An Analysis of Information Security Management Strategies in the Presence of Interdependent Security Risk*

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This study expands the current body of research by exploring multiple scenarios of insufficient and excessive IT security investments caused by interdependent risks and the interplay between IT security investments and cyber insurance. A key finding is that organizations experiencing interdependent risks with different types of cyber attacks (i.e., targeted and untargeted attacks) use different strategies in making IT security investment decisions and in purchasing cyber insurance policies for their information security risk management than firms that are facing independent risks. The study further provides an economic rationale for employing insurance mechanisms as a risk management solution for information security.

Keywords: IS Management, Interdependent Security Risks, Security Investments, Cyber Insurance, Externalities

^{*} This article is based in part on Chapter 3 of my dissrtation (Shim, 2010).

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I. Introduction

The rapid proliferation of information technologies (ITs) has changed the environment in which firms operate and the ways they do business. Most firms now store proprietary information in computer systems and transact with other firms via dedicated network connections as well as the Internet. While this rapid proliferation of information technologies has provided great benefits to organizations, it has also escalated their exposure to information security breaches. For example, in the U.S., TJX Companies, Inc. revealed that it had experienced a massive data breach caused by hackers breaking into its systems, and disclosed that an estimated 45.7 million credit and debit card records were stolen [Brodkin, 2007]. These security breaches, understandably, draw tremendous attention, notwithstanding the difficulty in calculating the exact amount of damages or losses from them.

While many organizations have begun to increase their investments in information security by continually adopting a range of more refined technical security solutions [Zhao, Xue, and Whinston, 2009], these masive investments only part of the overall solutions, and a residual risk remains because there is no system that is foolproof against all types of threats [Böhme, 2005; Bolot and Lelarge, 2008a]. For example, computer viruses can be designed to mutate in response to technical solutions being employed, and hackers learn from new security technologies and identify ways to circumvent them. Another reason for the existence of residual risk is the interdependence of information security risks: a firm's security investment not only affects its own security risks but also those of other firms [Grance, Hash, Peck, and Smith,

2002; Zhao et al., 2009]. This interdependence of IT security risks is the main focus of this study.

The interdependent feature of IT security risks generates externalities in various contexts. First, a firm's security investments often generate positive externalities for other firms.¹⁾ For example, if a firm raises its level of information security by investing more in technical security solutions, it may lower the chances of security breaches of the firm's business partners via its computer network. In contrast, a firm's security investment can also generate negative externalities such as the case where hacking attacks targeted at a highly secured server are diverted to other servers, and hence increase the risks of other firms. Therefore, a basic conclusion of the previous literature is that, without any mechanisms for internalizing externalities, self-interested firms' investment in IT security is likely to be below the socially optimal level (i.e., under-investment or under-provision) when security investments generate positive externalities, whereas the firms' investment in security tends to be above the socially optimal level (i.e., over-investment or over-provision) when security investments cause negative externalities [Camp and Wolfram, 2000; Lakdawalla and Zanjani, 2005; Muermann and Kunreuther, 2008; Zhao et al., 2009]. The question then is how to handle these externalities that result in inefficient security investments.

Researchers and practitioners in the field of

A typical example of a positive externality caused by an interdependent risk is Lojack, the auto theft response system. When Lojack is used by some cars, car owners who do not have Lojack benefit from a positive externality because theft against all autos is reduced by the fact that thieves cannot tell in advance which cars have Lojack protection [Camp and Wolfram, 2000].

information security have adopted an economic perspective to investigate how to internalize these externalities and overcome inefficiency [e.g., Gordon, Loeb, and Sohail, 2003; Kesan, Majuca, and Yurcik, 2005]. Some have argued that the enforcement of liability for losses due to security breaches can internalize security externalities [Ogut, Menon, and Raghunathan, 2005; Varian, 2000]. Since it is difficult, if not impossible, to determine who is responsible for the losses, however, the imposition of liability might be an infeasible option for internalizing the externalities [Zhao et al., 2009]. Other researchers [e.g., Bolot and Lelarge, 2008a; Gordon et al., 2003; Zhao et al., 2009] have instead suggested using cyber insurance, which can transfer the risk to an insurer who is willing to accept the risks, as an approach to address the externality problems. With cyber insurance, like other insurance products, insured firms may be able to overcome investment inefficiency by balancing their expenditures between security investments and cyber insurance. To date, however, there is a relative paucity of literature on cyber insurance itself.

This study intends to answer two research questions that arise from the above discussion: (1) How do externalities caused by interdependent security risks among organizations influence two widely employed security risk management strategies-information security investment and cyber insurance; and (2) How does cyber insurance affect a firm's decision regarding security investment, that is, is cyber insurance a complement or substitute for a firm's security investment? To answer these questions, the expected utility model is used with two firms to present the interplay between security investment and

cyber insurance in the context of independent and interdependent security risks. More specifically, the impact of externalities on the security investments of the firms with and without cyber insurance products being available is analyzed.

Unlike the previous literature which mostly focused on illustrating the problem of socially inefficient security investments caused by interdependent security risks, however, this study examines the effect of interdependent risks on decisions about both security investments and insurance coverage. Furthermore, this study illustrates how cyber risks caused by different types of cyber attacks including viruses, spyware and hacking could bring about different externality problems and give firms different incentives to employ diverse information security mechanisms. In the following section, I conceptualize that there are two broad classes of cyber risks, risks caused by targeted attacks and risks caused by untargeted attacks, and that these classes cause different types of investment inefficiency.

To the best of my knowledge, unlike other studies [e.g., Kunreuther and Heal, 2003; Ogut et al., 2005] which implicitly assume that interdependent security risks can result in either positive or negative externalities, this is the first study that links different types of cyber attacks (i.e., targeted and untargeted attacks) to a comprehensive mechanism of IT security risk management strategies that include both IT security investments and cyber insurance with interdependent risk.

Although the theoretical models are based on the expected utility theory, which is widely used in insurance research, this study derives unique propositions that have not been fully identified in other cyber security studies. A key finding is that organizations experiencing interdependent risks with different types of cyber attacks use different strategies in making IT security investment decisions and in purchasing cyber insurance policies for their information security risk management compared to firms that are facing independent risks.

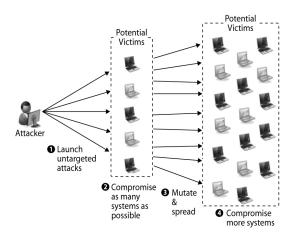
The remainder of the article is organized as follows; the next section presents several conceptual frameworks that address the characteristics of cyber attacks and security risks management strategies. Section three develops several theoretical models that tackle the issue of interdependent security risks and derive a number of new propositions that shows the effects of interdependent risks on security risk management strategies. Discussion and limitations of the research are presented in section four.

II. IT Security Risks and its Management Strategies

2.1 Targeted vs. Untargeted Attacks

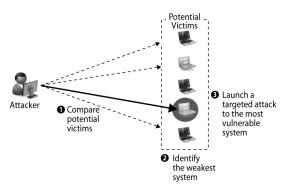
Cyber attacks can be categorized into targeted and untargeted attacks. "Untargeted" attacks aim at millions of potential victims, hoping to contaminate as many computer systems as possible [Dzung, Naedele, Von Hoff, and Crevatin, 2005; Tally, 2009]. Therefore, adversaries launching untargeted attacks intend to harm any vulnerable system which can be found on a network [Dzung et al., 2005; Turk, 2005]. Common examples of untargeted attacks include viruses, worms, trojan horses, and spyware. <Figure 1> shows untargeted attacks schematically. Since adversaries launching un-

targeted attacks do not target any specific system, an agent's increased investment for coping with untargeted attacks will decrease the risks faced by other agents connected to this agent's system. Therefore, investment in IT security against untargeted attacks is more likely to generate positive externalities.



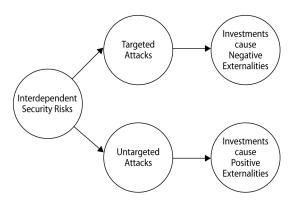
< Figure 1> Typical Untargeted Attack

"Targeted" attacks are designed to damage a particular communication system or a firm's information assets [Dzung et al., 2005; Tally, 2009]. Attackers using such strategies typically collect information about the target, customize attacks for each particular victim, and thus know who will be attacked [Dzung, et al., 2005; Turk, 2005]. Examples of targeted attacks are malicious hacking and whaling. The scheme of targeted attacks is depicted in <Figure 2>. Since targeted attacks are customized for an intended communication network of systems [Dzung, et al., 2005; Tally, 2009], an agent's increased investment in security against targeted attacks will increase the risks faced by other agents: adversaries launching targeted attacks will substitute less protected targets in place of their original targets, and thus the investment will generate negative externalities.²⁾ As a result, the relationship between the types of attack and the externality problem can be depicted, as shown in <Figure 3>.3)



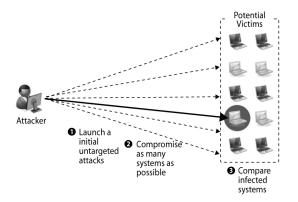
< Figure 2> Typical Targeted Attack

- 2) There might be hackers who are motivated by reputation in the hacking community. For example, some hackers try to break into computer networks of big companies such as Microsoft and Google because they will improve their own reputation if they succeed in breaking into networks which are extremely difficult to hack. In such cases, IT security investment of the firm will create a positive externality. This type of motivation, however, is only noted here and is not considered in this study.
- 3) Although not analyzed in this study, there is another type of attack: hybrid attacks. This type of attack involves the combination of a targeted and untargeted attack and has two stages. In the first stage, adversaries initiate untargeted attacks by spreading malicious software. In the second stage, the adversaries launch targeted attacks using two different types of schemes. First, the adversaries may launch targeted attacks by breaking into the computer system, which was infected in the first stage. Since some malicious software can create backdoors in infected systems, the adversaries can easily gain access to the systems. Second, the adversaries may attack particularly vulnerable systems using machines that were infected in the first stage. Some worms and viruses turn infected systems into remote-controlled zombie computers. These zombies are used by the adversaries to carry out DDoS attacks, sending out spam e-mails, etc. See Shim [2010] for more details.

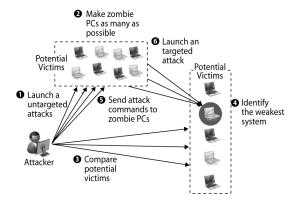


<Figure 3> Types of Attack and Externalities

The proposed categorization, which limits the types of cyber attacks to either targeted or untargeted attacks, has advantages and disadvantages. On the one hand, it simplifies the theoretical model and thereby enables a clearer understanding of the direct effects of each type of attack on firms' security risk management strategies. On the other hand, this study deals with targeted and untargeted attacks separately in order to keep the analysis simple and yet to obtain some transparent results. In reality, however, a lot of cyber attacks are likely to combine both targeted and untargeted attacks (i.e., hybrid attacks). For example, there can be cyber attacks which have two stages. In the first stage, adversaries initiate untargeted attacks by spreading malicious software. In the second stage, the adversaries launch targeted attacks using two different types of schemes. First, the adversaries may launch targeted attacks by breaking into the computer system which is infected in the first stage (see <Figure 4>). Since some malicious software can create backdoors on infected systems, the adversaries can easily gain access to the systems. Second, the adversaries may attack particularly vulnerable systems using infected machines in the first stage (see <Figure 5>). Some worms and viruses turn infected systems into remote-controlled zombie computers. These zombies are used by the adversaries to carry out distributed denial-of-service (DDoS) attacks, sending out spam e-mails, etc. As a result, the categorization used in this study would only allow a partial exploration of cases where the interaction between targeted and untargeted (i.e., hybrid attacks) cannot be taken into account.



<Figure 4> First Type of Hybrid Attacks



< Figure 5> Second Type of Hybrid Attacks

2.2 Self-Protection, Self-Insurance, and Cyber Insurance

Traditional security management strategies to hedge against losses from IT security breaches involve three different instruments: self-protection (to reduce the probability of a loss), self-insurance (to reduce the size of a loss) and insurance bought in the market.4) Recently, several studies [e.g., Doll, 2002; Ogut, 2006; Weiss, 2002] have questioned the effectiveness of sole dependence on the traditional security investment model, implemented by self-protection and self-insurance. This body of research claims that firms might not be able to fully protect their systems against cyber attacks or may even fail to detect the attacks since perpetrators continually use newer tactics which may not be detected by firms as most of the technical solutions are developed reactively in response to the detection of newer security flaws [Bandyopadhyay, 2006]. Moreover, this research argues that, because of integrated and interconnected information systems, security breaches of one organization can readily spread to other organizations. It therefore concludes that deficiencies in abilities for perfect detection and protection, together with the existence of interdependent security risks, have resulted in a considerable residual risk for organizations and they have started to demand alternative risk management mechanisms, most specifically market insurance that can make up for the weaknesses of traditional security management strategies.

Market insurance is a traditional instrument for shifting residual risks beyond due diligence [Bandyopadhyay, 2006]. In spite of its similarity to self-insurance in that both mechanisms intend

⁴⁾ As Bolot and Lelarge [2008b] indicated, it is somewhat artificial to distinguish self-protection and selfinsurance mechanisms since many IT security measures do both at the same time. Thus, in this study, I do not distinguish them and refer to them simply as self-protection.

to reduce the size of a loss, market insurance is offered by third party insurance companies. In the field of information security, insurance products (known as cyber insurance), which specifically dealt with losses from computer crimes, cover not only losses, such as physical damages that are addressed by traditional insurance products, but also provide coverage for intangible damages.

II. Theoretical Analysis

This section presents theoretical models that show how interdependence in cyber security affects firms' decisions regarding security investments and cyber insurance purchases. In the models, I consider identical firms with an initial wealth W and a utility function $U(\cdot)$. I assume that firms are rational and risk averse, implying that the utility function is concave (i.e., $U'(\cdot)$ > 0 and $U''(\cdot) < 0$), and constant absolute risk aversion (CARA) is given by r = -U''/U'. To simplify the illustration, this study assumes single-period probabilistic models for the risk, in which all firms' decisions and corresponding consequences occur in a simultaneous manner, such that firms invest in self-protection and/or purchase an insurance product in a single period.5) There are only two possible states for the firm: a good state, in which the firm does not experience any security breach, and a bad state in which the firm experiences such a breach. Firm i's breach probability (i.e., probability of loss or damage) is denoted by $B_i(\cdot)$ and can be decreased by the firm's investment in security (i.e., $B_i'(\cdot) < 0$).

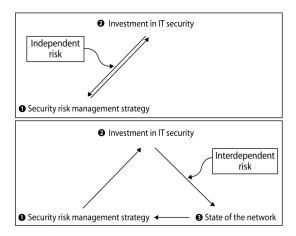
I assume that the breach probability has declining returns (i.e., $B_i^{"}(\cdot) > 0$). In the case of independent IT security risks, $B_i(\cdot)$ is only determined by firm i's level of security investment z_i , that is, $B_i(z_i)$. In contrast, the breach probability of a firm in the case of interdependent IT security risks is determined not only by the firm's own security investment, but also by those of other firms.⁶⁾ Similarly, a firm's investment in self-protection affects the breach probability at all firms. z_{-i} represents investment in self-protection of all firms except firm i. Consequently, in the interdependent case, firm i's breach probability is $B_i(z_i, z_{-i})$. If a security breach occurs at firm i, the firm incurs a loss of L_i .

3.1 Investment in Self-Protection without a Cyber Insurance Market

The effect of a firm's investment in IT security generally depends on whether security risks are independent or interdependent. According to Bolot and Lelarge [2008a], these different types of security risks create different feedback loops as shown in Figure 6. In this section, I first examine the baseline model in which security risks are independent and no cyber insurance product is available. I then consider cases in which breaches caused by untargeted and targeted attacks are interdependent, and thus generate positive and negative externalities, respectively.

⁵⁾ Therefore, this study does not take into account dynamic aspects which use game theoretic approaches.

⁶⁾ It can be argued that, ceteris paribus, a higher level of investment by a firm may increase the probability of a breach of other firms because hackers may focus their efforts on firms that are easier to attack. On the other hand, it can also be argued that a higher level of investment by a firm may reduce the breach probability of other firms since computers across firms are interconnected.



<Figure 6> Feedback Loop of IT Security Investment without Cyber Insurance(Figure Based on Bolot and Lelarge [2008a])

3.1.1 Baseline Model of Independent Risks without a Cyber Insurance Market

I assume that, when there is no insurance product available, all firms manage cyber risks by investing only in self-protection. The condition that maximizes the expected utility of firm i can be expressed as

$$\frac{\max_{z_i} B_i(\cdot) U(W_i - L_i - z_i)}{+ [1 - B_i(\cdot)] U(W_i - z_i)} \tag{1}$$

where $U(W_i-z_i)$ is firm i's utility without a security breach and $U(W_i-L_i-z_i)$ is its utility with a security breach. The first-order condition for IT security investment is

$$B_{i}'(\cdot) = \frac{B_{i}(\cdot)U_{L}' + [1 - B_{i}(\cdot)]U_{N}'}{[U_{L} - U_{N}]}$$
(2)

where $U_L = U(W_i - L_i - z_i)$ and $U_N = U(W_i - z_i)$. In order to assess this expression in a useful way, I use a Taylor series approximation which has been commonly used in the literature on uncertainty and insurance [e.g., Bhattacharya and Sood, 2006; Hau, 1999; Quaas and Baumgartner, 2008].⁷⁾ Using the first-order Taylor series approximation,⁸⁾ $U_N \approx U_L + U'_L L_i$ and $U'_N \approx U'_L + U_L L_i''$, equation (3.2) can be rewritten as:

$$B_{i}^{'}(z_{i}^{o}) = -\frac{1}{L_{i}} + r[1 - B_{i}(z_{i}^{o})]$$
(3)

where $r=-U_{L}^{''}/U_{L}^{'}$. The superscript o on z_{i} indicates the case in which security risks are independent and no cyber insurance product is available.

3.1.2 General Model of Interdependent Risks without a Cyber Insurance Market in the Context of Untargeted Attacks

Analyzed here are cases in which security risks are interdependent and IT security investments generate positive externalities due to untargeted cyber attacks. These attacks, which intend to harm large numbers of potential victims, generate positive externalities since the increased security investment of one firm will reduce the risks faced by other firms connected to this firm's computer system. For example, if a virus or a malware breaks into an unprotected system, it

According to Schoemaker [1982] and Hirshleifer [1970], any well-behaved utility function can be expanded by a Taylor series approximation.

⁸⁾ Hereinafter, I assume that a firm's initial wealth, W, is large enough to satisfy a condition for Taylor series approximation. In addition, I ignore the third and higher-order terms since, while they may exist, these derivatives will be multiplied by very small terms.

may be able to gain access to other systems in the network because many viruses and malware spread and proliferate among systems via trusted connections. Therefore, as shown in <Figure 7>, a firm's security investment reduces not only its own probability of security breach but also that of others, and thus firms have incentives to invest less in information security than they do in the case of independent IT security risks.



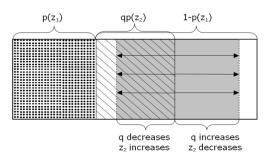
<Figure 7> Link between Untargeted Attacks and the Level of Investments

Following Ogut *et al.* [2005] and Zhao *et al.* [2009], I model positive externalities of security investments in the following manner. To simplify the model, I assume that there are only two symmetric firms with interdependent risks (i = 1, 2). Security investments have direct effects as well as indirect effects. Direct effects refer to the effects of security investment on a firm's security that change the breach probability caused by a direct attack made on the firm's information system. Indirect effects refer to the effects of other firms' security investment on the firm's security which affects the breach probability caused by an attack through other firms' systems.⁹

I model the breach probability under direct effects as $p(z_1)$ where z_1 is the security investment by firm 1 ($p'(\cdot) < 0$ and $p''(\cdot) > 0$). The breach probability caused by indirect effects is given by $q \cdot p(z_2)$, $0 \le q \le 1$ where the parameter q measures the probability that one firm has a security breach given that another firm has a security breach and vice versa. q models the degree of interdependency or externality between the two firms' IT security. A higher q indicates a higher degree of interdependence. $q \cdot p(z_2)$ represents the probability of malicious attacks breaking into firm 1's system through firm 2's system. Taken together, firm 1's breach probability can be expressed as:

$$\begin{split} B_1(z_1,\,z_2) &= p(z_1) + [1-p(z_1)] q p(z_2) \\ &= 1 - [1-p(z_1)] [1-q p(z_2)] \end{split} \tag{4}$$

<Figure 8> illustrates the breach probability of firm 1 in the case of positive externalities. If there are no externalities, the probability of breach is the dotted rectangle on the left. As positive externalities are considered, the oblique-lined rectangle in the center is added. The solid shaded rectangle represents the change of the breach probability resulted from the change of the degree of interdependence and firm 2's level of security investment.



<Figure 8> Illustration of Breach Probability with Positive Externalities

⁹⁾ Note that, according to Bandyopadhyay [2006], a security breach which occurs at a firm's own site incurs a higher loss to the firm (direct loss) than is the case when the loss caused by a breach arises at the partnering firm (indirect loss). He further argued that if the shared asset is compromised at both the firms, the losses are then superadditive and potentially higher than is the case when these firms experience separate security breaches.

From equation (3), the first order condition with respect to z_1 can be expressed as

$$\begin{split} B_{i}^{'}(z_{1},\,z_{2}) &= p'\,(z_{1})[1-qp(z_{2})] \\ &= -\frac{1}{L_{1}} + r[1-p(z_{1})][1-qp(z_{2})]. \end{split} \tag{5}$$

Therefore, if the cost of a breach is assumed to be equal to 1, the optimal level of security investment is the solution to the following equation:

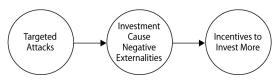
$$p'(z_1^p) = -\frac{1}{L_1[1 - qp(z_2^p)]} + r[1 - p(z_1^p)].$$
 (6)

The superscript p on z_1 indicates the case where security investments generate positive externalities and there is no cyber insurance product available.

3.1.3 General Model of Interdependent Risks without a Cyber Insurance Market in the Context of Targeted Attacks

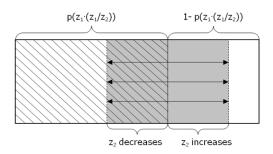
The model presented above implies that adversaries spread attacks across all possible targets. It can also be argued, however, that an adversary focuses all of his or her resources on a single target. Regardless of the underlying reasons for the attack, the focus on a single target may create instability in the network since it will cause something akin to an arms race among targets [Lakdawalla and Zanjani, 2005]. To see this outcome, consider a situation where a pool of malicious hackers chooses to attack the most vulnerable security system. Since firms know that the hackers will attack only one of them and will avoid firms with better protection than others, each firm has an incentive to deviate from Nash equilibrium by increasing investment in security protection by an infinitesimal amount. In other words, to make security investment effective, a firm should invest more in security compared to other firms. It would seem to follow then that a firm's security investment for coping with this type of targeted attacks, while reducing its own breach probability, increases the breach probabilities of other firms, and thus is likely to generate negative externalities.

Following Zhao et al. [2009], I model the negative externality of IT security investment in the following manner. A firm's breach probability is influenced not only by its own security investment but also by other firms' investments. If a firm's security investment is higher than the investment of other firms, its investment is more likely to drive away attacks targeted on it. In contrast, if a firm invests less than other firms, the firm is more likely to attract targeted attacks than are other firms. Therefore, to make security investment effective, a firm should invest more in security compared to other firms. Since this phenomenon gives firms incentives to make excessive security investments it may cause "destructive competition," which refers to situations when firms invest an extreme amount of resources in information security to avoid targeted attacks and, in so doing, may undermine their profits [Zhao, 2007]. <Figure 9> illustrates the link between targeted attacks and an incentive of excessive investment.



<Figure 9> Link between Targeted Attacks and the Level of Investment

I use the term z_1/z_2 to characterize the relative effectiveness of firm 1's security investment and model the breach probability as $B_1(z_1, z_2) =$ $p(z_1 \cdot (z_1/z_2))$. If firm 1 makes a higher security investment than firm 2 (i.e., $z_1/z_2 > 1$), we have $z_1 \cdot (z_1/z_2) > z_1$ and $p(z_1 \cdot (z_1/z_2)) < p(z_1)$. This implies that firm 1's security investment is more effective in decreasing its breach probability. For instance, if a firm invests more in security than do others, adversaries launching targeted attacks such as hacking and DDoS will substitute their initial target with a less protected target. Therefore, the breach probability of a firm increases corresponding to other firm's security investments, which captures the negative externality of security investment. < Figure 10> displays the information security risk in the case of negative externalities. Since the breach probability is determined not only by a firm's security investment but also by those of other firms, the breach probability changes as other firms changes the level of their security investments.



<Figure 10> Illustration of Breach Probability with Negative Externalities

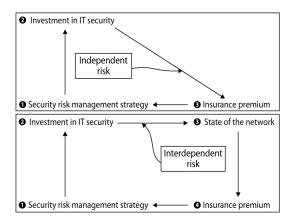
As was similarly the case in the previous section, I assume a case with two symmetric firms. Applying $B_1(z_1, z_2) = p(z_1 \cdot (z_1/z_2))$ and ∂B_1 $(z_1, z_2)/\partial z_1 = (2z_1/z_2)p'(z_1 \cdot (z_1/z_2))$ to equation (3), and using symmetric assumption where z_1 = z_2 , firm 1's equilibrium security investment is determined by

$$p'(z_1^n) = -\frac{1}{2L_1} + \frac{r[1 - p(z_1^n)]}{2}.$$
 (7)

The superscript n on z_1 indicates the case where security investments generate negative externalities and there is no cyber insurance product available.

3.2 Interplay between Self-Protection and Cyber Insurance

I now analyze the impact that cyber insurance has on the level of security investment in self-protection chosen by a firm. Several authors have proposed cyber insurance as an effective measure for internalizing externalities caused by interdependent IT security risks [e.g., Böhme, 2005; Bolot and Lelarge, 2008a; Kesan et al., 2005; Lakdawalla and Zanjani, 2005; Muermann and Kunreuther, 2008; Zhao et al., 2009]. They argued that firms can employ cyber insurance to cope with the security risks which are not prevented by investment in self-protection. If cyber insurance becomes available, <Figure 11> illustrated above would be changed to the following feedback loop situation:



<Figure 11> Feedback Loop of IT Security Investment with Cyber Insurance(Figure Modified from Bolot and Lelarge [2008a])

Based on Ogut et~al.~[2005], I model an insurance market, in this section, in the following manner. When a cyber insurance product is available, the insurance premium paid by firm i is $\pi_i I_i$ where π_i is the price of insurance coverage which shows the maximum willingness to pay to escape a loss from a security breach and I_i is indemnity paid by the insurer if a loss from a security breach is found. If firm i decides to purchase an insurance product, the firm pays the premium $\pi_i I_i$ at the beginning of the period and is paid an indemnity, I_i , at the end of the period if there is a security incident. I0

To take insurance market maturity into account, I use the loading factor, λ , and thus the insurance price can be expressed as $\pi_i = (1+\lambda)B_i$. That is, if competition in the insurance market is perfect (i.e., the insurance market is mature), the insurance price is actuarially fair, $\lambda = 0$, and the insurance companies make zero profit, $\pi_i = B_i$. In contrast, if competition in the insurance market is immature), the insurance market is immature), the insurance price is not actuarially fair, $\lambda > 0$, and the insurance companies make positive profits.¹¹)

3.2.1 Baseline Model of Independent Risks with a Cyber Insurance Market

Now assume that all firms can manage cyber security risks by investing in self-protection and/

or purchasing a cyber insurance product. Using the indemnity payment I_i and insurance premium $\pi_i I_i$, firm i's utility function is $U(W_i - L_i + [1 - \pi_i(z_i)]I_i - z_i)$ with a security breach, whereas the utility function is $U(W_i - \pi_i(z_i)I_i - z_i)$ with no security breach. Therefore, the maximization problem of firm i's expected utility can be presented as

$$\begin{array}{l} \max \limits_{z_{i},I_{i}} B_{i}(z_{i}) \, U_{i}(\, W_{i} - L_{i} + [1 - \pi_{i}(z_{i})] I_{i} - z_{i}) \\ \qquad + [1 - B_{i}(z_{i})] \, U_{i}(\, W_{i} - \pi_{i}(z_{i}) I_{i} - z_{i}) \end{array} \tag{8}$$

By using, $\pi_i(z_i) = [1+\lambda]B_i(z_i)$, and the first order Taylor series approximation, the first order conditions for IT security investment and cyber insurance can be written as:

$$B_{i}^{'}(z_{i}^{oI}) = -\frac{1}{(1+\lambda)L_{i}},$$
 (9)

and

$$I_{i} = L_{i} - \frac{\lambda}{r[1 - B_{i}(z_{i}^{oI})](1 + \lambda)}$$
 (10)

where $r=-U_{II}^{''}/U_{II}^{'}$. The superscript oI on z_i means that security risks are independent and there is a cyber insurance product available. When an insurance market is mature, the loading factor λ equals zero, a firm purchases full insurance coverage $(I_i=L_i)$ and the optimal level of investment is determined by $B_i^{'}(z_i^{oI})=-1/L_i$.

3.2.2 General Model of Interdependent Risks with a Cyber Insurance Market in the Context of Untargeted Attacks

I now consider the case in which a firm's security risk is interdependent and security investment has a positive externality. Using equations

To simplify the analysis, I again use simple oneperiod expected utility models, in which all decisions and outcomes occur simultaneously.

¹¹⁾ Currently, the cyber insurance market is not well developed [3]. There are only a small number of insurance companies offering cyber insurance products, and thus they are likely to make profits.

(9) and (10), the first order Taylor series approximation and a symmetric assumption (i.e., z_1 $= z_2$), the first order conditions for IT security investment and cyber insurance can be written as:

$$p'(z_1^{pI}) = -\frac{1}{[1 - qp(z_1^{pI})](1 + \lambda)L_1}$$
(11)

and

$$I_1 = L_1 - \frac{\lambda}{r(1+\lambda)[1-p(z_1^{pI})][1-qp(z_1^{pI})]} \tag{12} \label{eq:12}$$

where superscript pI on z_1 indicates positive externality and the existence of a cyber insurance market, and $r = -U_{II}^{"}/U_{II}^{'}$. Consequently, it can be seen that, as the insurance market becomes mature (i.e., as λ approaches to zero), firms are more likely to invest less in self-protection and, instead, buy full insurance coverage.

3.2.3 General Model of Interdependent Risks with a Cyber Insurance Market in the Context of Targeted Attacks

I now investigate the case in which investment in security measures causes negative externalities with considering the existence of a cyber insurance market. Using equation (9), firm 1's equilibrium security investment is determined by

$$p'(z_1^{nI}) = -\frac{1}{2(1+\lambda)L_1} \tag{13}$$

when $z_1 = z_2$. In addition, using equation (10), the optimal level of cyber insurance can be expressed as

$$I_{1} = L_{1} - \frac{\lambda}{r(1+\lambda)[1-p(z_{1}^{nI})]} \tag{14} \label{eq:14}$$

when $z_1 = z_2$. The superscript nI used in both equations (13) and (14) is used to indicate that security investments generate negative externalities and there is a cyber insurance product available.

3.3 Synthesis of the Theoretical Models: Impact of Externalities on Self-Protection and Cyber Insurance

To analyze the combined impact of interdependency and insurance market maturity on security investment and insurance coverage, I set forth security spending and insurance coverage in the cases of two identical firms in the following table.

Comparison of the solutions set forth above can provide valuable insight in understanding the issues of cyber security. I first compare the solutions for the baseline models with those for the general models of the cases of untargeted attacks (i.e., the existence of positive externality) and targeted attacks (i.e., the existence of negative externality).

From <Table 1>, it can be demonstrated that, when information security investment generates positive externalities, a firm's security investment reduces not only its breach probability but also those of others. For example, a firm which equips its computer systems with strong countermeasures against viruses and spyware will reduce the risks encountered by other firms connected to this firm's system. In the case of interdependent security risks with positive externalities, however, the risk controllable by firm 1's IT security investment is reduced from $p(z_1)$ to $p(z_1)[1-qp(z_2)]$ and the efficiency of its IT security investment, which is measured by the marginal reduction in breach probability result-

| | Insurance Market | No Insurance Market |
|----------------------|--|--|
| Independence | $\begin{aligned} p'(z_1^{oI}) &= -\frac{1}{(1+\lambda)L_1} \\ I_1^{oI} &= L_1 - \frac{\lambda}{r[1-p_1(z_1^{oI})](1+\lambda)} \end{aligned}$ | $p'(z_1^o) = \frac{1}{L_1} + r[1 - p(z_1^o)]$ |
| Positive Externality | $\begin{split} p'(z_1^{pI}) &= -\frac{1}{[1 - qp(z_1^{pI})](1 + \lambda)L_1} \\ I_1^{pI} &= L_1 - \frac{\lambda}{r(1 + \lambda)[1 - p(z_1^{pI})][1 - qp(z_1^{pI})]} \end{split}$ | $p'(z_1^p) = \frac{1}{L_1[1 - qp(z_1^p)]} + r[1 - p(z_1^p)]$ |
| Negative Externality | $\begin{split} p'(z_1^{nI}) &= -\frac{1}{2(1+\lambda)L_1} \\ I_1^{pI} &= L_1 - \frac{\lambda}{r(1+\lambda)[1-p(z_1^{pI})][1-qp(z_1^{pI})]} \end{split}$ | $p'(z_1^n) = -\frac{1}{2L_1} + \frac{r[1 - p(z_1^n)]}{2}$ |

<Table 1> Comparison of IT Security Investment and Insurance Coverage

ing from the investment, is also reduced from $|p'(z_1)|$ to $|p(z_1)[1-qp(z_2)]|$ (Ogut, Menon, *et al.*, 2005). As a result, taking together the reduced efficiency of IT security investment and the decreased controllability of security risk, firms may be discouraged from investing in IT security.

In contrast, in the case of negative externalities, we can observe that a negative externality caused by interdependency neither increases the breach probability nor reduces the risk controllability: that is, using two firms that are identical, it can be demonstrated that the overall security risk is unchanged since the probability of breach is the same whether firms' security risks cause a negative externality or no externality, i.e., $p(z_1) =$ $p(z_1 \cdot (z_1/z_2))$; the risk controllable by a firm's security investment also does not change for the same reason. On the other hand, the marginal decrease in security risk due to security investment, which is a measure of the efficiency of the investment, increases from $|p'(z_1)|$ to $|2p'(z_1)|$ in the case of identical firms. Therefore, from the firms' point of view, the increased efficiency of security investment along with the unchanged

overall risk gives them incentives to increase investment in IT security. This implies that firms have an incentive to invest more in cases where IT security investment generates negative externalities (i.e., targeted attack cases) and to invest less in cases where IT security investment generates positive externalities (i.e. untargeted attack cases) compared to the interdependent security risk case. Since this explanation holds true whether a cyber insurance market exists or not, taking these statements together, this leads us to the following proposition (a formal proof appears in the appendix):

Proposition 1: Regardless of the existence of a cyber insurance market, firms experiencing untargeted attacks invest less in self-protection than do firms experiencing the same number of targeted attacks.¹²)

In spite of the higher breach probability in the case of positive externalities compared to proba-

¹²⁾ Note that all propositions are stated under a 'ceteris paribus' assumption.

bility in situations of independent risks (i.e., $p(z_1) + \{1 - p(z_1)\} qp(z_2) > p(z_1)$, it can be demonstrated from Proposition 1 that positive externalities in IT security risks reduces a firm's incentive to invest in IT security. However, from the viewpoint of insurance companies, the higher breach probability in the case of positive externalities leads to a higher insurance premium charge for insureds, i.e., $(1+\lambda)[p(z_1)+\{1-p(z_1)\}$ $qp(z_2)$] > $(1+\lambda)p(z_1)$, which, in turn causes firms to reduce their insurance coverage. On the other hand, unlike the case of positive externalities, the total risk of firms experiencing targeted attacks is lower than that of firms experiencing untargeted attacks since firms experiencing targeted attacks generally invest more in self-protection than firms suffering untargeted attacks. Therefore, an insurance company might charge a lower insurance premium for the firms experiencing targeted attacks and this causes the firms to increase their insurance coverage. This leads us to the following proposition (a formal proof appears in the appendix):

Proposition 2: With a cyber insurance market, firms experiencing targeted attacks spend more on cyber insurance coverage than do firms experiencing the same number of untargeted attacks.

Consequently, from Propositions 1 and 2, one can infer that positive externalities in cyber security lead firms to decrease their level of IT security investment and insurance coverage.

I now discuss the impact of loss on firms' strategies through a comparative static analysis. For firms experiencing untargeted attacks, since $p'(z_1^{pI})$ $\{1-qp(z_2^{pI})\} = -1/(1+\lambda)L_1$, it can be seen that the efficiency of security investment increases

as the amount of security loss increases (i.e., $\partial p'(z_1^{pI})[1-qp(z_1^{pI})]/\partial L_1 = 1/(1+\lambda)L_1^2 > 0$). This increased efficiency, in turn, causes firms to invest more in their IT security. Similarly, in the case where firms experiencing targeted attacks, since the efficiency of security investment increases as the level of loss increases (i.e., $\partial 2p'(z_1^{nI})/\partial L = 1/2$ $(1+\lambda)L^2 > 0$), the increased efficiency leads firms to increase the investment in IT security. Therefore, we get (a formal proof appears in the appendix):

Proposition 3: With a cyber insurance market, firms increase security investments as the level of security risks rises, $\partial z/\partial L > 0$.

Similarly, an increase in loss also brings about an increase in insurance coverage. This relationship exists because an increase in loss raises the expected loss, which increased expected loss causes an increase in insurance coverage [Ogut, Menon et al., 2005]. Therefore,

Proposition 4: With a cyber insurance market, firms purchase more insurance coverage as loss from a security breach rises, $\partial I/\partial L > 0$ (See Appendix for proof).

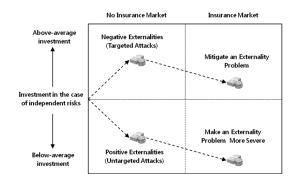
In addition, as mentioned earlier, cyber insurance is regarded as a remedy for the residual risk, and hence increases as security investments rise. As Ehrlich and Becker [1972] and Ogut [2006] have indicated, this implies that, for a given breach probability, cyber insurance and information security investments are also complements in the equilibrium. That is, for a given probability of breach, an increase in security investments causes an increase in insurance coverage, and vice versa. 13) This leads us to the following proposition:

Proposition 5: With a cyber insurance market, firms that make higher security investments in equilibrium will also cover more of the risk through cyber insurance, $\partial I^*/\partial L^* > 0$ (See Appendix for proof).

Lastly, I investigate the effect of cyber insurance on the demand for self-protection. If market insurance were available at an actuarially fair price, $\pi(z) = B(z)$, the optimal investment in IT security would be smaller than the amount spent in the absence of market insurance. That is,

Proposition 6: If a cyber insurance market is available and mature, firms invest less in cyber security when cyber insurance is available than when it is not (See Appendix for proof).

As argued by Powell [2005], Lakdawalla and Zanjani [2005] and Zhao, et al. [2009], Proposition 6 suggests that the employment of a cyber insurance market can only partially resolve the inefficient security investment problem in the case of targeted attacks by reducing the investment, whereas the insufficient security investment problem in the case of untargeted attacks becomes more severe. That is, even if the positive externality case is more problematic since it might cause higher security risks (due to less IT security investment and higher total risk), cyber insurance cannot solve this problem. The following figure illustrates how the adoption of a cyber insurance market affects firms' information security investments.



<Figure 12> Effect of the Adoption of a Cyber Insurance Market on the Level of Information Security Investment

IV. Discussion and Implications

The previous literature on IT security focused generally on the effectiveness of the adoption of technology-based security solutions or products as security management tools. While this approach helps in understanding security risk management, it cannot address the problem of residual risks caused by reactive development of security solutions in response to the detection of newly revealed security flaws, and interdependency of IT security risks. More recently, therefore, several studies [e.g., Bandyopadhyay, 2006; Majuca et al., 2006] have suggested that various security issues cannot be solely addressed through a technical lens and have begun to employ alternative risk management mechanisms that can complement the weaknesses of the traditional approach to technology-based security management. These new research approaches have generated various interesting and innovative proposals and this study adopted some of the approaches to investigate information security risk management strategies.

In this study, I investigated the effects of cy-

¹³⁾ Some researchers have argued that insurance coverage and security investments are substitutes: IT security investments would be discouraged by cyber insurance. This effect is generally referred to as "moral hazard" since policyholders buy less than full insurance coverage as they increase the level of security investments [Ogut, 2006].

ber attacks on security risk management strategies and their relationship from the newly proposed economic perspectives. More specifically, this study brought together issues of information security investment (i.e., the reduction of the probability of a loss) and cyber insurance (i.e., the reduction of the size of a loss) that jointly impact security risk management within a firm.

While I explored information security issues under conditions of an interdependent security environment, unlike the previous literature, this study not only took into account positive and negative externalities of IT security investments caused by interdependent security risks, but also explicitly illustrated how untargeted and targeted cyber attacks cause these externalities: targeted attacks might cause an overinvestment problem due to negative externalities and untargeted attacks might bring about an underinvestment problem because of positive externalities.

Several important implications emerged from the analysis. The first set of implications came from the perverse incentives to invest in IT security as the characteristics of interdependent information security risks distort firms' incentives for such investment. The analysis showed that when firms invest in IT security to protect their computer systems against untargeted attacks such as virus or spyware intrusion, the investments generate positive externalities and firms make insufficient investments in IT security. As a result, this may undermine a safe security environment. In contrast, when firms invest in IT security to protect their computer systems against targeted attacks such as hacking and DDoS attacks, the investment causes

negative externalities and firms invest excessively in IT security. This effect may lead firms to engage in "destructive competition" which implies situations when firms make redundant and excessive resource allocation on information security to avoid targeted attacks [Zhao, 2007]. Hence, these misaligned incentives may cause inefficient IT security management practices.

The second set of implications relate to whether the adoption of cyber insurance can mitigate the negative effects of interdependent IT security risks. While thorough evaluation of the characteristics of the security risks a firm is facing might be helpful for firms to effectively respond to various types of security risks, as explained at the outset, the interdependent security risks along with lagged development of security measures make firms difficult to determine how much residual security risk remains after security investments. Firms therefore have incentives to adopt cyber insurance which allows them to effectively transfer risks to third parties after the firms have deployed IT security measures [Richardson, 2008].

The analysis showed that the adoption of cyber insurance lowers the overall level of IT security investment regardless of firms' purchase of cyber insurance policies. Therefore, from a social planner's perspective, the adoption of cyber insurance can potentially improve social welfare by mitigating the problem of excessive investment in the case of negative externalities (i.e., a targeted attack case) whereas it may decrease a social surplus because the insufficient investment problem in the case of positive externalities (i.e., an untargeted attack case) might become more severe. This implies that when faced with targeted attacks, using a cyber insurance mechanism is a particularly useful strategy since it can mitigate the incentives of firms to over-invest in information security. As a result, firms can prioritize security spending and can divert resources which would be otherwise overinvested in information security to other areas which require investments. In contrast, cyber insurance may not be an appropriate resolution for mitigating the underinvestment problem since it will result in more severe problems of inefficient security investment. How to mitigate this problem can be found from the next implication.

As the last set of implications, this study identified the complementarity between investments in self-protection and the purchase of cyber insurance coverage. This implies that, although this study found that the adoption of cyber insurance might aggravate the insufficient security investment problem, the complementarity effect can potentially mitigate this problem and can improve social welfare. For example, due to the complementarity effect, government subsidies on organizations' purchase of cyber insurance policies, which cover damages caused by untargeted attacks, will induce the increase in organizations' purchase of the insurance policies as well as the level of IT security investments. Therefore, it can be inferred that additional mechanisms that take advantage of the complementarity effect could solve the insufficient investment problem resulting from the adoption of cyber insurance and lead to a better social outcome.

Its findings notwithstanding, this study has certain limitations, some of which are inherent in the assumptions and some are related to peculiarities of the theoretical model. The discussion of these limitations will include proposals for future research topics that may constitute interesting directions for independent research. First, this study did not consider hybrid attacks which combine targeted and untargeted attacks for a clear illustration of the theoretical analysis. Furthermore, this study did not take the dynamic features of cyber security: for instance, if a cyber attacker substitutes its target for another target, the targets' decision about security risk management strategies will be altered. Including these aspects in the analysis would be very helpful to understand cyber security issues.

Second, this study did not consider other security risk management mechanisms such as information sharing, markets for vulnerabilities, fines and subsidies, and liability rules. As indicated in the earlier sections, we found that cyber insurance can only offer a partial solution for inefficient security investment (i.e., the overinvestment problem). By considering alternative methods in combination, we would be able to propose the constellation of risk mitigation mechanisms which could result in a better social outcome.

Third, although this study investigated cyber insurance related issues, it did not include an analysis of implementation problems (i.e., moral hazard and adverse selection) which is an analysis widely conducted in the field of insurance economics. In the field of cyber security, since a market failure can occur not only because an inefficient level of information security investment but also because of moral hazard and adverse selection in a cyber insurance market, including this aspect in a model would be beneficial and yield potentially interesting results.

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(Appendix)

Proof of Proposition 1: Compare (6) with (3), if the cost of a breach is assumed to be equal to 1, $B_{1}^{'}(z_{1}^{o}) = -\frac{1}{L_{\text{\tiny I}}} + r[1 - B_{1}(z_{1}^{o})] > p^{'}(z_{1}^{p}) = -\frac{1}{L_{\text{\tiny I}}[1 - qp(z_{1}^{p})]} + r[1 - p(z_{1}^{p})] \text{ and } z_{1}^{o} > z_{1}^{p} \text{ since } B_{1}(z_{1}^{o}) = \ p(z_{1}^{o}) = p(z_$ and $p'(\cdot) < 0$. Similarly, compare (7) with (3), $B_1'(z_1^o) = -\frac{1}{L_1} + r[1 - B_1(z_1^o)] < p'(z_1^n) = -\frac{1}{2L_1} + r[1 - B_1(z_1^o)] < p'(z_1^n) = -\frac{1}{2L_1} + r[1 - B_1(z_1^o)] < 0$ $\frac{r[1-p(z_1^n)]}{2}$ and $z_1^o < z_1^n$. Therefore, it can be demonstrated that $z_1^n > z_1^o > z_1^p$. Similarly, compare (11) with (9), if the cost of a breach is assumed to be equal to 1, $p'(z_1^{oI}) = -\frac{1}{(1+\lambda)L_1} > p'(z_1^{pI}) =$ $-\frac{1}{[1-qp(z_1^{pI})](1+\lambda)L_1}$ and $z_1^{oI}>z_1^{pI}$. In contrast, compare (13) with (9), $p'(z_1^{nI})=-\frac{1}{2(1+\lambda)L_1}>$ $p'\left(z_{1}^{oI}\right)=-\frac{1}{(1+\lambda)L_{\scriptscriptstyle 1}} \ \ \text{and} \ \ z_{1}^{nI}>z_{1}^{oI}. \ \ \text{As a result,} \ \ z_{1}^{nI}>z_{1}^{oI}>z_{1}^{pI}.$

Proof of Proposition 2: Comparing equations (10), (12) and (14), it can be demonstrated that $L_1 - \frac{\lambda}{r(1+\lambda)[1-p(z_1^{nI})]} > L_1 - \frac{\lambda}{r[1-p_1(z_1^{oI})](1+\lambda)} \geq L_1 - \frac{\lambda}{r(1+\lambda)[1-p(z_1^{pI})][1-qp(z_1^{pI})]} \ . \ \ \text{As \ a \ result,}$ $I_1^{nI} > I_1^{oI} \ge I_1^{pI}$.

Proof of Proposition 3: In the presence of positive externalities, the impact of loss on firm 1's security investment can be expressed as:

$$\begin{split} &\frac{\partial p^{\prime}(z_{1}^{pI})[1-qp(z_{1}^{pI})]}{\partial L_{1}} = \frac{1}{(1+\lambda)L_{1}^{2}} \\ &\rightarrow \frac{\partial p^{\prime}(z_{1}^{pI})[1-qp(z_{1}^{pI})]}{\partial z_{1}^{pI}} \; \frac{\partial z_{1}^{pI}}{\partial L_{1}} = \frac{1}{(1+\lambda)L_{1}^{2}} \\ &\rightarrow \frac{\partial z_{1}^{pI}}{\partial L_{1}} = \frac{1}{(1+\lambda)L_{1}^{2}\{p^{\prime\prime}(z_{1}^{pI})[1-qp(z_{1}^{pI})]-p^{\prime\prime}(z_{1}^{pI})qp^{\prime\prime}(z_{1}^{pI})\}} > 0 \end{split}$$

Similarly, in the presence of negative externalities, the impact of loss on firm 1's security investment can be presented as:

$$\frac{\partial p'(z_1^{nI})}{\partial L_1} = \frac{1}{2(1+\lambda)L_1^2} \rightarrow \frac{\partial z_1^{nI}}{\partial L_1} = \frac{1}{2(1+\lambda)L_1^2 p''(z_1^{nI})} > 0$$

Proof of Proposition 4: In the presence of positive externalities, the impact of loss on firm 1's purchase of cyber insurance coverage can be expressed as:

$$\begin{split} \frac{\partial z_1^{pI}}{\partial L_1} &= 1 - \frac{1}{r(1+\lambda)} \ \frac{\partial \left\{ [1 - p(z_1^{pI})][1 - qp(z_1^{pI})] \right\}^{-1}}{\partial z_1^{pI}} \frac{\partial z_1^{pI}}{\partial L_1} \\ &= 1 + \frac{\lambda}{r(1+\lambda)} \ \frac{[-p'(z_1^{pI})(1 - qp(z_1^{pI})) - (1 - p(z_1^{pI}))qp'(z_1^{pI})]}{[1 - p(z_1^{pI})]^2[1 - qp(z_1^{pI})]^2} \frac{\partial z_1^{pI}}{\partial L_1} > 0 \end{split}$$

On the other hand, in the presence of negative externalities, the impact of loss on firm 1's purchase of cyber insurance coverage can be determined by:

$$\frac{\partial I_1^{nI}}{\partial L_1} = 1 - \frac{\lambda}{r(1+\lambda)} \cdot \frac{\partial [1 - p(z_1^{pI})]^{-1}}{\partial z_1^{nI}} \cdot \frac{\partial z_1^{nI}}{\partial L_1} = 1 - \frac{\lambda}{r(1+\lambda)} \cdot \frac{p'(z_1^{nI})}{[1 - p(z_1^{nI})]^2} \cdot \frac{\partial z_1^{nI}}{\partial L_1} > 0$$

Proof of Proposition 5: In the case of positive externalities, the relationship between firm 1' security investment and cyber insurance purchase can be determined by:

$$\begin{split} \frac{\partial I_{1}^{pf^{*}}}{\partial p(z_{1}^{pf^{*}})} &= -\frac{\lambda \left\{ r(1+\lambda)[(1-qp(z_{1}^{pf^{*}}))+q(1-p(z_{1}^{pf^{*}}))] \right\}}{\left\{ r(1+\lambda)[1-p(z_{1}^{pf^{*}})][1-qp(z_{1}^{pf^{*}})] \right\}^{-2}} < 0 \\ \frac{\partial p(z_{1}^{pf^{*}})}{\partial z_{1}^{pf^{*}}} &= -\frac{1}{[1-qp(z_{1}^{pf^{*}})](1+\lambda)L_{1}} < 0 \end{split}$$

Therefore, $\frac{\partial I_1^{p^k}}{\partial z_1^{p^k}} = \frac{\partial I_1^{p^k}}{\partial p(z_1^{p^k})} \frac{\partial p(z_1^{p^k})}{\partial z_1^{p^k}} > 0$. Similarly, in the presence of negative externalities, the relationship can be demonstrated by:

$$\begin{split} \frac{\partial I_1^{nF}}{\partial p(z_1^{nF})} &= \frac{\lambda(1+\lambda)r}{\{r(1+\lambda)[1-p(z_1^{nF})]\}^{-2}} < 0\\ \frac{\partial p(z_1^{nF})}{\partial z_1^{nF}} &= \frac{1}{2(1+\lambda)L_1} < 0 \end{split}$$

As a result,
$$\frac{\partial I_1^{nI}}{\partial z_1^{nI}} = \frac{\partial I_1^{nI}}{\partial p(z_1^{nI})} \frac{\partial p(z_1^{nI})}{\partial z_1^{nI}} > 0.$$

Proof of Proposition 6: From <Table I>, the comparison of optimal security investment for each cell leads us to the following results.

$$\begin{split} p'\left(z_{1}^{oI}\right) &= -\frac{1}{L_{1}} < p'\left(z_{1}^{o}\right) = -\frac{1}{L_{1}} + r[1 - p(z_{1}^{o})] \rightarrow z_{1}^{oI} < z_{1}^{o} \\ p'\left(z_{1}^{pI}\right) &= -\frac{1}{[1 - qp(z_{2}^{pI})]L_{1}} < p'\left(z_{1}^{p}\right) = -\frac{1}{L_{1}[1 - qp(z_{2}^{p})]} + r[1 - p(z_{1}^{p})] \rightarrow z_{1}^{pI} < z_{1}^{p} \\ p'\left(z_{1}^{nI}\right) &= -\frac{1}{2L_{1}} < p'\left(z_{1}^{n}\right) = -\frac{1}{2L_{1}} + \frac{r[1 - p(z_{1}^{n})]}{2} \rightarrow z_{1}^{nI} < z_{1}^{n} \end{split}$$

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