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Mathematical Thinking through Problem Solving and Posing with Fractions[1]

CHENG, Chun Chor Litwin
Department of Mathematics and Information Technology, Hong Kong Institute of Education,
10 Lo Ping Road, Tai Po, New Territories, Hong Kong; Email: twwong@ied.edu.hk

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One of the important aims in mathematics education is to enhance mathematical thinking for students. And students posing questions is a vital process in mathematical thinking as it is part of the reasoning and communication of their learning. This paper investigates how students develop their mathematical thinking through working on tasks in fractions and posing their own questions after successfully solved the problems. The teaching was conducted in primary five classes and the results showed that students' reasoning is related to their analogy with what previously learned. Also, posing their problems after solving the problem not only helps students to understand the structure of the problem, it also helps students to explore on different routes in solving the problem and extend their learning content.

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INTRODUCTION

Mathematical thinking is a vital element in many curriculums. How to bring out mathematical thinking? In general terms, problem solving and posing is deemed as effective approach in achieving this aim. Problem posing and solving together provide a solid platform for reasoning. And teaching tasks of mathematical structure is helpful for students in solving the tasks (Cheng, 2008). And problem posing is a kind of communication in mathematics structure.

Dienes (1971) proposed the Mathematical Variability Principle of learning mathematics, that concept learning and strengthening of concept is best through a learning process

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of variation of the content and the structure of mathematics. And learners can construct and extend their knowledge from such experience. In this sense, the posing of question by students allows the learning to happen with students' own variation on the questions they learned. What further evidence is how these variations happened in the posing of question enhance thinking. In English's suggestions (1998, 2004), these reasoning are in the form of analogy, analogy with the format of question and the analogy in the thinking process in the course of problem solving.

Fraction is a good topic for bringing out mathematical thinking. Using fraction as a tool to solve practical problems is common in many curriculums and Hong Kong is of no exception (CDC, 2002). Problem solving with fractions enable students to apply their knowledge in a wider areas so that the learning of fractions become more meaningful. Also, the extensions of knowledge on fractions help to relate more mathematical knowledge for understanding, making it a bigger domain of knowledge.

THE STUDY AND METHOD

The study is on classroom learning involving reasoning, problem solving and problem posing as communication. The reasoning element in this study is through the tool of equivalent fraction to investigate possible solutions on certain tasks. The problem solving part involves finding fraction under certain requirement. This will be enhanced by communication in mathematics through posing question and solving new problem with their learned and extended knowledge.

The teaching was conducted in two classes of primary 5. Both with three lessons, class A with problem solving alone on the tasks on equivalent fractions, class B will undergo the process of problem posing before working on problem solving. How students learn and how they solved the problem was observed. The study also compared their approaches in solving problem. The following questions were used is both class.

	Lesson 1	Lesson 2	Lesson 3
Question	$\frac{\square}{\square}=\frac{\square}{\square}, \frac{\square}{\square}=\frac{\square}{\square\square}$	$\frac{\square}{\square\square}=\frac{\square}{\square\square}, \frac{\square}{\square}=\frac{\square\square}{\square\square}$	$\frac{a+5}{b+5}\to\frac{3}{4}$
Class A	Problem Solving	Problem Solving	Problem Solving
Class B	Posing Question and Solving	Posing Question and Solving	Problem Solving

REASONING AND COMMUNICATION THROUGH FRACTIONS

Lesson 1 Reasoning with equivalent fractions

The two classes are given the same reasoning tasks. The objective of the tasks is to use their knowledge and skill of equivalent fractions to find the solution. For the unstructured activities, students are asked to think of any fractions that are equivalent fractions. Then it followed by a structured task fraction, with the same fraction format but required all numbers filled in are without repetitions.

Task 1 (Unstructured)	**Task 2 (Structured)**
Using any integers 1-9 to form equivalent fractions, $\frac{\square}{\square}=\frac{\square}{\square}$ (for example $\frac{1}{2}=\frac{2}{4}$).	Using integers 1-9 to form equivalent fractions $\frac{\square}{\square}=\frac{\square}{\square}$, the numbers used are without repetitions (for example $\frac{1}{2}=\frac{3}{6}$).

For the unstructured task, most of the solutions provided by both classes are

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6},\text{ or }\frac{2}{3}=\frac{4}{6}=\frac{6}{9}\text{ and }\frac{3}{4}=\frac{6}{8}=\frac{9}{12}\text{ etc.}$$

The unstructured task served as warm up activities for the students. For the structured task, they make use of the result of equivalent fraction

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$$

and select solutions:

$$\frac{1}{2}=\frac{3}{6},\frac{1}{2}=\frac{4}{8},\frac{3}{6}=\frac{4}{8},\ \frac{2}{4}=\frac{3}{6}.$$

and The next relation make used by students is

$$\frac{1}{3}=\frac{2}{6}=\frac{6}{9},$$

and some of the solutions obtained are listed below:

Fractions used	Solutions by equivalent fractions			
$\frac{1}{2},\ \frac{2}{4},\ \frac{3}{6}$	$\frac{1}{2}=\frac{3}{6}$	$\frac{1}{2}=\frac{4}{8}$	$\frac{2}{4}=\frac{3}{6}.$	$\frac{3}{6}=\frac{4}{8}$
$\frac{1}{3},\ \frac{2}{3}$	$\frac{1}{3}=\frac{2}{6}$	$\frac{2}{6}=\frac{3}{9}$	$\frac{2}{3}=\frac{4}{6}$	$\frac{2}{3}=\frac{6}{9}$
$\frac{1}{4},\ \frac{3}{4}$	$\frac{1}{4}=\frac{2}{8}$		$\frac{3}{4}=\frac{6}{8}$	

Solving problem versus posing and solving the problem

Students of class A is asked to solve the following problem. They discussed the tasks and then start to solve it. For class B, students are asked to pose a question similar to the structured task

$$\frac{\square}{\square} = \frac{\square}{\square} \text{ (finding fractions that satisfying the format).}$$

During the discussion of problem posing in class B, it was hinted that they could pose a question which require more numbers in the fraction expression. After the discussion, the same task as in class A was assigned to class B.

Question (used in both class A and class B)
Using the integers from 1 to 9 to fill in the boxes, without repetition to make two fractions, so that $\frac{\square}{\square} = \frac{\square}{\square\square}$.

Approaches used by students in the task investigation (class A and class B)

Apart from some trial and error approach, students of the two classes tackle the question with similar approaches, using equivalent fractions to work on the solution.

The first thought in students' minds is to make use of the relation

$$\frac{1}{2} = \frac{\square}{\square\square},$$

and obtain the answers

$$\frac{1}{2} = \frac{7}{14}, \frac{1}{2} = \frac{8}{16}, \frac{1}{2} = \frac{9}{18} \text{ etc.}$$

However, these results did not comply with the restriction of non-repeated use of numbers. Further, students observed that there is no solution of the form

$$\frac{1}{2} = \frac{a}{\square\square},$$

as the numerator a could only takes the values 6, 7, 8, 9, but all fractions repeated with the number 1

$$\left(\frac{1}{2} = \frac{6}{12} = \frac{7}{14} = \frac{8}{16} = \frac{9}{18}\right).$$

However, using the equivalent fraction of

$$\frac{1}{2}, \frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8},$$

and similarly with

$$\frac{1}{3}, \frac{2}{3}, \frac{3}{4}$$

and its equivalent, the following solutions were obtained .

Fraction	Solutions by equivalent fractions					
$\frac{1}{2}$	$\frac{2}{4}=\frac{8}{16}$	$\frac{2}{4}=\frac{9}{18}$	$\frac{3}{6}=\frac{9}{18}$	$\frac{3}{6}=\frac{7}{14}$	$\frac{4}{8}=\frac{6}{12}$	
$\frac{1}{3}$	$\frac{1}{3}=\frac{8}{24}$	$\frac{1}{3}=\frac{9}{27}$	$\frac{3}{9}=\frac{7}{21}$	$\frac{3}{9}=\frac{4}{12}$	$\frac{3}{9}=\frac{8}{24}$	$\frac{3}{9}=\frac{6}{18}$
$\frac{2}{3}, \frac{3}{4}$	$\frac{4}{6}=\frac{8}{12}$	$\frac{6}{9}=\frac{8}{12}$		$\frac{3}{4}=\frac{9}{12}$	$\frac{6}{8}=\frac{9}{12}$	

Equivalent Fraction as Ratio Thinking

Though both class of students use similar approaches of equivalent fractions to work on the solution, some students in class B started to make use of ratio thinking in doing this task. This is an unexpected difference between the approaches used by students in the two classes. Based on the mathematical fact that

$$\frac{a}{x}=\frac{b}{yz} \Leftrightarrow \frac{a}{b}=\frac{x}{yz},$$

students in both classes found that

$$\frac{3}{9}=\frac{6}{18} \Leftrightarrow \frac{3}{6}=\frac{9}{18}.$$

From then on, students found that whatever solutions they found, they can base on it to get another solution. The following list some of the examples of students' discovery.

Solution based on equivalent fraction $\frac{1}{4}$	$\frac{1}{4}=\frac{9}{36}$	$\frac{2}{8}=\frac{9}{36}$	$\frac{2}{8}=\frac{4}{16}$	$\frac{1}{4}=\frac{7}{28}$	$\frac{1}{4}=\frac{8}{32}$
$\frac{a}{x}=\frac{b}{yz} \Leftrightarrow \frac{a}{b}=\frac{x}{yz}$	$\frac{1}{9}=\frac{8}{72}$	$\frac{2}{9}=\frac{8}{36}$	$\frac{2}{4}=\frac{8}{16}$	$\frac{1}{7}=\frac{4}{28}$	$\frac{1}{8}=\frac{4}{32}$

$\frac{1}{9}, \frac{2}{9}$	$\frac{1}{9}=\frac{8}{72}$	$\frac{1}{9}=\frac{7}{63}$	$\frac{1}{9}=\frac{6}{54}$	$\frac{1}{9}=\frac{4}{36}$	$\frac{1}{9}=\frac{3}{27}$	$\frac{2}{9}=\frac{4}{18}$
Equivalent Solution	$\frac{1}{8}=\frac{9}{72}$	$\frac{1}{7}=\frac{9}{63}$	$\frac{1}{6}=\frac{9}{54}$	$\frac{1}{4}=\frac{9}{36}$	$\frac{1}{3}=\frac{9}{27}$	$\frac{2}{4}=\frac{9}{18}$

Table 1. Approaches in solving the problem in Class A and Class B

Format of fraction in Question: $\frac{\square}{\square}=\frac{\square}{\square\square}$	Class A	Class B
Trial and Error, then by simple fractions	5	6
Using simple fractions $\frac{1}{2},\frac{1}{3}$ etc. and its equivalent	26	26
Using simple fractions $\frac{1}{2},\frac{1}{3}$ etc. and ratio thinking	2	7
Number of students	31	32

PROBLEM POSING AND COMMUNICATION

Lesson 2 Problem solving for class A and Posing and solving problem for class B

For students in class A, they are given the following two questions. Both questions required to use the number 1 to 9 without repetitions to form equivalent fractions. The following is the question assigned.

> Question (assigned for class A)
>
> Using the integers from 1 to 9 to fill in the boxes, without repetition to make two fractions so that
>
> (1) $\frac{\square}{\square\square}=\frac{\square}{\square\square}$, or (2) $\frac{\square}{\square}=\frac{\square\square}{\square\square}$.

The learning activities in class B involve posing their new problem related to what they solved in lesson 1. Similar hint was given to students that they could use 6 numbers or more in their problem posing. Students posed their problem based on their analogy with the format of the question in lesson 1. Finally, students in class B posed the following problems, but they are requested to solve exactly the two same questions as in class A to start with.

> Questions (Posed by Class B)
>
> Using the integers from 1 to 9 to fill in the boxes, without repetition to make two fractions so that
>
> (1) $\frac{\square}{\square\square}=\frac{\square}{\square\square}$, (2) $\frac{\square}{\square}=\frac{\square\square}{\square\square}$, (3) $\frac{\square}{\square\square}=\frac{\square\square}{\square\square}$, or (4) $\frac{\square\square}{\square\square}=\frac{\square\square}{\square\square}$.
>
> [class B students are asked to do (1) and (2) at the start.]

The following describe the approaches used by students in both class A and class B in solving the tasks.

Student's general approach, using simple fractions and its equivalent

Students tackle the two questions individually. At the start, format (2)

$$\frac{\square}{\square} = \frac{\square\,\square}{\square\,\square}$$

get more attention from students than the format (1)

$$\frac{\square}{\square\,\square} = \frac{\square}{\square\,\square},$$

as students think that it is more easy to get solutions through equivalent fraction in format (2). As in their previous experience, students of both class A and B start with the simplest fraction

$$\frac{1}{2} \text{ and its equivalent } \frac{1}{2} = \frac{34}{68} = \frac{37}{74} = \frac{38}{76} = \frac{39}{78} = \frac{43}{86} = \frac{48}{96}$$

to look for solutions. For the case of

$$\frac{1}{2},$$

all denominators will be larger than 60 so as to avoid repetitions used of numbers. From this process of reasoning, solutions are selected from the list. Similarly, using the relation,

$$\frac{3}{4} = \frac{9}{12} = \frac{39}{52} = \frac{51}{68} = \frac{72}{96}, \text{ and } \frac{1}{7} = \frac{2}{14} = \frac{3}{21} = \frac{4}{28} = \frac{5}{35} = \frac{6}{42} = \frac{7}{49} = \frac{8}{56} = \frac{9}{63},$$

the following solutions are obtained.

Fraction	Solutions obtained					
$\frac{1}{2}$	$\frac{1}{2}=\frac{34}{68}$	$\frac{1}{2}=\frac{37}{74}$	$\frac{1}{2}=\frac{38}{76}$	$\frac{1}{2}=\frac{39}{78}$	$\frac{1}{2}=\frac{43}{86}$	$\frac{1}{2}=\frac{48}{96}$
$\frac{3}{4}$	$\frac{3}{4}=\frac{9}{12}$	$\frac{3}{4}=\frac{39}{52}$	$\frac{3}{4}=\frac{51}{68}$	$\frac{3}{4}=\frac{72}{96}$		
$\frac{1}{7}$	$\frac{2}{14}=\frac{7}{49}$	$\frac{2}{14}=\frac{8}{56}$	$\frac{2}{14}=\frac{9}{63}$	$\frac{7}{49}=\frac{8}{56}$		

Approaches for $\frac{\square}{\square\,\square} = \frac{\square}{\square\,\square}$, observing numerators and denominators

There are two approaches used by students in dealing this format, the first one is focusing on the ratio of the two numerators, and the second one is focusing on the two denominators.

The reason for considering the numerators is straight forward; it is easier to look at the numerators than the denominators. Students posed the question also provide the following solution.

$$\frac{1}{34}=\frac{2}{68},\ \frac{1}{38}=\frac{2}{76},\ \frac{1}{39}=\frac{2}{78}.$$

This is the simplest ratio 1 to 2 for the denominators. Such reasoning provides students the direction of thinking. In fact, some cases of ratio were left out by students. For example, the ratio of numerators 2:4 is equivalent to 1:2, but this was left out earlier. However, once this is known, students start to explore other solution such as

$$\frac{2}{18}=\frac{4}{36},\frac{2}{19}=\frac{4}{38},\ \frac{2}{38}=\frac{4}{76},\ \frac{2}{39}=\frac{4}{78}\ \text{ etc.}$$

The ratio of numerators in 3:6 and 4:8 provide more solutions, such as

$$\frac{3}{14}=\frac{6}{28},\frac{3}{29}=\frac{6}{58},\ \frac{4}{13}=\frac{8}{26},\ \frac{4}{16}=\frac{8}{32},\ \frac{4}{31}=\frac{8}{62}\text{ etc.}$$

The working continues to include other ratios such as 1:3

(Solutions $\frac{1}{26}=\frac{3}{78},\frac{1}{29}=\frac{3}{87},\frac{2}{18}=\frac{6}{54},\frac{2}{19}=\frac{6}{57}$), and 1:4 ($\frac{2}{14}=\frac{8}{56}$).

Another consideration is to observe the denominator, by the solution

$$\frac{1}{43}=\frac{2}{86},$$

students found that it is easier to start with a prime number at the denominator and look for a ratio, such as

$$\frac{2}{19}=\frac{6}{57}\text{ and }\frac{2}{19}=\frac{8}{76}.$$

Using

$$\frac{1}{17}=\frac{2}{34}=\frac{3}{51}=\frac{4}{68},$$

solutions such as

$$\frac{1}{17}=\frac{2}{34},\ \frac{1}{17}=\frac{4}{68},\ \frac{3}{51}=\frac{4}{68}$$

could be selected.

Using structure transfer based on ratio thinking

Later on, some students in both classes A and B recognize that the two formats in the questions were equivalent, that is,

$$“\frac{\square}{\square\square}=\frac{\square}{\square\square}\Leftrightarrow\frac{\square}{\square}=\frac{\square\square}{\square\square}”. \text{ As } “\frac{a}{b}=\frac{pq}{xy} \Leftrightarrow \frac{a}{pq}=\frac{b}{xy}”,$$

solution with the format

$$\frac{\square}{\square}=\frac{\square\square}{\square\square}, \text{ such as } \frac{3}{4}=\frac{51}{68}$$

could be expressed as

$$\frac{3}{51}=\frac{4}{68}, \text{ the format of } \frac{\square}{\square\square}=\frac{\square}{\square\square}.$$

This helps to obtain new solutions in both forms. The discovery enables students to see that the two questions are of the same structure and they could use it to find equivalent solutions. When asked how the discovery was made, students responded that they were using analogy of the thinking in lesson 1, in mathematics term, it is

$$\frac{a}{x}=\frac{b}{yz}\Leftrightarrow\frac{a}{b}=\frac{x}{yz}.$$

However, the experience of posing question in class B enable students to search for the knowledge they got and it is related to their analogy in recently learned knowledge.

Difference with problem solving and problem posing and solving (class B)

The other difference is posing problems allow students to extend their thinking. For class B, after posing the two questions and work on it, some students continue to work on different formats

$$(3)\ \frac{\square}{\square\square}=\frac{\square\square}{\square\square} \text{ and } (4)\ \frac{\square\square}{\square\square}=\frac{\square\square}{\square\square}$$

they posed earlier, which involve more numbers. Though neither the solutions given by students in class B were complete nor the ways of tackling the problem contain new approaches, the new problem posed and the solutions given by students enable them to be more confidence in their own learning process. The approach they used in solving questions in this format is also using simple fractions such as

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$$

and its equivalent. The following described the problem and some of the solutions students provided:

Format of posed question	Solution using fraction $\frac{1}{2}$	Solution using fraction $\frac{1}{3}$	Solution using fraction $\frac{1}{4}$
$\frac{\square}{\square\square}=\frac{\square\square}{\square\square}$	$\frac{7}{14}=\frac{28}{56}\Leftrightarrow\frac{7}{28}=\frac{14}{56}$	$\frac{9}{27}=\frac{16}{48}\Leftrightarrow\frac{9}{16}=\frac{27}{48}$	$\frac{8}{32}=\frac{14}{56}\Leftrightarrow\frac{8}{14}=\frac{32}{56}$
$\frac{\square\square}{\square\square}=\frac{\square\square}{\square\square}$	$\frac{18}{36}=\frac{27}{54}\Leftrightarrow\frac{18}{27}=\frac{36}{54}$	$\frac{18}{54}=\frac{23}{69}\Leftrightarrow\frac{18}{23}=\frac{54}{69}$	

The following table summarized the approaches taken by the students in the two classes in lesson 2.

Table 2. Approaches taken by the students in Class A and Class B

Approaches in solving the problem	$\frac{\square}{\square\square}=\frac{\square}{\square\square}$		$\frac{\square}{\square}=\frac{\square\square}{\square\square}$	
	Class A	Class B	Class A	Class B
Using trial and error	0	0	0	0
Using simple equivalent fractions	21	22	22	22
Using ratio of numerators	10	14	10	14
Using ratio of denominators	4	6	5	8
Using structure transfer	2	10	2	10
Total number of students	31	32	31	32

It is noted that no student use trial and error approaches after their first lesson, noting that they have a focus in dealing with the problem. And that the class with posing questions got more focused on the structure explorations.

PROBLEM SOLVING WITH FRACTIONS

Lesson 3 Problem solving with equivalent fractions

The two classes are asked to solve three problems, with context of fractions. The following questions are used.

> Question 1
>
> The denominator of the fraction $\frac{a}{b}$ is 4 larger than the numerator, and the fraction could be simplified to $\frac{15}{17}$, find this fraction.

In this question, the given fraction

$$\frac{15}{17}$$

has a denominator 2 larger than the numerator. Students obtained the answer

$$\frac{30}{34}$$

through the relation

$$\frac{15}{17} = \frac{30}{34}.$$

Many students in class A used algebraic equation to solve the problem. They let the number of numerator be x, and get

$$\frac{x}{x+4} = \frac{15}{17},$$

which gives 17x = 15(x+4). Solving the equation gives 2x = 60, and x = 30, the solution is

$$\frac{30}{34}.$$

Less student in class B use the equation approach.

> Question 2
>
> The sum of the denominator and numerator of the fraction $\frac{a}{b}$ is 52, the fraction can be simplified to $\frac{6}{7}$, find the fraction.

Again, students in class A tends to use equations in their first attempt, while students in class B focus on equivalent fractions. Based on equation, some students in both classes can provide the equation

$$\frac{52-a}{a} = \frac{6}{7},$$

but only a few could obtain the solution. More students in class A then started to use method of equivalent fractions in their first attempt and fewer students begin to use algebraic equation to solve the problem. Students start with the fraction

$$\frac{6}{7} \text{ (sum = 13)}$$

and obtain its equivalent

$$\frac{12}{14} \Rightarrow \frac{18}{21} \Rightarrow \frac{24}{28} \text{ (sum =52).}$$

Also, some students note for the fraction

$$\frac{12}{14},$$

the sum of the numerator and the denominator add up to 26, and doubling both the numerator and denominator will arrive at the solution.

Question 3
When both the denominator and numerator are increased by 5, the fraction $\frac{a}{b}$ could be simplified to $\frac{3}{4}$, find this fraction.

Fewer students used equation approach for this question. However, the question could not be solved by using equivalent fraction alone, it has to solve with extra observation. By the fraction

$$\frac{3}{4} \text{ and its equivalent } \frac{6}{8},$$

students work backward to obtain

$$\frac{6-5}{8-5} = \frac{1}{3}.$$

And

$$\frac{1+5}{3+5} = \frac{6}{8} = \frac{3}{4}$$

is the solution. Also, using

$$\frac{3}{4} = \frac{9}{12} \text{ and } \frac{9-5}{12-5} = \frac{4}{7},$$

another solution

$$\frac{4}{7}$$

could also be obtained. Similarly, many more solutions could be obtained by this approach.

In solving these three problems, the general observation is that students in class A tends to use equation and class B more focus on using equivalent fraction as a tool. The different approach may lie in the past learning experience of posing problems in equivalent fractions. The results of their different approaches in solving the problem are summarized in the following table.

Table 3. Approaches taken by students in tackling the problem (first attempt)

Approaches in problem solving	Question 1		Question 2		Question 3	
	Class A	Class B	Class A	Class B	Class A	Class B
Using trial and error	3	4	3	2	-	-
Using equivalent Fractions	15	26	18	28	-	-
Using trial and error with equivalent Fractions	-	-	-	-	22	20
Using backward reasoning and equivalent Fractions	-	-	-	-	3	8
Using Equations	13	2	10	2	6	4
Number of students	31	32	31	32	31	32

CONCLUSION

As mentioned in the above discussion, posing questions is a vital process in mathematical thinking. It could help students to develop new questions format and the process allow them to study and make use of the structure of the problem. The study showed that students' reasoning is related to their analogy in what they previously learned. In the case of class B that students posed their question, their way of solving the problem in lesson 3 is more flexible as shown in using the backward method but still its thinking is related to with what previously learned in lesson 2. Posing their problems after solving the problem helps students to explore on different routes in solving the problem.

This process of reasoning and problem solving is based on their communication of thinking in term of posing problem. Posing problem is the application of formal mathematics in terms of students own symbolism, which has been advocated in Gray et. al

(1999), when they proposed that mathematics learning gone through the three stages of embodiment, symbolism and formal mathematics.

The results also echo the work of Gholson et.al (1997), that children's' development of analogical skill are related to problem-solving skills. However, there should be some communication between reasoning and problem solving. Such communication can based on analogical skills in comparing what has been solved in the old problem and what is need in posing the new problem.

Reasoning	Communication $\leftrightarrow$ Problem Posing	Problem solving

Mathematics knowledge and skills goes together (Zhang, 2008), and the skills and knowledge of mathematics are intertwined. In the process of posing problem, both the knowledge and skills in mathematics are enhanced. It is important to know how students think and how they work; as it provides the hint for the design of tasks in lesson. The posing process allow student to learn partly by themselves and partly through teachers' intervention, so that mathematical thinking happens.

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