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Bayesian reliability estimation of bivariate Marshal-Olkin exponential stress-strength model

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Abstract. In this article we attempted reliability analysis of a component under the stress-strength pattern with both classical as well as Bayesian techniques. The main focus is made to develop the theory for dealing the reliability problems in various circumstances for bivariate environmental set up in context of Bayesian paradigm. A stress-strength based model describes the life of a component which has strength (Y) and is subjected to stress(X). We develop the Bayes and moment estimators of reliability of a component for each of the three possible conditions, under the assumption that the two stresses (i.e. X_1 and X_2) on a component are dependent and follow a Bivariate exponential (BVE) of Marshall-Olkin distribution, the strength of a component (Y) following exponential distribution is independent of the stresses. The simulation study is performed with Markov Chain Monte Carlo technique via Gibbs sampler to obtain the estimates of Bayes estimators of reliability, are compared with moment estimators of reliabilities on the basis of absolute biases.

Key Words: *Reliability analysis, Bayes estimator, moment estimator, bivariate exponential, stress-strength model, Monte Carlo integration, Gibbs sampling, MCMC simulation*

1. INTRODUCTION

The reliability of a component is the probability that the random strength(Y) exceeds the random stress(X).Thus $\mathrm{R} = P_{X\,Y}\left[(x, y)/\, y > x\right]$ is a measure of component reliability.

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Many researchers have extensively used this approach for the measurement of R, which has wide application in real life problems like engineering system, aerospace, nano technology, telecommunication, information technology and even in bioinformatics, biotechnology, biosciences and stress-biology. Some more applications are attracted by Nadarajah and Kotz (2006), Nadarajah (2005), Kotzet.al. (2003) and Jeevanand (1998).

Plenty of literature on estimation of system reliability as well as reliability of a single component stress-strength model are available. The associated problems of reliability estimation has been considered by several researchers such as Bhattacharyya and Johnson (1974, 1975), Johnson (1988), Pandey and Chandra (2004) with the assumption that the component's stresses are independent and identically distributed random variables and these are independent of their strength. All of these follow some specified univariate life distributions.

Apart from the above assumptions many authors have considered the case where stresses are dependent, and are independent to the strength of the component. Chandra and Owen (1975)have discussed the estimates of reliability of a component subjected to several different stresses. They obtained the reliability function as R=Pr[Max $(X_1, X_2, \ldots, X_k) < Y$], where $(X_1, X_2, \ldots, X_k)$ are stresses applied on a component. Jana (1994) obtained estimators of the reliability of a component under stress-strength environment, when the stress and strength are dependent with Marshall-Olkin of BVE model. Hanagal (1995) obtained the estimate of R of the component (of strength (Y)) subjected to the random stress (X). It is assumed that (X,Y) follows a BVE of Gumbel (1960) and Block-Basu (1974). The estimates of R, are based on either proportion of count in the sample of size n or maximum likelihood estimates (mle). Hanagal (1999) has attempted an estimate of reliability of a component R = Pr $[(X_1 + X_2) < Y]$, when only two stresses (X_1, X_2) applied are dependent BVE. The BVE models considered by him are (i) proposed by Marshall-Olkin (1976), (ii) Block-Basu (1974), (iii) Freund(1961) and (iv) Proschan-Sullo (1974), strength follows exponential distribution with failure rate μ independent of stresses. Jeevanand (1998) derived the Bayes estimate of reliability(R) under quadratic loss function for BVE model (Marshall-Olkin) and this estimate is the Mann-Whitney's estimate of R (see Awadet al.1981).

In this paper, we obtain the Bayes as well as moment estimators of reliability of a component subjected to two stresses (X_1, X_2) when the stresses X_1 and X_2 are dependent random variables having BVE model proposed by Marshall-Olkin (1976). The strength (Y) of the component is independent of the stresses and Y follows exponential distribution with failure rate μ. The comparison has been done on the basis of the absolute biases of the Bayes estimator of reliability with that of the moment estimator of reliability.

This article may be the first attempt for numerical investigation of the Bayes as well as moment estimators of reliability of a component following the Marshall-OlkinBVE model of stress-strength set up, for three different conditions for possible variation of parameters. Hanagal (1999) has developed the possible reliability functions for the said three conditions of this article. He has only suggested that the mle of reliability can be evaluated by either Newton-Raphson or Fisher method of scoring. The conditions under which these should be advantageously used, have been given in this paper, not in any earlier paper.

There are plenty of real life examples suitable where a unit / component are subjected to multiple (two or more dependent) stresses simultaneous in nature, which may cause the failure of component. In this article few example are given for two dependent stresses.

1) A scooter/bike tube may fail when an accumulated amount of two dependent stresses (X_1: environmental temperature and X_2: air pressure) subjected to tube and exceeds its strength(Y).
2) An electronic tube light may fail when an accumulated amount of two dependent stresses(X_1: voltage(volt) and X_2: current(amp)) subjected to it and exceeds its strength(Y).

The random variables X_1 and X_2 are said to follow the BVE distribution of Marshall-Olkin (1976) with respect to the Lebesgue measure on R_2.The joint pdf of X_1 and X_2 is given by

$$f(x_1,x_2) = \begin{cases} \lambda_1(\lambda-\lambda_1)\exp\{-[(\lambda_1 x_1 + \lambda_2 x_2) + \lambda_3 x_2]\} & ; x_2 > x_1 > 0 \\ \lambda_2(\lambda-\lambda_2)\exp\{-[(\lambda_1 x_1 + \lambda_2 x_2) + \lambda_3 x_1]\} & ; x_1 > x_2 > 0 \\ \lambda_3 \exp(-\lambda x) & ; x_1 = x_2 = x > 0 \\ \lambda_1, \lambda_2 > 0,\ \lambda_3 \geq 0 \text{ and } \lambda = \lambda_1 + \lambda_2 + \lambda_3, & \end{cases} \tag{1.1}$$

The third term of $f(x_1, x_2)$ is a density with respect to the Lebesgue measure on R_1.

In section 2, the reliability expressions under dependent stresses for conditions I, II and III (stated in next section) have been derived. The likelihood function is given in section 3 and in section 4, we obtain Bayes estimators of reliability for all the said three conditions, of the parameters of proposed model. In section 5, we introduced the moment estimators of reliability for each of said conditions. Numerical comparison of the proposed estimators of reliability with respect to moment estimators of reliability on the basis of absolute biases for each of the condition I, II and III are shown in section 6. Finally, conclusion of the proposed study is given in section 7.

2. RELIABILITY EXPRESSIONS UNDER DEPENDENT STRESSES

In order to obtain the estimate of reliability (R) of a component with strength Y when the stresses (X_1, X_2) follow BVE of Marshall-Olkin (1976) defined in (1.1), first we find the pdf of U (= X_1+X_2) under each of the following three conditions.

$$\left.\begin{array}{ll} 1) & \lambda_1 \neq \lambda_2 + \lambda_3 \text{ and } \lambda_2 \neq \lambda_1 + \lambda_3 \\ 2) & \lambda_1 = \lambda_2 + \lambda_3 \text{ and } \lambda_2 \neq \lambda_1 + \lambda_3 \\ 3) & \lambda_1 \neq \lambda_2 + \lambda_3 \text{ and } \lambda_2 = \lambda_1 + \lambda_3 \end{array}\right\} \tag{2.1}$$

The pdf of the random variable U for the conditions (I), (II) and (III), are derived as

$$h_1(u) = \begin{cases} \dfrac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)}\left\{\exp(-\lambda u/2) - \exp[-(\lambda_2+\lambda_3)u]\right\}; & x_2 > x_1 \\ \dfrac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_2-\lambda_1-\lambda_3)}\left\{\exp[-(\lambda_1+\lambda_3)u] - \exp(-\lambda u/2)\right\}; & x_2 < x_1 \\ \dfrac{\lambda_3}{2}\exp(-\lambda u/2); & x_1 = x_2 \\ & \text{if } \lambda_1 \neq \lambda_2+\lambda_3,\ \lambda_2 \neq \lambda_1+\lambda_3 \end{cases} \tag{2.2}$$

$$h_{II}(u) = \begin{cases} \frac{\lambda_1}{2}(\lambda_2+\lambda_3)\, u\, \exp(-\lambda_1 u) & ;\, x_2 > x_1 \\ \frac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_2-\lambda_1-\lambda_3)}\left\{\exp\left[-(\lambda_1+\lambda_3)u\right]-\exp\left(-\lambda u/2\right)\right\} & ;\, x_2 < x_1 \\ \frac{\lambda_3}{2}\exp(-\lambda u/2) & ;\, x_1 = x_2 \end{cases} \tag{2.3}$$

if $\lambda_1 = \lambda_2 + \lambda_3$, $\lambda_2 \neq \lambda_1 + \lambda_3$

$$h_{III}(u) = \begin{cases} \frac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)}\left\{\exp\left(-\lambda u/2\right)-\exp\left[-(\lambda_2+\lambda_3)u\right]\right\} & ;\, x_2 > x_1 \\ \frac{\lambda_2}{2}(\lambda_1+\lambda_3)\, u\, e^{-(\lambda_1+\lambda_3)u} & ;\, x_2 < x_1 \\ \frac{\lambda_3}{2}\, e^{-\lambda u/2} & ;\, x_1 = x_2 \end{cases} \tag{2.4}$$

if $\lambda_2 = \lambda_1 + \lambda_3$, $\lambda_1 \neq \lambda_2 + \lambda_3$

The expression of reliability of the component subjected to BVE stresses has been derived under the three conditions given below.
Assume that the strength (Y) has exponential distribution with survival function, given by

$$\overline{H}(y) = P(Y>y) = \exp(-\mu y), \tag{2.5}$$

where, μ is failure rate. Now, the expressions of reliability of a component subjected to BVE stresses, for Conditions (I), (II) and (III), are obtained as:

$$R_1 = \left\{\frac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)}\left[\frac{1}{(\lambda/2+\mu)}-\frac{1}{(\lambda_2+\lambda_3+\mu)}\right] + \frac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_1+\lambda_3-\lambda_2)}\left[\frac{1}{(\lambda/2+\mu)}-\frac{1}{(\lambda_1+\lambda_3+\mu)}\right]+\frac{\lambda_3}{2(\lambda/2+\mu)}\right\}$$

if $\lambda_1 \neq \lambda_2 + \lambda_3$, $\lambda_2 \neq \lambda_1 + \lambda_3$ (2.6)

$$R_2 = \left\{\frac{\lambda_1(\lambda_2+\lambda_3)}{2(\lambda_1+\mu)^2}+\frac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_1+\lambda_3-\lambda_2)}\left[\frac{1}{(\lambda/2+\mu)}-\frac{1}{(\lambda_1+\lambda_3+\mu)}\right]+\frac{\lambda_3}{2(\lambda/2+\mu)}\right\}.$$

if $\lambda_1 = \lambda_2 + \lambda_3$, $\lambda_2 \neq \lambda_1 + \lambda_3$ (2.7)

$$R_3 = \left\{\frac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)}\left[\frac{1}{(\lambda/2+\mu)}-\frac{1}{(\lambda_2+\lambda_3+\mu)}\right]+\frac{\lambda_2(\lambda_1+\lambda_3)}{2(\lambda_1+\lambda_3+\mu)^2}+\frac{\lambda_3}{2(\lambda/2+\mu)}\right\}$$

if $\lambda_1 \neq \lambda_2 + \lambda_3$, $\lambda_2 = \lambda_1 + \lambda_3$ (2.8)

3. LIKELIHOOD FUNCTION

Let (x_{1k}, x_{2k}) and y_k, k = 1, 2, …, n, be i.i.d. random sample of size n. Let $n = n_1 + n_2 + n_3$, where n_1, n_2 and n_3 are the number of observations with $(x_1 < x_2)$, $(x_1 > x_2)$ and $(x_1 = x_2)$, respectively. The likelihood function of the sample is given by

$$L(data\,|\,\lambda_1,\lambda_2,\lambda_3\mu)=\prod_{k=1}^{n} f(x_{1k},x_{2k}).\ \prod_{k=1}^{n} f(y_k)$$

$$=(\mu\lambda_3)^n\left(\frac{\lambda_1}{\lambda_3}(\lambda-\lambda_1)\right)^{n_1}\left(\frac{\lambda_2}{\lambda_3}(\lambda-\lambda_2)\right)^{n_2} e^{-z}\,. \quad (3.1)$$

where,

$$z=\sum_{k=1}^{n}\left(\sum_{i=1}^{2}\lambda_i x_{ik}+\mu y_k\right)-\lambda_3\left(\sum\nolimits_1 x_{2k}+\sum\nolimits_2 x_{1k}\right)-\sum\nolimits_3\left(\sum_{i=1}^{2}\lambda_i x_{ik}+\lambda x_k\right) \quad (3.2)$$

$\sum_1, \sum_2$ and $\sum_3$ denote the sum over the n_1 sets of observations with $x_1<x_2$,n_2 sets of observations with $x_1>x_2$ and n_3 (=n-n_1-n_2) sets of observations with $x_1=x_2=x$, respectively, such samples are treated as 'data'.

4. BAYESIAN ESTIMATION OF RELIABILITY

For the Bayes estimation, we consider the independent non-informative prior distribution of random variables λ_1, λ_2, λ_3 and μ. The joint prior distribution of λ_1, λ_2, λ_3 and μ are given by

$$h(\lambda_1, \lambda_2, \lambda_3, \mu)=\frac{1}{(\lambda_1\,\lambda_2\,\lambda_3\,\mu)} \quad ;\ 0<\lambda_1<a, 0<\lambda_2<b,\ 0<\lambda_3<c,\ 0<\mu<d \quad (4.1)$$

Bayes theorem yields the joint posterior probability distribution of λ_1, λ_2, λ_3 and μ given by

π (λ_1, λ_2, λ_3, μ/data)

$$=\frac{(\mu\lambda_3)^{n-1}\,\lambda_1^{n_1-1}\,\lambda_2^{n_2-1}\left(\frac{\lambda-\lambda_1}{\lambda_3}\right)^{n_1}\left(\frac{\lambda-\lambda_2}{\lambda_3}\right)^{n_2} e^{-z}}{\int_0^a\int_0^b\int_0^c\int_0^d(\mu\lambda_3)^{n-1}\,\lambda_1^{n_1-1}\,\lambda_2^{n_2-1}\left(\frac{\lambda-\lambda_1}{\lambda_3}\right)^{n_1}\left(\frac{\lambda-\lambda_2}{\lambda_3}\right)^{n_2} e^{-z}\,d\lambda_1\,d\lambda_2\,d\lambda_3\,d\mu} \quad (4.2)$$

Under the squared error loss function (SELF), Bayes estimator of reliability is defined as the posterior means of R_i (i=1,2,3) given as

$$R_i^{**}=E(R_i/\text{data})=N_{r_i}/D_r\ ,\ i=1,2,3 \quad (4.3)$$

The derived Bayes estimators of reliability for conditions (I), (II) and (III), are given in (4.4), (4.7) and (4.9), respectively.

Table4.1. The expressions of the Bayes estimators of reliability for the possible conditions (I), (II) and (III).

Conditions	Bayes estimators of reliability
$\lambda_1 \neq \lambda_2 + \lambda_3$, $\lambda_2 \neq \lambda_1 + \lambda_3$	$R_1^{**} = \frac{Nr_1}{Dr}$ (4.4) $Nr_1 = \int_0^a \int_0^b \int_0^c \int_0^d \left\{ \frac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)} \left[\frac{1}{(\lambda/2+\mu)} - \frac{1}{(\lambda_2+\lambda_3+\mu)} \right] + \frac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_1+\lambda_3-\lambda_2)} \left[\frac{1}{(\lambda/2+\mu)} - \frac{1}{(\lambda_2+\lambda_3+\mu)} \right] + \frac{\lambda_3}{2(\lambda/2+\mu)} \right\} . (\mu\lambda_3)^{n-1} \lambda_1^{n_1-1} \lambda_2^{n_2-1} \left(\frac{\lambda-\lambda_1}{\lambda_3} \right)^{n_1} \left(\frac{\lambda-\lambda_2}{\lambda_3} \right)^{n_2} e^{-z}\, d\lambda_1\, d\lambda_2\, d\lambda_3\, d\mu$ (4.5) and $Dr = \int_0^a \int_0^b \int_0^c \int_0^d \left[(\mu\lambda_3)^{n-1} \lambda_1^{n_1-1} \lambda_2^{n_2-1} \left(\frac{\lambda-\lambda_1}{\lambda_3} \right)^{n_1} \left(\frac{\lambda-\lambda_2}{\lambda_3} \right)^{n_2} e^{-z} \right] d\lambda_1\, d\lambda_2\, d\lambda_3\, d\mu$ (4.6)
$\lambda_1 = \lambda_2 + \lambda_3$, $\lambda_2 \neq \lambda_1 + \lambda_3$	$R_2^{**} = \frac{Nr_2}{Dr}$, (4.7) $Nr_2 = \int_0^a \int_0^b \int_0^c \int_0^d \left\{ \frac{\lambda_1(\lambda_2+\lambda_3)}{2(\lambda_1+\mu)^2} + \frac{\lambda_2(\lambda_1+\lambda_3)}{(\lambda_1+\lambda_3-\lambda_2)} \left[\frac{1}{(\lambda/2+\mu)} - \frac{1}{(\lambda_2+\lambda_3+\mu)} \right] + \frac{\lambda_3}{2(\lambda/2+\mu)} \right\} (\mu\lambda_3)^{n-1} \lambda_1^{n_1-1} \lambda_2^{n_2-1} \left(\frac{\lambda-\lambda_1}{\lambda_3} \right)^{n_1} \left(\frac{\lambda-\lambda_2}{\lambda_3} \right)^{n_2} . e^{-z}\, d\lambda_1\, d\lambda_2\, d\lambda_3\, d\mu$, (4.8)
$\lambda_1 \neq \lambda_2 + \lambda_3$, $\lambda_2 = \lambda_1 + \lambda_3$	$R_3^{**} = \frac{Nr_3}{Dr}$, (4.9) $Nr_3 = \int_0^a \int_0^b \int_0^c \int_0^d \left[\frac{\lambda_1(\lambda_2+\lambda_3)}{(\lambda_2+\lambda_3-\lambda_1)} \left\{ \frac{1}{(\lambda/2+\mu)} - \frac{1}{(\lambda_2+\lambda_3+\mu)} \right\} + \frac{\lambda_2(\lambda_1+\lambda_3)}{2(\lambda_1+\lambda_3+\mu)^2} + \frac{\lambda_3}{2(\lambda/2+\mu)} \right] (\mu\lambda_3)^{n-1} \lambda_1^{n_1-1} \lambda_2^{n_2-1} \cdot \left(\frac{\lambda-\lambda_1}{\lambda_3} \right)^{n_1} \left(\frac{\lambda-\lambda_2}{\lambda_3} \right)^{n_2} e^{-z}\, d\lambda_1\, d\lambda_2\, d\lambda_3\, d\mu$ (4.10)

5. MOMENT ESTIMATORS OF RELIABILITY

According to Bemis et al. (1972), MLE's are in general biased and less efficient than moment type estimators for small samples. Therefore, the following moment estimators are obtained as

$$\lambda_{1m} = \left[\frac{n}{A_1} - \frac{n_3}{A_2}\right] \Big/ \left[1 + \frac{n_3}{n}\right] \tag{5.1}$$

$$\lambda_{2m} = \left[\frac{n}{A_2} - \frac{n_3}{A_1}\right] \Big/ \left[1 + \frac{n_3}{n}\right] \tag{5.2}$$

$$\lambda_{3m} = n_3 \left[\frac{1}{A_1} - \frac{1}{A_2}\right] \Big/ \left[1 + \frac{n_3}{n}\right] \tag{5.3}$$

$$\mu_m = \bar{x} \tag{5.4}$$

where,

$A_1 = \sum_1$: denote the sum of n_1 sets of observations with $x_2 > x_1$

$A_2 = \sum_2$: denote the sum of n_2 sets of observations with $x_2 < x_1$

$A_3 = \sum_3$: denote the sum of n_3 sets of observations with $x_2 = x_1 = x$

The moment estimators R_{1m}^*, R_{2m}^* and R_{3m}^* of reliability for the conditions (I), (II) and (III) are obtained by replacing λ_1, λ_2, λ_3 and μ by λ_{1m}, λ_{2m}, λ_{3m} and μ_m respectively, in equations (2.6), (2.7) and (2.8), respectively.

6. NUMERICAL STUDIES

6.1 Simulation studies

The advance simulation tools like Markov Chain Monte Carlo technique via Gibbs sampler is performed due to multiple integrals involved in the proposed estimators with the help of Monte Carlo integration method. The numerical investigation helps to know the performance of the Bayes estimators of reliability. The comparison of Bayes estimators of reliability has been done with the moment estimators of reliability. The Biases of Bayes estimators R_1^{**}, R_2^{**}, R_3^{**} of reliabilities are compared with that of the moment estimators of reliabilities viz., R_{1m}^*, R_{2m}^* and R_{3m}^*. The variation of λ_i(i=1,2,3) has been shown in Table 6.1, for condition(I), rest of the parameters values are supposed to be fixed.

The true reliability (R1) is observed almost 80% for such possible variations in each of λ_i (for i =1,2,3) and for particular variation of $\lambda 1$, it is observed that for the $\lambda 1<0.50$, Bayes estimator have less bias in comparison with moment estimator. For the varied values of $\lambda_1 > 0.50$, the moment estimator is less biased. It is found that Bayes estimator is less (or more) biased for varied value of $\lambda_2<3.00$(or > 3.00) and for the variation of $\lambda_3<3.5$(or >3.5), the Bayes estimator is less(or more) biased.

The simultaneous variation of λ_1 , λ_2 and λ_3 has been shown in Table6.2 for both the conditions (II) and (III) while values of other parameter μ, n_1 , n_2 , n_3 , n as well as the prior parameters(a, b, c, d) are considered to be fixed.

Table 6.1 Absolute Biases of Bayes(R_1^{**}) and Moment(R_{1m}^{*}) estimators for Condition-I

Variation of λ_1				Variation of λ_2				Variation of λ_3			
$\lambda_2 = 3.00, \lambda_3 = 3.00, \mu=0.50$, $n_1=8$, $n_2=5$, $n_3=2$, n= 15				$\lambda_1 = 0.25, \lambda_3 = 3.00, \mu=0.50$, $n_1=8$, $n_2=5$, $n_3=2$, n= 15				$\lambda_1 = 0.25, \lambda_2 = 3.00, \mu=0.50$, $n_1=8$, $n_2=5$, $n_3=2$, n= 15			
λ_1	R_1	ABSOLUTE BIASES		λ_2	R_1	ABSOLUTE BIASES		λ_3	R_1	ABSOLUTE BIASES	
		R_1^{**}	R_{1m}^{*}			R_1^{**}	R_{1m}^{*}			R_1^{**}	R_{1m}^{*}
0.21	0.87	0.138	0.146	0.75	0.77	0.225	0.291	2.25	0.77	0.124	0.307
0.22	0.86	0.044	0.143	1.00	0.78	0.280	0.286	2.50	0.78	0.180	0.299
0.23	0.85	0.032	0.163	1.50	0.79	0.239	0.280	2.75	0.79	0.143	0.327
0.25	0.84	0.148	0.163	2.00	0.79	0.274	0.281	3.25	0.81	0.223	0.297
0.50	0.82	0.539	0.178	2.50	0.79	0.220	0.280	3.50	0.82	0..283	0.296
0.75	0.82	0.611	0.191	3.00	0.80	0.148	0.298	3.75	0.83	0.332	0.295
1.00	0.81	0.643	0.219	3.50	0.81	0.808	0.304	4.00	0.84	0.372	0.294
2.00	0.80	0.695	0.298	4.00	0.81	0.812	0.313	4.25	0.85	0.436	0.292
3.00	0.80	0.718	0.324	4.50	0.82	0.815	0.319	4.50	0.86	0.462	0.291
4.00	0.80	0.870	0.316	4.75	0.82	0.817	0.422	4.75	0.88	0.424	0.321
5.00	0.80	0.870	0.310	5.00	0.84	0.818	0.436	5.00	0.88	0.451	0.315

Table 6.2 Absolute Biases of Bayes and Moment estimators for both the Conditions (II) & (III)

Condition-II :Variation of λ_1, λ_2 and λ_3						Condition-III :Variation of λ_1, λ_2 and λ_3					
$\mu = 0.50$, $n_1=8$, $n_2=5$, $n_3=2$, n= 15, a= 2.0, b=3.0, c = 2.0, d= 5.0						$\mu = 0.25$, $n_1=8$, $n_2=5$, $n_3=2$, n= 15, a=2.0, b=3.0, c=2.0, d=5.0					
λ_1	λ_2	λ_3	R_2	ABSOLUTE BIASES		λ_1	λ_2	λ_3	R_3	ABSOLUTE BIASES	
				R_2^{**}	R_{2m}^{*}					R_3^{**}	R_{3m}^{*}
0.50	0.15	0.35	0.60	0.045	0.142	0.50	2.00	1.50	0.85	0.152	0.419
0.50	0.35	0.15	0.73	0.249	0.264	1.00	3.00	2.00	0.87	0.453	0.537
0.75	0.25	0.50	0.81	0.105	0.115	2.00	3.00	1.00	0.92	0.375	0.431
0.75	0.50	0.25	0.71	0.123	0.268	3.00	5.00	2.00	0.92	0.156	0.431
1.00	0.70	0.30	0.80	0.207	0.233	3.00	7.00	4.00	0.93	0.299	0.395
2.00	1.50	0.50	0.76	0.116	0.298	4.00	7.00	3.00	0.93	0.380	0.421
3.00	0.50	1.50	0.75	0.292	0.517	2.00	7.00	5.00	0.94	0.223	0.401
4.00	2.00	2.00	0.90	0.365	0.582	5.00	7.00	2.00	0.94	0.125	0.431
4.50	2.00	2.50	0.92	0.225	0.561	5.00	6.00	1.00	0.92	0.389	0.418
5.00	3.00	3.00	0.92	0.115	0.573	2.00	5.00	3.00	0.84	0.577	0.366
5.50	4.00	3.50	0.92	0.278	0.531	5.00	8.00	3.00	0.94	0.290	0.425
6.00	5.00	4.00	0.93	0.324	0.5.45	6.00	9.00	3.00	0.94	0.792	0.410
6.50	6.00	5.00	0.93	0.332	0.436	3.00	8.00	5.00	0.95	0.551	0.370

Table 6.2 shows that the value of R_2 is between 60-90%, Bayes estimator of reliability has smaller bias in comparison to moment estimator of reliability for condition-II. For condition-III, the true reliability is high i.e.95% and Bayes estimator of reliability has less bias in comparison to moment estimator of reliability for most of the chosen values of parameters λ_1, λ_2 and λ_3 (refer to Table 6.2).

7. CONCLUDING REMARKS

In this article, we developed the Bayes estimators of reliability of a BVE distribution of Marshall-Olkin for a component subjected to two dependent "shocks" for three possible conditions. It is observed that the proposed Bayes estimators of reliability are better than the moment estimators of reliability for each of the considered conditions. It is justifiable through the numerical studies that the proposed Bayes estimators of reliability for all three said conditions may be recommended to use in real life situation for testing the reliability of failure data of such bivariate environmental set up. Overall, the estimator under condition (III) is more advantageous to use in compare to others.The reliability estimation aspects for bivariate exponential proposed by Block-Basu (1974), Freund (1971) and Proschan-Sullo (1974) are being searched. The further research studies are, also left for other bivariate life distributions viz. Bivariate Weibull, bivariate gamma, bivariate lognormal etc..

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