theorem 1* is similar to the proof presented in [21] and has been omitted due to space limitations. ■

From *Theorem 1*, each user j 's level of satisfaction is represented by the S-NBS-based utility function $f_j(\psi) - u_j^0$, where the overall system's level of satisfaction as $\prod_{j \in J} (f_j(\psi) - u_j^0)$. Accordingly, each user j 's level of satisfaction in terms of data ratecan be expressed by the utility function $\left(\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij} \right) - u_j^0$; the overall system's level of satisfaction is $\prod_{j \in J} \left(\left(\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij} \right) - u_j^0 \right)$. Consequently, we can reformulate optimization problem (8) as:

$$\max_{\mathcal{S}_{K \times N_F}[H_{K \times N_F}, Q_{K \times 1}], \mathcal{P}_{K \times N_F}[H_{K \times N_F}, Q_{K \times 1}]} E\left[\prod_{j \in J} \left(\left(\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij} \right) - u_j^0 \right) \right] \Rightarrow$$
$$\max_{\mathcal{S}_{K \times N_F}[H_{K \times N_F}, Q_{K \times 1}], \mathcal{P}_{K \times N_F}[H_{K \times N_F}, Q_{K \times 1}]} E\left[\sum_{j=1}^{K} \ln \left(\left(\sum_{i=1}^{N_F} s_{ij} \cdot r_{ij} \right) - u_j^0 \right) \right]. \tag{9}$$

Utility optimization problem (9) aims to maximize the overall users' level of satisfaction over the subcarrier and power allocation policies $\mathcal{S}_{K \times N_F}[\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}]$ and $\mathcal{P}_{K \times N_F}[\mathbf{H}_{K \times N_F}, \mathbf{Q}_{K \times 1}]$, respectively. The overall users' level of satisfaction is represented by the aggregate utility of the allocated subcarriers to each user j. In addition, it is easy to seethat maximization problem (9) fully complies with all of the four S-NBS Axioms in terms of subcarrier and power allocations.

In the following section, we rely on (9) to formulate a cross-layer optimization problem subject to the system conditions determined by (2), (3) and (7); we derive the optimal solutions forthe new problems utilizing convex optimization.

4. Problem Formulation and Convex Optimization-based Scheduling Strategies

In this section we will formulatethe primary cross-layer utility optimization problem and transform it to a convex strategy through applying time-sharing relaxation of the optimization variables. In addition, we derive the optimal solutions by means of final formulas via thenovel solution methodology introduced in Appendix B.

4.1 The Cross-Layer Optimization Problem

As we have mentioned, our aim is to maximize the overall users' level of satisfaction subject to the OFDMA system's characteristics.In other words, we rely on (9) to formulate a cross-layer optimization problem in order to determine the optimalresource allocation policiesand simultaneously maintain the OFDMA physical layer constraints regarding the subcarrier selection, transmission power, and QoS requirements as determined by (2), (3) and (7), respectively. This cross-layer optimization problem is formulated as:

Find the optimal subcarrier, power and data rate allocation policies, i.e., $\boldsymbol{\mathcal{S}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[s^*_{ij}\right]$, $\boldsymbol{\mathcal{P}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[p^*_{ij}\right]$ and $\boldsymbol{\mathcal{R}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[r^*_{ij}\right]$, *respectively*

$$\text{such that: } \max_{\boldsymbol{\mathcal{S}}_{K\times N_F}\left[H_{K\times N_F},Q_{K\times 1}\right],\boldsymbol{\mathcal{P}}_{K\times N_F}\left[H_{K\times N_F},Q_{K\times 1}\right]} \sum_{j=1}^{K}\ln\left(\left(\sum_{i=1}^{N_F}s_{ij}\cdot r_{ij}\right)-u^0_j\right), \tag{10}$$

$$\text{subject to: } s_{ij}\in\{0,1\}, \tag{11}$$

$$\sum_{j=1}^{K}s_{ij}\le 1, \tag{12}$$

$$\sum_{j=1}^{K}s_{ij}\le 1, \tag{13}$$

$$E\left[\ln\left(\left(\sum_{i=1}^{N_F}s_{ij}\cdot r_{ij}\right)-u^0_j\right)\right]\ge q_j\left(B,T^{\max}_j,\lambda_j\right), \tag{14}$$

$$E\left[\sum_{j=1}^{K}\sum_{i=1}^{N_F}s_{ij}\cdot p_{ij}\right]\le P_{TOTAL}. \tag{15}$$

In the cross-layer problem of (10)-(15), constraints (11) and (12) ensure that each subcarrier can be occupied by only one user per timeslot. Constraint (13) certifies that the power can possessonly positive values, (14) expresses the average delay boundof each user, and (15) the average total power limitation of the system.It is a giventhat the cross-layer problem is a mixed combinatorial problem,sincethe variables $\{s_{ij}\}$ are discrete and $\{p_{ij}\}$ are continuous. In such problems, the optimal $\{p_{ij}\}$ and their corresponding r_{ij}s can be calculated for a selected user over a subcarrier for each possible combination of $\{s_{ij}\}$. The total system throughput can be then evaluated for all cases by enumerating all of the possible combinations of $\{s_{ij}\}$; the one that gives the largest throughput is the optimal solution.This means that there would be K^{N_F} possible subcarrier assignments,sinceeach subcarrier can be used by only one user.The above solution methodologyleads to impracticable optimal solutions, especially for real-time systems, i.e. for $N_F=2048$, $K=200$, due to the high complexity of the allocation strategies.

Since we wish to avoid the above complexity we transform the cross-layer problem laid out in (10)-(15) into a convex one based on thetechnique presented in [22] and [23]. More specifically, we introduce the factor $\tilde{s}_{ij}\in(0,1]$ to transform the subcarrier allocation constraint (11) in terms of time-sharing. The new variable $\tilde{s}_{ij}$ indicates the portion of time that subcarrier i is assigned to user j during a transmission frame. The non-integer fractional part of $\tilde{s}_{ij}\in(0,1]$ is given by the fractional function of $\tilde{s}_{ij}$, i.e., $frac\left(\tilde{s}_{ij}\right)=s_{ij}-\lfloor s_{ij}\rfloor$ for $0<frac\left(\tilde{s}_{ij}\right)\le 1$. Nevertheless, although the introduction of thetime-sharing factor $\tilde{s}_{ij}\in(0,1]$, the utility optimization objective (10) is not convex over $\left(\tilde{s}_{ij},p_{ij}\right)$.To bypass this problem we introduce the continues variable $\tilde{p}_{ij}=p_{ij}\cdot\tilde{s}_{ij}$ and the maximum achievable instantaneous data rate r_{ij} in (6) is now denoted as $\tilde{r}_{ij}=\log_2\left(1+\frac{\tilde{p}_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\tilde{s}_{ij}\cdot\sigma^2_z}\right)$. The cross-layer problem in (10)-(15) can be then transformed into a convex problem as:

Find $\boldsymbol{\mathcal{S}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{s}^*_{ij}\right]$, $\boldsymbol{\mathcal{P}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{p}^*_{ij}\right]$ and $\boldsymbol{\mathcal{R}}^*_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{r}^*_{ij}\right]$,

$$\text{such that: } \max_{\boldsymbol{\mathcal{S}}_{K\times N_F}\left[H_{K\times N_F},Q_{K\times 1}\right]:\left\{\tilde{s}_{ij}\in(0,1],\sum_{j=1}^{K}\tilde{s}_{ij}\le 1\right\},\boldsymbol{\mathcal{P}}_{K\times N_F}\left[H_{K\times N_F},Q_{K\times 1}\right]:\left\{\tilde{p}_{ij}\ge 0\right\}} E\left[\sum_{j=1}^{K}\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}\cdot\tilde{r}\right)-u^0_j\right)\right], \tag{16}$$

$$\text{subject to: } E\left[\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}\cdot\tilde{r}_{ij}\right)-u^0_j\right)\right]\ge q_j\left(B,T^{\max}_j,\lambda_j\right), \tag{17}$$

$$E\left[\sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{p}_{ij}\right]\le P_{TOTAL}. \tag{18}$$

Proposition1: The cross-layer problem in (16)-(18) is convex over a feasible convex set within the region specified by $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$.

Proof: The proof of *Proposition 1* is presented in 0. ■

By using *Proposition 1*, we can now derive the optimal allocation policies of the cross-layer problem (16)-(18) through the utilization ofconvex optimization.

4.2 Convex Optimization-based Solutions

After definitionof the Lagrangian function and the Karush-Kuhn-Tucker (KKT) conditions of the cross-layer optimization problemlaid out in(16)-(18), the optimal subcarrier, power and data rate allocation policiesare signified as:

Theorem 2: The optimal S-NBS-based subcarrier allocation policy $\boldsymbol{\mathcal{S}}^{*}_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{s}^{*}_{ij}\right]$ has individual matrix elements.The optimal subcarrier allocation index $\tilde{s}^{*}_{ij}$ is given by:

$$\tilde{s}^{*}_{ij}=\begin{cases}0, \text{ if } \nu^{*}_{i}>H_{ij}\left(\xi^{*}_{j},\mu^{*}\right)\\ 1, \text{ otherwise}\end{cases}, \tag{19}$$

where ξ^{*}_{j}, μ^{*} and ν^{*}_{i} represent the optimal Lagrangian multipliers associated with the QoS constraint (17), the power allocation constraint (18) and the subcarrier allocation constraint $\sum_{j=1}^{K}\tilde{s}_{ij}\le 1$, respectively; function $H_{ij}\left(\xi^{*}_{j},\mu^{*}\right)$ is presented in Appendix B due to space limitations. The optimal user j^{*} can be then defined by decoupling $\boldsymbol{\mathcal{S}}^{*}_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]$ among the N_F system's subcarriers and by applying the following searching process:

$$\text{For } i=1 \text{ to } N_F \quad j^{*}=\arg\max_{j\in[1,K]} H_{ij}\left(\xi^{*}_{j},\mu^{*}\right) \quad \text{and} \quad \tilde{s}^{*}_{ij}=\begin{cases}1, \text{ if } j=j^{*}\\ 0, \text{ if } j^{*} \text{ does not exist}\end{cases}. \tag{20}$$

Proof: The proof of *Theorem 2* is presented in Appendix B. ■

Theorem3: The optimal S-NBS-basedpower allocation policy $\boldsymbol{\mathcal{P}}^{*}_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{p}^{*}_{ij}\right]$ has individual matrix elements. The instantaneous optimal power $\tilde{p}^{*}_{ij}$ of user j on subcarrier i is given by:

$$\tilde{p}^{*}_{ij}=\begin{cases}\dfrac{\sigma_z^2}{\left|h_{ij}\right|^2\eta_{ij}}\left[2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\frac{\left(\xi^{*}_{j}+1\right)\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{2^{u_j^0}\cdot\mu^{*}\cdot\sigma_z^2\cdot\ln(2)}}\right)\right)\right)-1\right]^{+} & \text{, if } \tilde{s}^{*}_{ij}=1\\ 0 & \text{, if } \tilde{s}^{*}_{ij}=0\end{cases}, \tag{21}$$

where $W(\cdot)$ denotes the Lambert-W function [24] and the notation $(x)^{+}$ means $\max(0,x)$.

Proof: The proof of *Theorem 3* is presented in Appendix B. ■

Relying on *Theorems 2 & 3*, the optimal S-NBS-based optimal throughput allocation policy $\boldsymbol{\mathcal{R}}^{*}_{K\times N_F}\left[\mathbf{H}_{K\times N_F},\mathbf{Q}_{K\times 1}\right]=\left[\tilde{r}^{*}_{ij}\right]$ has individual matrix elements for the optimal data rate $\tilde{r}^{*}_{ij}$ allocated to user j on subcarrier i given by $\tilde{r}^{*}_{ij}=\log_2\left(1+\dfrac{\tilde{p}^{*}_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\sigma_z^2}\right)$.

5. The Implementation Process and the Evaluation of the Optimal Results

In this section, we examine the theoretical performance of an efficient root-finding iteration process utilized to compute the optimal Lagrangian multipliers introduced in *Theorems 2 & 3*. We also study the feasibility of our solutions in terms of implementation complexity,minimum required transmitting power, and the algorithm's convergence.

Iteration ProcessDetails: In order to define the optimal solutions $\tilde{s}^{*}_{ij}$ and $\tilde{p}^{*}_{ij}$ in (19) and (21), respectively, we initially need to obtain the sets of the optimal Lagrangian multipliers $\left\{\xi^{*}_{j}\right\}$ and $\left\{\mu^{*}\right\}$ sothat the QoS constraint (14) of the problem laid out in (10)-(15) is satisfied for all users. In other words, the role of the Lagrangian multipliers $\left\{\xi^{*}_{j}\right\}$ and $\left\{\mu^{*}\right\}$ in (19) and (21) is to calibrate the subcarrier and power allocation,ensuring that the minimum, per user, required throughput, e.g., $q_j\left(B,T_j^{\max},\lambda_j\right)$ [4] is provided.

The aforementioned Lagrangian multipliers $\left\{\xi^{*}_{j}\right\}$ and $\left\{\mu^{*}\right\}$ are defined by an iterative searching algorithm with its main process described by:

[4] In reality, there are instances where some users have significantly worse channel conditions or higher QoS requirements (or even both) than others. In such cases, we have observed that traditional cross-layer schemes, e.g. [3, 4, 5, 12, 13, 18], allocate resources to users with good channel conditions without satisfying those who are in poor conditions. In other words, the QoS constraint (14) is not satisfied for each individual user resulting inproblematic and unfair allocation of the available resources.

$$\begin{cases} P\left(\left\{\xi_j^*\right\},\left\{\mu^*\right\}\right) = P_{TOTAL} - E\left[\sum_{i=1}^{N_F}\sum_{j=1}^{K}\frac{\tilde{s}_{ij}^*\cdot\sigma_z^2}{\left|h_{ij}\right|^2\cdot\eta_{ij}}\cdot\left[2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\frac{\left(\xi_j^*+1\right)\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{2^{u_j^0}\cdot\tilde{s}_{ij}^*\cdot\mu^*\cdot\sigma_z^2\cdot\ln(2)}}\right)\right)\right)-1\right]^+\right] = 0, \forall j \\ F_j\left(\left\{\xi_j^*\right\},\left\{\mu^*\right\}\right) = E\left[\xi_j^*\cdot\sum_{i=1}^{N_F}\ln\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}^*\cdot\log_2\left(2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\frac{\left(\xi_j^*+1\right)\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{2^{u_j^0}\cdot\tilde{s}_{ij}^*\cdot\mu^*\cdot\sigma_z^2\cdot\ln(2)}}\right)\right)\right)\right)-u_j^0\right)-q_j\left(B,T_j^{\max},\lambda_j\right)\right] = 0 \end{cases} \quad (22)$$

In (22), if $F_j\left(\left\{\xi_j^*\right\},\left\{\mu^*\right\}\right)=0, \forall \xi_j^* \in \left\{\xi_j^*\right\}$ then the QoS constraint (14) is ensured for each user, whereas if $P\left(\left\{\xi_j^*\right\},\left\{\mu^*\right\}\right)>0$ means that part of the supplied power P_{TOTAL} remains unexploited. Solving (22) through the Semi-Implicit-Root (SIR) finding approach [25, 26], we straight forwardly obtain the optimal multipliers $\left\{\xi_j^*\right\}$ and $\left\{\mu^*\right\}$, and consequently the optimal solutions $\tilde{s}_{ij}^*$ and $\tilde{p}_{ij}^*$ from (19) and (21), respectively. At this point we can additionally find the minimum required power $P_{\min}$ required to support all of the delay constraints of the heterogeneous system users by solving the following system of equations.

$$\begin{cases} P_{\min} = E\left[\sum_{i=1}^{N_F}\sum_{j=1}^{K}\frac{\tilde{s}_{ij}^*\cdot\sigma_z^2}{\left|h_{ij}\right|^2\cdot\eta}\cdot(\kappa-1)^+\right] \\ E\left[\sum_{i=1}^{N_F}\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}^*\cdot\tilde{r}_{ij}^*\right)-u_j^0\right)\right] = q_j\left(B,T_j^{\max},\lambda_j\right) \end{cases}.$$

We notethat it is useful to find $P_{\min}$ prior to the allocation process in order to determineif the total supplied power P_{TOTAL} is sufficientfor our system given the current channel conditions, i.e., if $P_{\min} > P_{TOTAL}$ it becomesmeaningless to proceed.

Comments on the Scheme's Convergence and Feasibility: The convergence of the introduced scheme is guaranteed for the following reason. For a single system user j, as ξ_j^* increases the function $H_{ij}\left(\xi_j^*,\mu^*\right)$ decreases $\forall i \in N_F$. Hence, more of the $\tilde{s}_{ij}^*$ s in (19) become one and the term $\sum_{i=1}^{N_F}\tilde{s}_{ij}^*\cdot\tilde{p}_{ij}^*$ increases. During the change of $\tilde{s}_{ij}^*$ some of the other $\tilde{s}_{ij}^*$, i.e., $\tilde{s}_{ij'}^*$ s change from one to zero and by their turn they decrease the throughput for other users. However, for all system users, as all the possible ξ_j^* s increase, the optimal powers $\tilde{p}_{ij}^*$ s increase accordingly. Consequently, the algorithm converges to a unique solution that satisfies all the constraints found in (16)-(18)[5]. In addition, since the problem in (16)-(18) is a convex optimization problem over a convex set, the unique optimal solution is also sufficient to satisfy the set of all of the necessary conditions.

In order to examine the feasibility of the introduced scheme, we focus on the subcarrier allocation constraints from (11) and (12). From *Theorem 2* it is easy to conclude that for each subcarrier i, the function $H_{ij}\left(\xi_j^*,\mu^*\right)$ is different for all users. From the search in (20) only the user with the largest $H_{ij}\left(\xi_j^*,\mu^*\right)$ can use aspecific subcarrier, i.e., $\tilde{s}_{ij^*}^*=1$ and $\tilde{s}_{ij}^*=0$ for all $j \neq j^*$. Also, as the ergodic realizations $\left\{\left|h_{ij}\right|^2\right\}$ are i.i.d for different users, the optimal search in (20) is always feasible since the chance for the function $H_{ij}\left(\xi_j^*,\mu^*\right)$ to be the same for different users happens only with probability 0.

The Complexity of the Proposed Method: One shouldrecall that the primary purpose of utilizing the time-sharing method is to avoid the mixed combinatorialsearch needed bythe problem laid out in (10)-(15). The implementation complexity of the introduced scheme dependsonly on the subcarrier allocation search (20) in *Theorem 2*. It is easy to seethat the theoretical complexity of (20) is linearto the number of users and subcarriers, i.e., $\mathcal{O}\left(N_F\cdot K\right)$, with $\mathcal{O}(\cdot)$ to denote the big-*O* notation [26]. Contrarily, the mixed combinatorial solution of (10)-(15) has the exponential complexity of $\mathcal{O}\left(K^{N_F}\right)$, significantly higher than our proposal. To obtain a clearer view on our scheme's complexity, we examined the implementation process of our optimal policies given via the dual decomposition method. In the latter case the theoretical complexity is significantly higher compared tothe convex optimization based solutions, since only the ellipsoid method utilized in the dual decomposition converges in $\mathcal{O}\left((K+1)^2\right)$ iterations [27]. In addition, we compared our optimal solutions with those given in [28], where the widely adopted Hungarian method wasapplied. We found that the Hungarian method hasa computational complexity of $\mathcal{O}\left(N_F^4\right)$, which is also notably higher than the complexity of our solution.Similar conclusions are obtained whenexamining the further advancements of the Hungarian algorithm developed in [29]. For a more well documented view, we compared the complexity of a similar scheme presented in [13] with ours and found that its complexity is $\mathcal{O}\left(N_F\cdot K^2\cdot\log_2\left(N_F\right)+K^4\right)$ when multiple us-

[5] A similar mechanism is presented in [22], where more information regardingits convergence can be found in detail. Also, for the system of non-linear equations in (22), we remark that we observed rapid convergence towards its roots when we applied the SIR [25], which seems to perform significantly better than Newton's-Raphson, Bisection, Secant and Brent's methods [26].

ers participate in the system and $\mathcal{O}\left(N_F \cdot K \cdot \log_2\left(N_F\right)\right)$ for the two-users case.In conclusion, in all our comparisons the proposed scheme has the lowest theoretical complexity amongst allof the relative examined approaches.

6. Simulation Results

In this section we examine the performance of our proposed resource allocation strategies by means of the trade-off between the required power, the data rate, and the provided fairness. To clarify our evaluations, we performed-simulation comparisonswith three other relevant approaches; the M-R [7, 13, 14], the Fixed-Rate (F-R) [18], and the Max-Min (M-M) fairness schemes [9].

6.2 The Simulation Model

We considered a single-cell OFDMA system with $BW = 80KHz$, $N_F = 64$, and $t_s = 0.002sec$. The frequency selective fading channel wasspecified according to the ISU-3 model for pedestrian and vehicular mobility in urban environments. We also assumed that the data packets hada fixed sizeof $B = 80bits$ [6]. Additionally, we set the heterogeneous users into four different classes,given as (K_1, K_2, K_3, K_4), where the K_1 class 1 users have higher QoS requirements than the K_2 class 2 users, which have higher QoS requirements than the K_3 class 3 users. The K_4users are the un-classed users who have high delay tolerance, e.g., $T_{\{K_4\}}^{\max} \to \infty$. Each user class's parameters are shown in Table 1.

Table 1. The Heterogeneous Users Characteristics

Parameters	Class 1	Class 2	Class 3	Un-Classed
Maximum delay tolerance T_j (timeslots)	**2**	**4**	**8**	∞
Poisson arrival rate λ_j (packets/timeslot)	0.5	0.3	0.2	-
Packet size (bits)	80	80	80	80
Data Rate (bits/sec/Hz)	5.8564	3.0384	1.6	-

6.3 The Simulation Results

Fig. 2 depicts a comparison of the minimum required power versus the number of class 1 users possessing different allocation schemes. As can be seen, the S-NBS scheme consumes 2.35*dB* more than the M-R, whereas at the same time the M-M and F-R require even more power in order to meet the desired QoS levels. In general, the power requirements of all of the schemes increasedlinearly as the number of class 1 users increased. As was expected, the S-NBS requires more power than the M-R in order to provide fairness amongst users, due to the fact that some users may be in a deep fade. In addition, the M-M and F-R schedulers demand more power, with the F-R being the most power demanding.

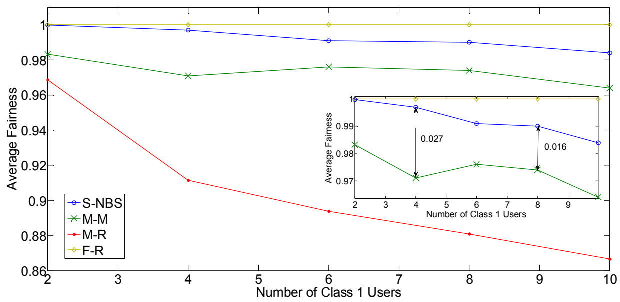

Fig. 3. The average fairness vs. the total number of system users (Class 1 users)

Fig. 3 illustrates the fairness performance of the compared schemes versus the number of class 1 users. In order to quantify the fairness level foreach case, we adopted the fairness index F from [30] as $F = \left(\sum_{j=1}^{K} R_j / R_j^{\min}\right)^2 / \left(K \cdot \sum_{j=1}^{K}\left(R_j / R_j^{\min}\right)^2\right)$ where R_j represents the allocated rate to user j and $R_j^{\min}$ the user's minimum required rate for each of the examined schemes. The F-R scheme allocates a fixed rate to each class 1 user, meaning that the achieved fairness index F is equal to one, i.e., $F = 1$, when homogeneous QoS support is required [18][7].The higher power consumption and the lower overall rate amongst all of the schemes is the price that F-R pays for perfect fairness. S-NBS scheme achieves a fairness of $F = 0.993$, which is the best of the remainingschemes, as the M-M and M-R have fairness indexes of $F = 0.967$ and $F = 0.911$, respectively. Therefore, from Fig. 3 it can be seen that the S-NBS provides significantly higher fairness than the opportunistic M-R scheme, and it also accomplishes similar power consumption and overall throughput performances as theM-R.

A comparison of the average delay of each scheme versus different numbers of class 1 users is shown in Fig. 4. To perform themetrics, every time that twousers wereadded to the system, we increased the initial value of the supplied power $P_{TOTAL} = 3.5dB$ by $\Delta P_{TOTAL} = 2dB$. As more users wereadded to the system the overall QoS requirements increased. Through these settings we aimedto examine each scheme's behaviour over different power starvation conditions in terms of average delay, namely, when the total supplied power P_{TOTAL} is not enough to support the required QoSthe users will suffer from delays. It can be

[6] We recall that data packets from physical and higher layers are multiplexed and served at the cross-layer through a realistic but exhaustive cross-layer system service [20].

[7] By definition $F = 1$ indicates perfect fairness provision to homogenous users [18]. In the case of heterogeneous users, the F-R scheme still allocates a fixed rate to each user meaning that on one hand the F-R will achieve the maximum value of the fairness index F but on the other hand F will be less than one, i.e., $F < 1$, since the minimum QoS will be different for each user.

seen that even in the case where 10 class 1 users participated in the system, the M-R and S-NBS methods werecapable ofproviding average delays below the minimum QoS threshold. The price for the overall system's performance due to S-NBS fairness provision is an average of $2.46 timeslots$ more delay than the opportunistic M-R scheme. Nevertheless, the M-M and F-R schemes fail to provide the minimum delay requirements when $K > 6$ as the required power is less than the available P_{TOTAL}.

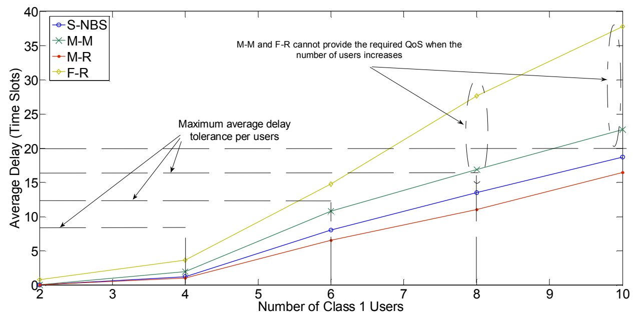

Fig. 4. The average delays vs. the total number of system users (Class 1 users)

Fig. 5 depicts the average throughput versus the average transmit power performance foreach scheme. As expected, we can seethat the opportunistic M-R scheme achieved a higher throughput than the S-NBS in all cases. For example, the M-R reachedan average throughput of $58.56 bits/sec/Hz$ when the average transmitting power was $13.58dB$, whereasthe S-NBS scheme neededan additional of $2.87dB$ more power to attainthe same throughput level. However, the extra $2.87dB$ is an inconsiderable amount when accountingfor the fair allocation provided by the S-NBS scheme. Moreover, the other two schemes, the M-M and F-R, attainedsignificantly lower performance than the S-NBS and M-R. For instance, when $P_{TOTAL} = 26dB$. the of M-M and F-R throughput was $74.16 bits/sec/Hz$ and $71.65 bits/sec/Hz$ whereas theM-R and S-NBS achieved throughputs of $105.68 bits/sec/Hz$ and $98.77 bits/sec/Hz$, respectively.

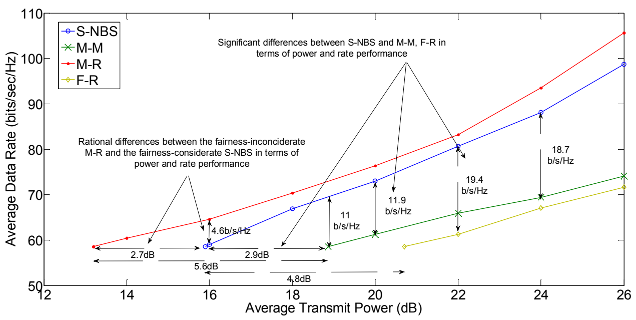

Fig. 5. The average data rate vs. the average transmitting power

In Fig. 6, we investigatethe average data rate per user vesus the overall transmit power in a system consisting of three users each with different classes, i.e., $(K_1, K_2, K_3, K_4,) = (1,1,1,0)$. As can be seen in sub-figures 6.a, 6.b, 6.c and 6.e, the S-NBS scheme performed the best amongst the others, offering the highest tradeoff between fairness and efficiency. In particular, each heterogeneous user wasallocated with at least hisminimum QoS requirment. When the supplied power P_{TOTAL} was greater thanits minimum value, i.e., $P_{TOTAL} > 5.956dB$ (more resources available for distribution), the S-NBS allocated the same data rates to all of the users. In other words, the proposed scheme has the ability to share the extra resources equaly among the heterogeneous users offering close to ideal fairness. On the other hand, in sub-figure 6.b the M-R scheme alloates the extra resources opportunistically, totally ignoring the fairness parameter.In sub-figure 6.c, the M-M requires more resourcesthan the S-NBS to satisfy each user'sminimum QoS requirements, whereasusers remained unsatisfied at thelarge power region. In sub-figure 6.d, although the F-R scheme offers close to perfect fairness,it had asignificantlylower data rates than all of the others.To further clarify our sheme's performance, we summarized in sub-figure 6.e the overall allocated data rate versus the overall supplied power under heterogeneous QoS considerations. The proposed S-NBS achieves the highest rate performance amongstall of the fairness considerate schemes,including the M-M and F-R.

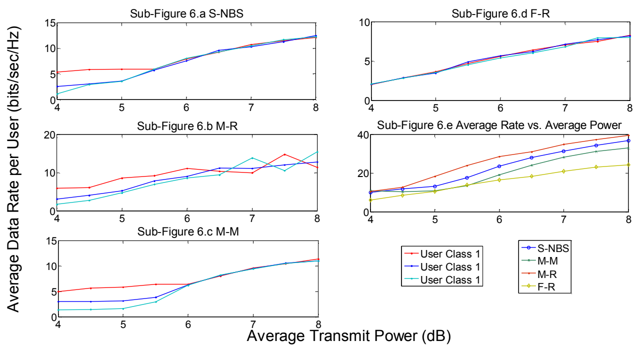

Fig. 6. The average data rate per user vs. the average transmit power

In summery,from the above simulation results we can conclude that the S-NBS-schemeoffersa profitable solution that significantly improvesthe trade-off between the fairness provision and throughput/power performance compared to the relevant approaches, such as the M-R, M-M and F-R schemes.

7. Conclusion

In this paper,we presenta game theoretic cross-layer design for OFDMA networks that allocates subcarriers and poweraccording tothe S-NBS property.Initially, we correlated the system's regulations forboth the physical layer and network layers in order to express them fromthe cross-layer perspective. In addition, we utilized utility theory to express each user's level of satisfaction by means of throughput. Using these factors, we formulated a cross-layer problem that aimed to maximize the overall level of satisfaction subject to the subcarrier, power and QoS constraints. Furthermore, we proved that the cross-layer problem can be transformed in a convex manner over a convex feasible set. Through the utilizationof a novel solution methodology, we then applied convex optimization to obtain the optimal allocation strategies by means of final formulas. We show that our solutions can be implemented by a low complexity iteration process that converges rapidly to the global optimal. Finally, we demonstrated with simulation comparisons torelevant approachesthat the proposed designacquiressignificantly bettertrade-off performance between the QoS support and fairness provision with a comparably greaterthroughput increase.

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Appendix A – The Convesity of the Cross-Layer Problem (Proof of *Proposition 1*)

We initially prove that the optimization objective function in (16)-(18) is convex. The system's utility objective (16), is a summation of utilities of the form $f\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)=\ln\left(\tilde{s}_{ij}\cdot\log_2\left(1+\left(\left(\tilde{p}_{ij}\cdot A\right)/\tilde{s}_{ij}\right)\right)-C\right)$, where A and C are positive constraints. The Hessian matrix of $f\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$ can be easily found to be negative semi-definite. Hence each $f\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$ is a concave function over the $K\cdot N_F+K\cdot N_F$ dimensional space $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$. Therefore, the system's utility objective (16) is also a concave function, since any positive linear combination of concave functions is also a concave function.

Secondly we prove that the cross-layer problem (16)-(18) is determined over a feasible convex set that satisfies all of the involved constraints. The inequality constraints (17) are straightforward convex, whereas constraints (18) are all affine. This means that the set defined by all constraints as well as the system's utility objective (16) is convex as it is well known that the intersection of convex sets is also convex. Additionally, the cross-layer problem laid out in (16)-(18) is convex and soa unique global optimalexists, which is obtained in polynomial time.

Finally, we verify that the feasible set that satisfies constraints (11), (12), (13), (17) and (18) is non-empty. Assuming that $u_j^0=q_j$ the convex set over $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$ is non-empty since, i.e. for $\tilde{p}_{ij}=0$ and $\tilde{s}_{ij}=1$, all constraints are satisfied. Let us denote S_1 as the feasible set over $\tilde{s}_{ij}$ that satisfies the subcarrier allocation constraints (11), (12), (17) and $\tilde{s}_{ij}\in\left(0,1\right]$, and S_2 as the feasible set over $\tilde{p}_{ij}$ that satisfies the power constraints (13) and (18). Then in the $K\cdot N_F+K\cdot N_F$ dimensional space $\left(\tilde{s}_{ij},\tilde{p}_{ij}\right)$, constraints (11), (12), (17) and $\tilde{s}_{ij}\in\left(0,1\right]$ of $\tilde{s}_{ij}$ verify a cylinder with base S_1. Similarly constraints (13) and (18) of $\tilde{p}_{ij}$ verify another cylinder with base S_2. The intersection of the two cylindersobviously determines a non-empty set over $\left(\tilde{s}_{ij}, \tilde{p}_{ij}\right)$, which is also convex due to the convexity and affinity of all the constraints. This completes the proof of *Proposition 1*. ■

Appendix B – Optimal Allocation Strategies (Proofs of *Theorems 2&3*)

The Lagrangianfunction $\tilde{L}\left(\{\tilde{p}_{ij}\},\{\tilde{s}_{ij}\},\xi,\mu,v\right)$ of the cross-layer problem (16)-(18) is written as:

$$\tilde{L}\left(\{\tilde{p}_{ij}\},\{\tilde{s}_{ij}\},\xi,\mu,v\right)=E\left[\sum_{j=1}^{K}\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}\cdot\log_2\left(1+\frac{\tilde{p}_{ij}\cdot|h_{ij}|^2\cdot\eta_{ij}}{\tilde{s}_{ij}\cdot\sigma_z^2}\right)\right)-u_j^0\right)\right]+$$
$$E\left[\sum_{j=1}^{K}\xi_j\cdot\left(\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}\cdot\log_2\left(1+\frac{\tilde{p}_{ij}\cdot|h_{ij}|^2\cdot\eta_{ij}}{\tilde{s}_{ij}\cdot\sigma_z^2}\right)\right)-u_j^0\right)-q_j\left(B,T_j^{\max},\lambda_j\right)\right)\right]-$$
$$\mu\cdot\left(\left(\sum_{j=1}^{K}\sum_{i=1}^{N_F}\tilde{p}_{ij}\right)-P_{TOTAL}\right)-\sum_{i=1}^{N_F}v_i\cdot\left(\left(\sum_{j=1}^{K}\tilde{s}_{ij}\right)-1\right),\tag{23}$$

where the vectors $\xi=\left(\xi_1,...,\xi_j,...,\xi_K\right)\geq 0$, $\mu\geq 0$ and $v=\left(v_1,...,v_i,...,v_{N_F}\right)$ represent the Lagrangian multipliers for the QoSconstraint (17), the power constraint (18) and the subcarrier allocation constraint (12), respectively. By applying the KKT conditions, the boundary constraints $\tilde{s}_{ij}\in(0,1]$ and $\tilde{p}_{ij}\geq 0$ are absorbed and the sufficient conditions for the optimal instantaneous power $\tilde{p}_{ij}^*$ and the optimal subcarrier allocation index $\tilde{s}_{ij}^*$ to be global maxima are obtained by:

$$\left.\frac{\partial\tilde{L}\left(\{\tilde{p}_{ij}\},\{\tilde{s}_{ij}\},\xi,\mu,v\right)}{\partial\tilde{p}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\xi_j,\mu,v_i\right)=\left(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\xi_j^*,\mu^*,v_i^*\right)}\begin{cases}\text{infeasible, } \tilde{p}_{ij}^*<0\\ \leq 0,\ \tilde{p}_{ij}^*=0\\ =0,\ \tilde{p}_{ij}^*>0\end{cases},\tag{24}$$

$$\left.\frac{\partial\tilde{L}\left(\{\tilde{p}_{ij}\},\{\tilde{s}_{ij}\},\xi,\mu,v\right)}{\partial\tilde{s}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\xi_j,\mu,v_i\right)=\left(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\xi_j^*,\mu^*,v_i^*\right)}\begin{cases}<0,\ \tilde{s}_{ij}^*=0\\ =0,\ \tilde{s}_{ij}^*>0\end{cases},\tag{25}$$

$$\xi_j^*\cdot E\left[\ln\left(\left(\sum_{i=1}^{N_F}\tilde{s}_{ij}^*\cdot\log_2\left(1+\frac{\tilde{p}_{ij}^*\cdot|h_{ij}|^2\cdot\eta_{ij}}{\tilde{s}_{ij}^*\cdot\sigma_z^2}\right)\right)-u_j^0\right)-q_j\left(B,T_j^{\max},\lambda_j\right)\right]=0.\tag{26}$$

When $\tilde{s}_{ij}^*=0$ then the result is infeasible or it gives a local maxima, i.e., $\left(\tilde{p}_{ij}^*\cdot\frac{\partial\tilde{L}}{\partial\tilde{p}_{ij}}\right)+\left(\tilde{s}_{ij}^*\cdot\frac{\partial\tilde{L}}{\partial\tilde{s}_{ij}}\right)\leq 0$, $\forall\tilde{s}_{ij}\in(0,1]$, $\tilde{p}_{ij}>0$. When $\tilde{s}_{ij}^*\neq 0$ then $\tilde{p}_{ij}^*>0$ meaning that the KKT condition (24) will resolveto a global maximum solution over $\tilde{p}_{ij}$ as:

$$\left.\frac{\partial\tilde{L}\left(\{\tilde{p}_{ij}\},\{\tilde{s}_{ij}\},\xi,\mu,v\right)}{\partial\tilde{p}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\xi_j,\mu,v_i\right)=\left(\tilde{p}_{ij}^*,\tilde{s}_{ij}^*,\xi_j^*,\mu^*,v_i^*\right)}=0\quad\overset{a=\frac{(\xi_j^*+1)\cdot|h_{ij}|^2\cdot\eta_{ij}}{\mu^*\cdot\sigma_z^2\cdot\ln(2)}}{\Rightarrow}$$
$$\left(1+\frac{\tilde{p}_{ij}^*\cdot|h_{ij}|^2\cdot\eta_{ij}}{\tilde{s}_{ij}^*\cdot\sigma_z^2}\right)\cdot\left(\left(\tilde{s}_{ij}^*\cdot\log_2\left(1+\frac{\tilde{p}_{ij}^*\cdot|h_{ij}|^2\cdot\eta_{ij}}{\tilde{s}_{ij}^*\cdot\sigma_z^2}\right)\right)-u_j^0\right)=a.\tag{27}$$

It is easy to seethat (27) is a transcendental algebraic equation,meaning that its explicit solution is very hard and perhaps impossible to be defined. Usually, recursive searches are utilized to approximate the solutions resulting incomplex and time-consuming procedures. Contrary tothe traditional way, we present a new methodology for solving such equations that allows us to derive their explicit solutions.

Let us define $b=u_j^0$ and $1+\left(\left(\tilde{p}_{ij}^*\cdot|h_{ij}|^2\cdot\eta_{ij}\right)/\left(\tilde{s}_{ij}^*\cdot\sigma_z^2\right)\right)=x$. Then (27) has the form of $x\cdot\left(\tilde{s}_{ij}^*\cdot\log_2(x)-b\right)=a$ or

$$\tilde{s}_{ij}^*\cdot\log_2\left(x^x\right)=bx+a.\tag{28}$$

If we set $b=\log_2 c$, $c>1$ then equation (28) becomes $\tilde{s}_{ij}^*\cdot\log_2\left(x/c\right)^x=a$ and by multiplying both its sides with $1/c$ it becomes:

$$\log_2\left(\frac{x}{c}\right)^{\frac{x}{c}}=\frac{a}{\tilde{s}_{ij}^*\cdot c}.\tag{29}$$

In addition, by defining that $\varphi=x/c$ then we can denote (29) as $\log_2(\varphi)^\varphi=a/\left(\tilde{s}_{ij}^*\cdot c\right)$, which gives that $\varphi^\varphi=2^{a/\left(\tilde{s}_{ij}^*\cdot c\right)}$. Based on Lambert-*W* function's properties [24], the latter equation resolves into $\varphi=\exp\left(W\left(\ln\left(2^{a/\left(\tilde{s}_{ij}^*\cdot c\right)}\right)\right)\right)$, where $W(\cdot)$ denotes the Lambert-*W* function [24]. By substituting φ we obtain:

$$x=c\cdot\exp\left(W\left(\ln\left(2^{\frac{a}{\tilde{s}_{ij}^*\cdot c}}\right)\right)\right),\ 2^{\frac{a}{\tilde{s}_{ij}^*\cdot c}}>1,\tilde{s}_{ij}^*\in(0,1].\tag{30}$$

With further substitutions of x, a, b and c into (30) the optimal power allocation policy $\mathcal{P}^{*}_{K\times N_F}\left[\mathbf{H}_{K\times N_F}\right]=\left[\tilde{p}^{*}_{ij}\right]$ has individual matrix elements defined as:

$$\tilde{p}^{*}_{ij}=\frac{\tilde{s}^{*}_{ij}\sigma_z^2}{\left|h_{ij}\right|^2\eta_{ij}}\left[2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\frac{\left(\xi_j^*+1\right)\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{2^{u_j^0}\cdot\tilde{s}^{*}_{ij}\cdot\mu^*\cdot\sigma_z^2\cdot\ln(2)}}\right)\right)\right)-1\right]. \tag{31}$$

Furthermore, we use the KKT condition (25) to derive the optimal subcarrier allocation as:

$$\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\xi,\mu,v\right)}{\partial s'_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\xi_j,\mu,v_i\right)=\left(\tilde{p}^*_{ij},\tilde{s}^*_{ij},\xi^*_j,\mu^*,v^*_i\right)}=0\Rightarrow$$

$$\left(1+\xi_j^*\right)\cdot\left(\frac{\log_2\left(1+\frac{\tilde{p}^*_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\tilde{s}^*_{ij}\cdot\sigma_z^2}\right)-\frac{\tilde{p}^*_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\tilde{s}^*_{ij}\cdot\sigma_z^2\cdot\ln(2)\left(1+\frac{\tilde{p}^*_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\tilde{s}^*_{ij}\cdot\sigma_z^2}\right)}}{\left(\tilde{s}^*_{ij}\cdot\log_2\left(1+\frac{\tilde{p}^*_{ij}\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}}{\tilde{s}^*_{ij}\cdot\sigma_z^2}\right)\right)-u_j^0}\right)-v_i^*=0. \tag{32}$$

With the substitution of the optimal power allocation $\tilde{p}^*_{ij}$ in (31) then (32) becomes:

$$\underbrace{\left(1+\xi_j^*\right)\cdot\left(\frac{\log_2\left(2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\kappa}\right)\right)\right)\right)-\frac{2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\kappa}\right)\right)\right)-1}{\ln(2)\cdot\left(2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\kappa}\right)\right)\right)\right)}}{\left(\tilde{s}^*_{ij}\cdot\log_2\left(2^{u_j^0}\cdot\exp\left(W\left(\ln\left(2^{\kappa}\right)\right)\right)\right)\right)-u_j^0}\right)}_{H_{ij}\left(\xi_j^*,\mu^*\right)}-v_i^*=0, \tag{33}$$

with the variable κ used for brevity and denoted by $\kappa=\left(\left(\xi_j^*+1\right)\cdot\left|h_{ij}\right|^2\cdot\eta_{ij}\right)/\left(2^{u_j^0}\cdot\tilde{s}^*_{ij}\cdot\mu^*\cdot\sigma_z^2\cdot\ln(2)\right)$. We now assume that all of the time sharing factors $\left\{\tilde{s}^*_{ij}\right\}$ in (33) are equal to each other, i.e., $\tilde{s}^*_{ij}=0.5,\forall i,j$. The reason we make this assumption is to make (33) independent from the unknown $\left\{\tilde{s}^*_{ij}\right\}$ and perform the S-NBS decision regarding thesubcarrier selection relying on the players' cooperation and each channel's physical conditions. In other words, given that the time-sharing factors $\left\{\tilde{s}^*_{ij}\right\}$ are homogeneous, then through $H_{ij}\left(\xi_j^*,\mu^*\right)$ we can indicate whichof the N_F sub-carriers isappropriateto be allocated to each of the K users, according to each channel's conditions and the S-NBS-based cooperation between users[8]. Consequently, condition (25) becomes

$$\left.\frac{\partial\tilde{L}\left(\left\{\tilde{p}_{ij}\right\},\left\{\tilde{s}_{ij}\right\},\xi,\mu,v\right)}{\partial\tilde{s}_{ij}}\right|_{\left(\tilde{p}_{ij},\tilde{s}_{ij},\xi_j,\mu,v_i\right)=\left(\tilde{p}^*_{ij},\tilde{s}^*_{ij},\xi^*_j,\mu^*,v^*_i\right)}\begin{cases}=0, & \text{if } 0<\tilde{s}^*_{ij}<1\\ >0, & \text{if } \tilde{s}^*_{ij}=1\end{cases}$$

meaning that the optimal subcarrier index $\tilde{s}^*_{ij}$ willbeequal to one for $H_{ij}\left(\xi_j^*,\mu^*\right)>v_i^*$ otherwise the allocation will-not occur. This completes the proofs of *Theorems 2 &3*.

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[8] The value of $H_{ij}\left(\xi_j^*,\mu^*\right)$ is usually small for large channel realizations $\left\{\left|h_{ij}\right|^2\right\}$. This means that subcarriers in good conditions (large $\left|h_{ij}\right|^2$) require less time (small $H_{ij}\left(\xi_j^*,\mu^*\right)$) to transfer the same amount of information than subcarriers in bad conditions (small $\left|h_{ij}\right|^2$ and large $H_{ij}\left(\xi_j^*,\mu^*\right)$). However, this does not always hold as $H_{ij}\left(\xi_j^*,\mu^*\right)$ is also dependent by the S-NBS-based cooperation.

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