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Operational behaviour and reliability measures of a viscose staple fibre plant including deliberate failures

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Abstract. This Paper deals with the stochastic behavior and failure analysis of a Viscose Staple Fibre Plant which produces fibre for making clothes. The fibre making plant is a complex system with various subsystems as: Vendor (supplies Charcoal and Sulphur, raw materials for the process), Carbon di sulphide Plant, Acid Plant, Pulp Plant and Processing Plant. The considered system can completely fail due to failure of any of the subsystems. The Carbon di Sulphide Plant can fail in two different ways, due to lack of Sulphur or Charcoal. Processing Plant has the configuration 5-out-of -10: d and 6-out -of-10: f. It is also assumed that the system can fail due to workers strike and catastrophic failure. All failures follow exponential time distribution whereas all repairs follow general time distribution. Preventive Maintenance policy has been applied to reduce the failure in the system. Various reliability characteristics such as transition state probabilities, steady state behavior, reliability, availability, M.T.T.F and the cost analysis have been obtained using supplementary variable technique and Gumbel-Hougaard copula methodology.

Key words: *Supplementary variable technique, reliability, availability, MTTF, asymptotic behavior, markov process*

1. INTRODUCTION

In the present scenario of competitive market, speedy and economical production is a prime requirement for survival. To achieve this goal the concept of production line is becoming popular day by day. In this line, non-identical machines are arranged logically to perform desired process to convert raw materials into the finished good. This helps to achieve the mass production at minimum cost with international quality standards. Furthermore, real-time and embedded systems are now a central part of our lives. Reliable functioning of these systems is of paramount concern to the millions of users that depend on these systems every day. Unfortunately most embedded systems still fall short of user's expectation of reliability.

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Basically a system is a combination of elements forming a planetary whole i.e. there is a functional relationship between its components. The properties and behavior of each component ultimately affects the properties of the system. Any system has a hierarchy of components that pass through the different stages of operations which can be operational, failure, degraded or in repair. Failure doesn't mean that it will always be complete; it can be partial as well. But both these types affect the performance of system and hence the reliability [Bazovsky (1961), Barlow (1965)]. Majority of the systems in the industries are repairable. The performance of these systems can influence the quality of product, the cost of business, the service to the customers, and thereby the profit of enterprises directly. Modern repairable systems tend to be highly complex due to increase in convolution and automation of systems.

As far as the production-operations are concerned, not only reliability but also steady state availability analysis is essential, again on account of increased complexity and cost of present day equipment. Also the markets are getting globalized and more competitive. Penalties for delayed deliveries have been increased. Sometimes the orders are cancelled and defaulting plants are not favoured with orders. To overcome these types of problems, reliability and steady state availability analysis is necessary for performance studies in the area of discrete manufacturing systems. Many researcher [Chandrashekar (1996), Dhillon (1991), Kumar (1993) and Gopalan (1982)] discussed reliability and steady state analysis of Manufacturing Plant by using different approaches.

The purpose of the present paper is to compute the reliability characteristics of Viscose Staple Fibre Plant which produces fibre for making clothes. A Viscose Staple Fibre (VSF) Plant consists of five subsystems connected in series, namely Vendor (who supplies raw materials Charcoal and Sulphur), Carbon di sulphide (CS_2) Plant, Acid Plant, Pulp Plant, and Processing Plant. In the Processing Plant ten parallel machines are involved in doing the same job. This subsystem follows 5-out -of-10:d and 6-out-of-10:f configurations which specifies that if 5 machines of the Processing Plant are not working then the system is in reduced efficiency state and not able to fulfil the required target and if more than 5 machines are failed then the system is failed [Chung (1988)]. Also the subsystem CS_2 can fail in two different ways viz, due to the lack of Charcoal (supplied by Vendor) and due to lack of Sulphur (supplied by Vendor) as they both are the raw materials for the process. Preventive maintenance is one of the important aspects of production companies; it is possible by providing rest to all the machines one by one for a particular period of time as per maintenance schedule. This fact has been taken into consideration in the present studies.

METHODOLOGY USED:

Supplementary variable technique is used to estimate the reliability measures of the considered industrial problem. Copula methodology has also been incorporated to evaluate the joint probability distribution of repairs in CS_2 Plant. This method provides an easy way to estimate the variation in different system performance in terms of reliability with respect to time 2.

APPROACH:

The mathematical model of Viscose Staple Fibre (VSF) Plant has been developed using Markov Process with the help of supplementary variable technique and copula methodology. The differential equations are solved using Laplace Transforms. Maple - program have been developed to study the variations of reliability, MTTF, and availability of the system with respect to time.

NOVELTY:

Industrial implications of the results have been discussed.

2. SYSTEM DESCRIPTION

Fibre making process is one of the best examples to understand the implications of production process of an industry. VSF is a man-made, biodegradable fibre with characteristics akin to cotton. As an extremely versatile and easily bendable fibre, VSF is widely used in apparel, home textiles, dress material, knitted wear and non-woven applications.

VISCOSE:

There are two main categories of man-made fibres: those that are made from natural products (cellulose fibres) and those that are synthesized solely from chemical compounds (non-cellulose polymer fibres). Rayon is a natural-based material that is made from the cellulose of wood pulp or cotton. This natural base gives it many of the characteristics low cost, diversity, and comfort that have led to its popularity and success. This regular Rayon is also called Viscose Rayon. The first patent on the Viscose process was granted to Cross and Bevan in England in 1893. By 1908, the fibre spun from Viscose dope had been accepted as a key component of the burgeoning textile industry.

Various Plants involved in producing fibre are as follows:

CS_2 PLANT:

CS_2 Plant comparsis of: Electric Furnace, Raw sulphur Pits, Calciners, Recovered sulphur pits, CS_2 Refineries, Oil Scrubbers, Tertiary condensers, gas holders and CS_2 Storage tanks. In the process of making CS_2, Charcoal is heated to red hot into electric furnace, raw Sulphur is then mixed into the furnace results into emission of raw CS_2 gas which is then condensed using primary, secondary and tertiary condensers. Raw liquid CS_2 is then sent to refinery to obtain refined CS_2 as a final product.

H_2SO_4 PLANT (ACID PLANT):

H_2SO_4 Plant comparsis of: Sulphur pit, Metering pump, Furnace, Boiler, Circulation tank, Main air blower, Acid towers Heat exchanger, Acid storage tank and Converters. In the process of making Sulphuric acid (H_2SO_4), initially liquid Sulphur is heated into the boilers and then it is converted into the gas. This gas blows to the catalyst, then by mixing the gas and acid H_2SO_4 is obtained.

PULP PLANT:

Pulp which is the most important raw material for making fibre is produced by using different types of woods, but now a day it is also manufactured through the waste of Sugar plant, which is a good example of waste management.

PROCESS:

In a Fibre making industry there are five plants arranged in series and these five plants have been divided into five subsystems to understand the logical sequence of the processes viz Vendor(who supplies raw materials Sulphur and Charcoal), Carbon disulphide Plant, Acid Plant, Pulp Plant, Processing Plant. Vendor, supplies the raw materials Charcoal and Sulphur for making CS_2 and Acid (Sulphuric Acid). CS_2 Plant, Acid Plant, Pulp Plant produces Carbon di sulphide, H_2SO_4 and pulp respectively. In the process of making fibre, initially Sulphur and Charcoal (supply by the Vendor) goes to the CS_2 Plant, where Charcoal is heated up to 680^0C in the furnace when it becomes red hot, liquid Sulphur is then mixed with it. Then CS_2 is obtained. To produce H_2SO_4 initially liquid Sulphur is heated into the boilers and then it is converted into the gas. The gas blows to the catalyst, and then H_2SO_4 is obtained.

The main heart of Fibre making Plant is Processing Plant, where fibre is produced. The whole Process of making fibre is discussed below. In the Processing Plant initially the Slurry which is a mixer of CS_2 and pulp converted into a Viscose which is looking like as Honey. This Viscose is then passes through H_2So_4 then by the process of spinning regular fibre is obtained. Then washing of this regular fibre is done by after treatment method. Finally cutting and then packing. The transition state diagram describing the system is shown in Figure 2.1 and states description is given by Table 2.1.

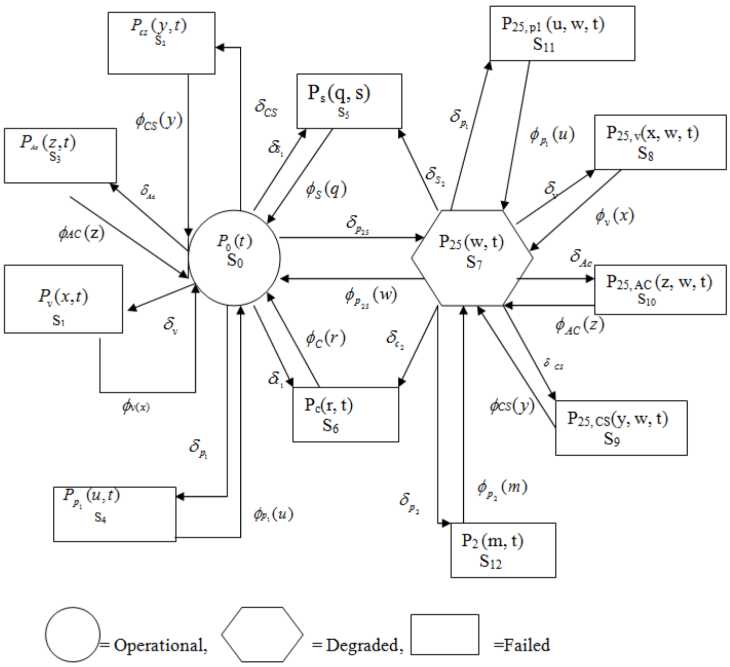


Figure 2.1 State-transition diagram

The following assumption has been taken into the considerations in this study.

- Initially at t=0, all plants are operating well.
- Failures are statistically independent.
- The repair time of the plants are assumed to be arbitrarily distributed.
- Repaired subsystem/ plant(s) works like new.
- All failures follow exponential time distribution.
- Processing Plant has 5-out-of-10: d and 6-out-of-10: f configuration.
- The whole system can also fail due to deliberate failures like; workers strike and catastrophic failures.
- Joint probability distribution has been applied in CS_2 Plant for repair as the plant can fail due the lack of Charcoal as well as Sulphur, the raw materials supplied by the Vendor [Pandey (2008)].

RELIABILITY IMPROVEMENT FEATURES USED:

1. Use of SS 316L pipeline for CS2 application.
2. Increase in thickness of CS2 storage tank to enhance life.
3. Use of Teflon lined ms pipe for dilute acid application.
4. Two acid strength controllers provided for each plant.

STATES SPECIFICATION:

Table 2.1 shows the state specifications of the transition diagram.

Table 2.1 State specifications table

State	Description	System state
S_0	The state when the system is in fully operational condition.	G
S_1	The state when the system is in failed state due to the failure of Vendor.	F_R
S_2	The state when the system is in failed state due to the failure of CS_2 Plant.	F_R
S_3	The state when the system is in failed state due to the failure of Acid Plant.	F_R
S_4	The state when the system is in failed state due to the failure of Pulp Plant.	F_R
S_5	The state when the system is in failed state due to the workers strike.	F_R
S_6	The state when the system is in failed state due to the catastrophic failure.	F_R
S_7	The state when the system is in reduced efficiency state due to the failure of 5(out of 10) machines of Processing Plant (p_2).	D_R
S_8	The state when the system is in failed state from the degraded state S_7 due to the failure of Vendor.	F_R
S_9	The state when the system is in failed state from the degraded state S_7 due to the failure of CS_2 Plant.	F_R
S_{10}	The state when the system is in failed state from the degraded state S_7 due to the failure of Acid Plant.	F_R
S_{11}	The state when the system is in failed state from the degraded state S_7 due to the failure of Pulp Plant.	F_R
S_{12}	The state when the system is in failed state from the degraded state S_7 due to the failure of other machines of Processing Plant.	F_R

Note: G: Good state; D_R: Degraded State and under repair; F_R= Failed state and under repair.

3. NOMENCLATURE

Pr	Probability
$P_0(t)$	Pr{at time t the system is in state S_0}
$P_{25}(w,t)$	Pr {the system is in degraded state at time t due to the failure of 5machines of the processing Plant (p_2) and elapsed repair time lies between *w* and *w+Δ*}.
$P_i(k,t)$	Pr {the system is in failed state due to the failure of the i^{th} subsystemat time t and elapsed repair time lies between *k* and *k+Δ*}, where *i*=V, CS_2, Acid, Pulp(p_1), Degraded Processing plant(p_{25}),Processing plant(p_2), Strike and catastrophic failure and *k=x, y, z, u, w, m, q, r*.
δ_v / δ_{cs} / δ_{Ac}/δ_{p_1}	Failure rate of Vendor/ CS_2 Plant/ Acid Plant/ Pulp Plant.
K	Elapsed repair time, where *k=x,y,z,u,w,m,q,r*.
$\delta_{p_{25}}$/ δ_{p_2}	Failure rate of the 5 machines of the processing Plant/Failure rate of the other (more than 5) machines of the Processing Plant.
δ_{s_1} / δ_{s_2}	Failure rate due to strike (from the operating state S_0/from the degraded state S_7).
$\delta_{c_1}/\delta_{c_2}$	Failure rate of Catastrophic failure (from the operating state S_0/from the degraded state S_7).
$\phi_i(k)$	General repair rate of i^{th} system in the time interval (*k, k+Δ*), where *i*=V, CS_2, Acid, Pulp (p_1), Degraded Processing (p_{25}), Processing plant (p_2), Strike and catastrophic failure and *k=x, y, z, u, w, m, q, r*.
$\phi_{p_{25}}$/ ϕ_{p_2}	General repair rate of 5 machines of Processing Plant (p2)/other (more than 5) machines of Processing Plant.
$P_{25,i}$(k,w,t)	Pr (at time t system is failed due to the failure of the i^{th} subsystem while 5 machines of the Processing Plant are already failed). Elapsed repair time for i^{th} subsystem lies between (*k, k+Δ*) and for Processing Plant it lies between(*w ,w+Δ*),where *i*= V, CS_2, Acid, Pulp(p_1) and *k= x, y, z, u.*
K_1, K_2	Revenue per unit time and service cost per unit time respectively.
$\overline{S}_i(j)$	$\int_0^\infty \phi_i(j)\exp[-sj-\int_0^i \phi_i(j)dj]dj$, for $i=V,CS_2,AC,p_1,S,C$ & $j=x,y,z,u,w,m,q,r$.

Let $u_1=e^y$ and $u_2=\phi_{CS}(y)$ then the expression for joint probability according to Gumbel-Hougaard family of copula is given as

$$\phi_{cs}=\exp\left[y^\theta+[\log\phi_{cs}(y)]^\theta\right]^{1/\theta}.$$

4. FORMULATION OF MATHEMATICAL MODEL

Probabilistic considerations and limiting procedure yield the following integro-differential equations satisfying the model

$$\left[\frac{d}{dt}+\delta_v+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{c_1}+\delta_{s_1}+\delta_{P_{25}}\right]P_0(t)=\int_0^\infty P_v(x,t)\phi_v(x)dx+\int_0^\infty P_{cs}(y,t)\exp\left[y^\theta+(\log\phi_{cs}(y))^\theta\right]^{1/\theta}dy+$$
$$\int_0^\infty P_{Ac}(z,t)\phi_{Ac}(z)dz+\int_0^\infty P_{p_1}(u,t)\phi_{p_1}(u)du+\int_0^\infty P_c(r,t)\phi_c(r)dr+\int_0^\infty P_s(q,t)\phi_s(q)dq+\int_0^\infty P_{25}(w,t)\phi_{P_{25}}(w)dw. \tag{4.1}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_v(x)\right]P_v(x,t)=0 \tag{4.2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_{CS}(y)\right]P_{CS}(y,t)=0 \tag{4.3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_{Ac}(z)\right]P_{Ac}(z,t)=0 \tag{4.4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\phi_{p_1}(u)\right]P_{p_1}(u,t)=0 \tag{4.5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\delta_v+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{p_2}+\delta_{s_2}+\delta_{c_2}+\phi_{p_{25}}(w)\right]P_{25}(w,t)=\int_0^\infty P_{25,v}(x,w,t)\phi_v(x)dx+$$
$$\int_0^\infty P_{25,cs}(y,w,t)\phi_{cs}(y)dy+\int_0^\infty P_{25,Ac}(z,w,t)\phi_{Ac}(z)dz+\int_0^\infty P_{25,p_1}(u,w,t)\phi_{p_1}(u)du \tag{4.6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\phi_v(x)\right]P_{25,v}(x,w,t)=0. \tag{4.7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\phi_{cs}(y)\right]P_{25,cs}(y,w,t)=0 \tag{4.8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\phi_{Ac}(z)\right]P_{25,Ac}(z,w,t)=0. \tag{4.9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial u}+\phi_{p_1}(u)\right]P_{25,p_1}(u,w,t)=0 \tag{4.10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\phi_{p_2}(m)\right]P_2(m,t)=0 \tag{4.11}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial q}+\phi_s(q)\right]P_s(q,t)=0. \tag{4.12}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial r}+\phi_c(r)\right]P_c(r,t)=0 \tag{4.13}$$

Boundary Conditions:

$$P_v(0,t)=\delta_v P_0(t) \tag{4.14}$$

$$P_{cs}(0,t)=\delta_{cs}P_0(t) \tag{4.15}$$

$$P_{Ac}(0,t) = \delta_{Ac} P_0(t) \tag{4.16}$$

$$P_{p_1}(0,t) = \delta_{p_1} P_0(t) \tag{4.17}$$

$$P_s(0,t) = \delta_{s_1} P_0(t) + \delta_{s_2} P_{25}(w,t) \tag{4.18}$$

$$P_c(0,t) = \delta_{c_1} P_0(t) + \delta_{c_2} P_{25}(w,t) \tag{4.19}$$

$$P_{25}(0,t) = \delta_{p_{25}} P_0(t) + \int_0^\infty P_2(m,t)\phi_{p_2}(m)dm \tag{4.20}$$

$$P_{25,v}(0,w,t) = \delta_v P_{25}(w,t). \tag{4.21}$$

$$P_{25,cs}(0,w,t) = \delta_{cs} P_{25}(w,t) \tag{4.22}$$

$$P_{25,Ac}(0,w,t) = \delta_{Ac} P_{25}(w,t) \tag{4.23}$$

$$P_{25,p_1}(0,w,t) = \delta_{p_1} P_{25}(w,t) \tag{4.24}$$

$$P_2(0,t) = \delta_{p_2} P_{25}(t) \tag{4.25}$$

Initial Condition:
$P_0(0) = 1$, otherwise zero. (4.26)

5. SOLUTION OF THE MODEL

Solving equations (4.1) through (4.13) by taking Laplace transform and using initial and boundary conditions, one may obtain following transition state probabilities of the system.

$$\overline{P_0}(s) = \frac{1}{K(s)} \tag{5.1}$$

$$\overline{P_{25}}(s) = \frac{I(s)}{K(s)} \tag{5.2}$$

$$\overline{P_v}(s) = \frac{\delta_v}{K(s)} J_v(s) \tag{5.3}$$

$$\overline{P_{cs}}(s) = \frac{\delta_{cs}}{K(s)} J_{cs}(s) \tag{5.4}$$

$$\overline{P_{Ac}}(s) = \frac{\delta_{Ac}}{K(s)} J_{Ac}(s) \tag{5.5}$$

$$\overline{P_{p_1}}(s) = \frac{\delta_{p_1}}{K(s)} J_{p_1}(s) \tag{5.6}$$

$$P_{25,v}(s) = \frac{\delta_v}{K(s)} J_v(s) \left[\delta_{p_{25}} + \delta_{p_2} \overline{S}_{p_2}(s) I(s)\right] J_{p_{25}}(A) \tag{5.7}$$

$$P_{25,cs}(s) = \frac{\delta_{cs}}{K(s)} J_{cs}(s)\left[\delta_{p_{25}} + \delta_{p_2}\overline{S}_{p_2}(s)I(s)\right]J_{p_{25}}(A) \tag{5.8}$$

$$P_{25,Ac} = \frac{\delta_{Ac}}{K(s)} J_{Ac}(s)\left[\delta_{p_{25}} + \delta_{p_2}\overline{S}_{p_2}(s)I(s)\right]J_{p_{25}}(A) \tag{5.9}$$

$$P_{25,p_1}(s) = \frac{\delta_{p_1}}{K(s)} J_{p_1}(s)\left[\delta_{p_{25}} + \delta_{p_2} S_{p_2}(s)I(s)\right]J_{p_{25}}(A) \tag{5.10}$$

$$\overline{P}_2(s) = \delta_{p_2}\frac{I(s)}{K(s)} J_{p_2}(s) \tag{5.11}$$

$$\overline{P}_S(s) = \left[\delta_{S_1} + \delta_{S_2} I(s)\right]\frac{J_S(s)}{K(s)} \tag{5.12}$$

$$\overline{P}_C(s) = \left[\delta_{C_1} + \delta_{C_2} I(s)\right]\frac{J_C(s)}{K(s)} \tag{5.13}$$

where,

$$\begin{aligned}K(s) &= s + \delta_v + \delta_{cs} + \delta_{Ac} + \delta_{p_1} + \delta_{C_1} + \delta_{S_1} + \delta_{p_{25}} - \delta_v\overline{S}_v(s) - \delta_{cs}\overline{S}_{cs}(s) - \delta_{Ac}\overline{S}_{Ac}(s) - \\ &\quad \delta_{p_1}\overline{S}_{p_1}(s) - \left[\delta_{C_1} + \delta_{C_2} I(s)\right]\overline{S}_C(s) - \left[\delta_{S_1} + \delta_{S_2} I(s)\right]\overline{S}_s(s) - \left[\delta_{p_{25}} + \delta_{p_2}\overline{S}_{p_2}(s)I(s)\right]\overline{S}_{p_{25}}(A)\end{aligned} \tag{5.14}$$

$$I(s) = \frac{\delta_{p_{25}} J_{p_{25}}(A)}{1 - \delta_{p_2}\overline{S}_{p_2}(s) J_{p_{25}}(A)} \tag{5.15}$$

$$J_{p_{25}}(A) = \frac{1 - \overline{S}_{p_{25}}(A)}{A} \tag{5.16}$$

$$J_i(s) = \frac{1 - \overline{S_v(s)}}{s}, \text{for } i = v, cs, Ac, p_1, S, C \tag{5.17}$$

$$\overline{S}_i(j) = \int_0^\infty \phi_i(j)\exp[-s_j - \int_0^i \phi_i(j)dj]dj, \ \ for\ i = v, cs, Ac, p_1, S, C \ \&\ j = x, y, z, u, w, m, q, r \tag{5.18}$$

$$\phi_{cs} = \exp\left[y^\theta + \left[\log\phi_{cs}(y)\right]^\theta\right]^{1/\theta} \tag{5.19}$$

Verification:

$$\overline{P}_{up}(s) + \overline{P}_{down}(s) = \frac{1}{s} \tag{5.20}$$

6. STEADYSTATE BEHAVIOR OF THE SYSTEM

Using Abel's lemma in Laplace transforms, viz;

$$\lim_{s\to 0} s\bar{f}(s) = \lim_{t\to\infty} f(t) = f(\text{say}) \tag{6.1}$$

provided the limit on the right hand side exists, the time independent operational probabilities are obtained as follows.

$$P_0 = \frac{1}{K^{'}(0)} \tag{6.2}$$

$$P_v = \frac{\delta_v}{K^{'}(0)} T_v \tag{6.3}$$

$$P_{cs} = \frac{\delta_{cs}}{K^{'}(0)} T_{cs} \tag{6.4}$$

$$P_{Ac} = \frac{\delta_{Ac}}{K^{'}(0)} T_{Ac} \tag{6.5}$$

$$P_{p_1} = \frac{\delta_{p_1}}{K^{'}(0)} T_{p_1} \tag{6.6}$$

$$P_{25,v} = \frac{\delta_v}{K^{'}(0)} T_v \left[\delta_{p_{25}} + \delta_{p_2} I(0)\right] J_{p_{25}} (\delta_{S_2} + \delta_{C_2}) \tag{6.7}$$

$$P_{25,cs} = \frac{\delta_{cs}}{K^{'}(0)} T_{cs} \left[\delta_{p_{25}} + \delta_{p_2} I(0)\right] J_{p_{25}} (\delta_{S_2} + \delta_{C_2}) \tag{6.8}$$

$$P_{25,Ac} = \frac{\delta_{Ac}}{K^{'}(0)} T_{Ac} \left[\delta_{p_{25}} + \delta_{p_2} I(0)\right] J_{p_{25}} (\delta_{S_2} + \delta_{C_2}) \tag{6.9}$$

$$P_{25,p_1} = \frac{\delta_{p_1}}{K^{'}(0)} T_{p_1} \left[\delta_{p_{25}} + \delta_{p_2} I(0)\right] J_{p_{25}} (\delta_{S_2} + \delta_{C_2}) \tag{6.10}$$

$$P_2 = \delta_{p_2} \frac{I(0)}{K^{'}(0)} T_{p_2} \tag{6.11}$$

$$P_S = \left[\delta_{S_1} + \delta_{S_2} I(0)\right] \frac{T_S}{K^{'}(0)} \tag{6.12}$$

$$P_C = \left[\delta_{C_1} + \delta_{C_2} I(0)\right] \frac{T_C}{K^{'}(0)} \tag{6.13}$$

$$P_{25} = \frac{I(0)}{K^{'}(0)} \tag{6.14}$$

where,

$$K^{'}(0) = \left[\frac{d}{ds} K(s)\right]_{s=0} \tag{6.15}$$

$T_i = -\overline{S}_i(0)$ =Mean time to repair the i^{th} failure (6.16)

$$I(0)=\frac{\delta_{p_{25}}J_{p_{25}}(\delta_{S_2}+\delta_{C_2})}{1-\delta_{p_2}J_{p_{25}}(\delta_{S_2}+\delta_{C_2})} \tag{6.17}$$

7. PARTICULAR CASES

When repairs follow exponential distribution. Let $S\phi_i(j)=\frac{\phi_i}{j+\phi_i}$, for all *i* and *j* in equations (5.1) through (5.13), one may obtain the following transition state probabilities,

$$\overline{P}_0(s)=\frac{1}{R(s)} \tag{7.1}$$

$$\overline{P}_p(s)=\frac{\delta_v}{R(s)}\frac{1}{[s+\phi_v]} \tag{7.2}$$

$$\overline{P}_{cs}(s)=\frac{\delta_{cs}}{R(s)}\frac{1}{[s+\phi_{cs}]} \tag{7.3}$$

$$P_{Ac}=\frac{\delta_{Ac}}{R(s)}\frac{1}{[s+\phi_{Ac}]} \tag{7.4}$$

$$\overline{P}_{p_1}(s)=\frac{\delta_{p_1}}{R(s)}\frac{1}{[s+\phi_{p_1}]} \tag{7.5}$$

$$\overline{P}_{25,v}(s)=\frac{\delta_v}{R(s)}\frac{1}{[s+\phi_v]}\left[\delta_{p_{25}}+\delta_{p_2}D(s)\frac{\phi_{p_2}}{s+\phi_{p_2}}\right]\frac{1}{A+\phi_{p_{25}}} \tag{7.6}$$

$$\overline{P_{25,}cs}(s)=\frac{\delta_{cs}}{R(s)}\frac{1}{[s+\phi_{cs}]}\left[\delta_{p_{25}}+\delta_{p_2}D(s)\frac{\phi_{p_2}}{s+\phi_{p_2}}\right]\frac{1}{A+\phi_{p_{25}}} \tag{7.7}$$

$$\overline{P_{25,}}_{Ac}(s)=\frac{\delta_{Ac}}{R(s)}\frac{1}{[s+\phi_{Ac}]}\left[\delta_{p_{25}}+\delta_{p_2}D(s)\frac{\phi_{p_2}}{s+\phi_{p_2}}\right]\frac{1}{A+\phi_{p_{25}}} \tag{7.8}$$

$$\overline{P_{25,p_1}}(s)=\frac{\delta_{p_1}}{R(s)}\frac{1}{[s+\phi_{p_1}]}\left[\delta_{p_{25}}+\delta_{p_2}D(s)\frac{\phi_{p_2}}{s+\phi_{p_2}}\right]\frac{1}{A+\phi_{p_{25}}} \tag{7.9}$$

$$\overline{P_2}(s)=\frac{\delta_{p_2}}{R(s)}\frac{D(s)}{[s+\phi_{p_2}]} \tag{7.10}$$

$$\overline{P}_S(s)=\frac{1}{R(s)}\frac{[\delta_{S_1}+\delta_{S_2}D(s)]}{[s+\phi_S]} \tag{7.11}$$

$$\overline{P}_C(s) = \frac{1}{R(s)} \frac{[\delta_{C_1} + \delta_{C_2} D(s)]}{[s + \phi_C]} \tag{7.12}$$

$$\overline{P_{25}}(s) = \frac{D(s)}{R(s)} \tag{7.13}$$

where,

$$K(s) = s + \delta_v + \delta_{cs} + \delta_{Ac} + \delta_{p_1} + \delta_{C_1} + \delta_{S_1} + \delta_{p_{25}} - \delta_v \overline{S}_v(s) - \delta_{CS}\overline{S}_{CS}(s) - \delta_{Ac}\overline{S}_{Ac}(s) - \delta_{p_1}\overline{S}_{p_1}(s) -$$
$$[\delta_{C_1} + \delta_{C_2} I(s)]\overline{S}_C(s) - [\delta_{S_1} + \delta_{S_2} I(s)]\overline{S}_S(s) - [\delta_{p_{25}} + \delta_{p_2}\overline{S}_{p_2}(s) I(s)]\overline{S}_{p_{25}}(A)$$

$$D(s) = \frac{\delta_{p_{25}}}{A + \phi_{p_{25}} - \dfrac{\delta_{p_2}\phi_{p_2}}{s + \phi_{p_2}}} \tag{7.14}$$

$$\phi_{cs} = \exp\left[y^\theta + [\log \phi_{cs}(y)]^\theta\right]^{1/\theta}$$

7.1 NON REPAIRABLE SYSTEM

If the system is non repairable then the probabilities will be independent of x and repair rates are zero then the reliability function is obtained as mentioned below.

The Laplace transform of the reliability when all repair rates of the system are zero, then from equation (5.1), we have

$$\overline{P_0}(s) = \frac{1}{K(s)}$$

When all repair rates of the system are zero.

$$\overline{\mathrm{R}}(\mathrm{s}) = \frac{1}{s + \delta_v + \delta_{cs} + \delta_{Ac} + \delta_{p_1} + \delta_{C_1} + \delta_{S_1} + \delta_{p_{25}}}$$

where, R(s) is the Laplace transform of the reliability function. The reliability of the transit system is obtained as:

$$R(t) = e^{\{-(\delta_v + \delta_{CS} + \delta_{AC} + \delta_{p_1} + \delta_{C_1} + \delta_{S_1} + \delta_{p_{25}})t\}} \tag{7.15}$$

7.2 AVAILABILITY OF THE SYSTEM

$$\overline{P}_{up}(s) = \overline{P}_0(s) + \overline{P}_{25}(s) = \frac{1}{K(s)}[1 + I(s)]$$

Taking inverse Laplace transforms, we have

$$P_{up}(t) = e^{\left\{-(\delta_V+\delta_{cs}+\delta_{AC}+\delta_{p_1}+\delta_{C_1}+\delta_{S_1}+\delta_{p_{25}})^*t\right\}} - \frac{\delta_{p25}}{(\delta_{C_1}-\delta_{C_2})+(\delta_{S_1}-\delta_{S_2})+\delta_{p_{25}}} e^{\left\{-(\delta_V+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{C_1}+\delta_{S_1}+\delta_{p_{25}})^*t\right\}}$$

$$-\frac{\delta_{p25}}{(\delta_{C_1}-\delta_{C_2})+(\delta_{S_1}-\delta_{S_2})+\delta_{p_{25}}} e^{\left\{-(\delta_V+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{C_2}+\delta_{S_2})^*t\right\}} \quad (7.16)$$

7.3 MTTF OF THE SYSTEM

The mean time to failure (MTTF) is given as under

$$\text{MTTF} = \lim_{s \to 0} \bar{P}\text{up}(s) = \int_0^\infty R(t)dt = \frac{1}{\delta_v + \delta_{cs} + \delta_{Ac} + \delta_{p_1} + \delta_{C_1} + \delta_{S_1} + \delta_{p_{25}}} \quad (7.17)$$

7.4 COST EFFECTIVENESS OF THE SYSTEM

Cost function for considered system is given by

$$G(t) = K_1 \cdot \int_0^t P_{up}(t)dt - K_2 t$$

where, K_1 and K_2 are revenue and repair costs per unit time, respectively.
Also

$$\int_0^t P_{up}(t)dt = \int_0^t e^{(-(\delta_v+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{c_1}+\delta_{s_1}+\delta_{p_{25}})t)} + \frac{\delta_{p_{25}} e^{(-(\delta_v+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{c_1}+\delta_{s_1}+\delta_{p_{25}})t)}}{[\delta_{c_1} - \delta_{c_2} + \delta_{S_1} - \delta_{S_2} + \delta_{p_{25}}]} - \frac{\delta_{p_{25}} e^{(-(\delta_v+\delta_{cs}+\delta_{Ac}+\delta_{p_1}+\delta_{c2}+\delta_{s_2}+\delta_{p_{25}})t)}}{[\delta_{c_1} - \delta_{c_2} + \delta_{S_1} - \delta_{S_2} + \delta_{p_{25}}]} \quad (7.18)$$

7.5 SENSITIVITY ANALYSIS FOR R (t)

First we perform sensitivity analysis for changes in the R (t) resulting from changes in system parameters δ_V and δ_{S_1} **.** Differentiating equation (7.15) with respect to ψ_V , we obtain

$$\frac{\partial R(t)}{\partial \delta_V} = -te^{(-(\delta_V+\delta_{CS}+\delta_{Ac}+\delta_{C1}+\delta_{S1}+\delta_{p_{25}}+\delta_{p_1})t} \quad (7.19)$$

Using the same procedure we can get $\dfrac{\partial R(t)}{\partial \delta_{S_1}}$

8. NUMERICAL COMPUTATION

For a more concrete study of the system's behavior, we calculate the values of reliability, availability and cost function of the system with respect to time and keeping the other parameter fixed and MTTF of the system for different failure rates.

8.1 RELIABILITY ANALYSIS

Consider $\delta_v = .005, \delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .004$ in equation (7.15) and by putting different values of t such as 0, 1, 2, 3, 4,…, one can obtain the output as shown in Figure-2.

8.2 AVAILABILITY analysis

Setting, $\delta_v = .005, \delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .004, \delta_{S_2} = .02, \delta_{C_2} = .03$ in equations (7.16). Availability of the system is obtained as given in Figures-3.

8.3 MTTF ANALYSIS

8.3.1 MTTF for different values of Vendor failures

Setting, $\delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .04$ in equation (7.17) and put δv =.001,.002,.003. One can get Figure-4 which exhibits the variation of MTTF for different values of Vendor failure.

8.3.2 MTTF for different values of CS_2 Plant Failures

Consider, $\delta_v = .005, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .04$ in equation (7.17) and put δ_{CS}=.001, .002,.003. One can obtain the MTTF for different values of CS_2 Plant failure as shown in Figure-4.

8.3.3 MTTF for different values of Workers Strike

Putting, $\delta_{cs} = 0.007, \delta_{Ac} = 0.006, \delta_{C1} = 0.03, \delta_{p1} = 0.007, \delta_{V} = 0.005, \delta_{p25} = 0.04$, $\delta_{S_1} = 0.01, 0.02, 0.03, 0.04, .$ in equation (7.17),we getFigure-5 that gives us the changes of MTTF for various values of workers strike.

8.3.4 MTTF for different values of catastrophic Failures

Assuming, $\delta_v = .005, \delta_{CS} = .007, \delta_{Ac} = .006, \delta_{s_1} = .02, \delta_{p_1} = .007, \delta_{p_{25}} = .04$ in equation (7.17) and take δ_{C_1} = .01, .02, .03, .04,…We have Figure-5 which shows the variation of MTTF for a range of values of catastrophic failure.

8.4 COST analysis

Let, $\delta_v = .005, \delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .004, \delta_{S_2} = .02, \delta_{C_2} = .03$, K_1=*3*, K_2=7 in equations (7.18). Cost of the system is obtained as given in Figures-6.

8.5 SENSITIVITY analysis

8.5.1 For Vendor failure rate

Putting, $\delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{p_1} = .007, \delta_{S_1} = .02, \delta_{p_{25}} = .004$ and δ_v =.001, .005 and .01in equation (7.19) one can obtain the figure-7 which shows the sensitivity of the system reliability with respect to Vendor failure rate.

8.5.2 For Strike rate

Putting, $\delta_v = .005, \delta_{CS} = .007, \delta_{Ac} = .006, \delta_{C_1} = .03, \delta_{P_1} = .007, \delta_{P_{25}} = .004$ and δ_{S_1} =.01, .05 and .1in equation (7.19) one can obtain the figure-8 which shows the sensitivity of the system reliability with respect to Strike rate.

9. RESULTS AND DISCUSSION

Figure 9.1 shows the trends of reliability of the system with respect to time when all the failure rates and all the repair rates have some fixed values. From the graph we conclude that the reliability of the system decreases with passage of time when all failures follows exponential time distribution.

Analysis of the Figure 9.2 gives us the idea of the availability of the system with respect to time t. Critical examination of Figure 9.2 yields that the values of the availability decreases approximately in a steady manner with increase in time.

A critical examination of Figure 9.3 shows that MTTF decreases smoothly with increase in Vendor failure rate and CS_2 Plant failure rate. Figure 9.4 shows thatas failure rates increase, MTTF decreases with respect to workers strike rate and catastrophic failure rate. Figure 9.5 represents the graph of the "Cost function vs. time. Fixing the revenue cost K_1 per unit time at value of RS. 20 and varying service cost K_2 as 10, 15, and 20. The graphs reveal an important conclusion that increasing service cost leads decrement in expected profit.

The sensitivities of the system reliability R (t) with respect to system parameters δ_V and δ_{S_1} are shown in figures 9.6 and 9.7. It can easily be observed that the biggest impact almost happened at the same time for all the system parameters. Moreover, we find that δ_{S_1} are most prominent parameters and almost have the equal sensitive effect on the system reliability. δ_V is the second in magnitude.

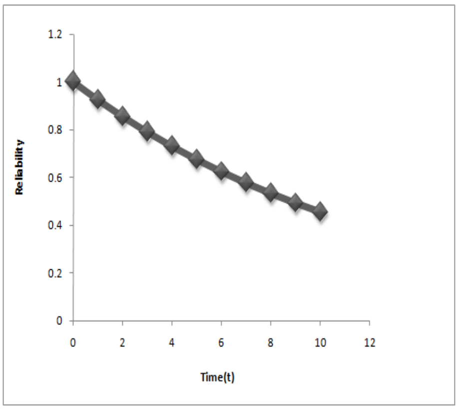


Figure 9.1 Reliability Vs Time

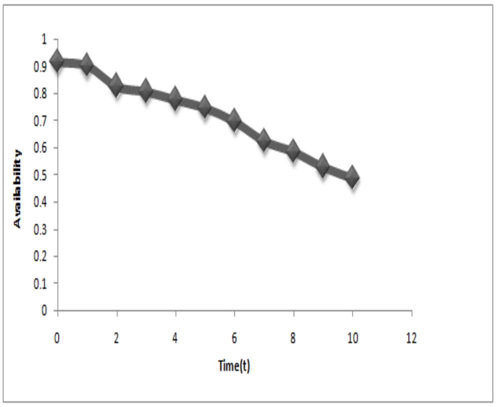


Figure9.2 Availability Vs Time

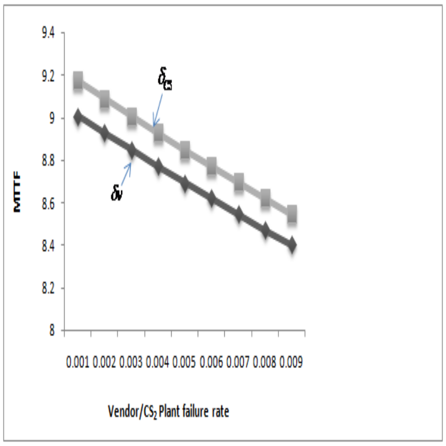


Figure 9.3: MTTF Vs Vendor /CS_2 plant failure rate

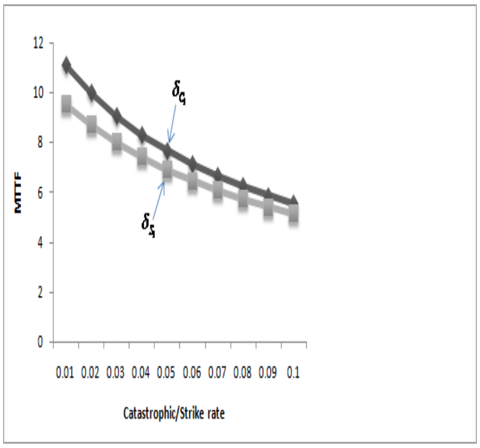


Figure 9.4 MTTF Vs Catastrophic Failure/Strike rate

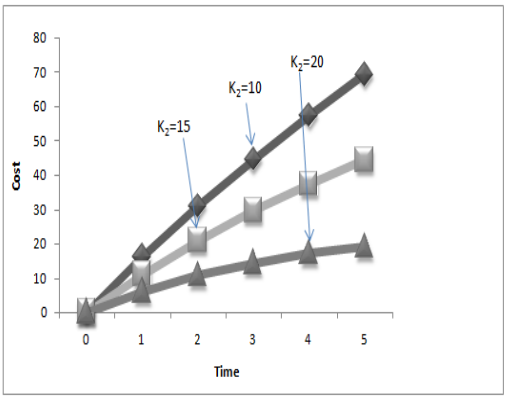
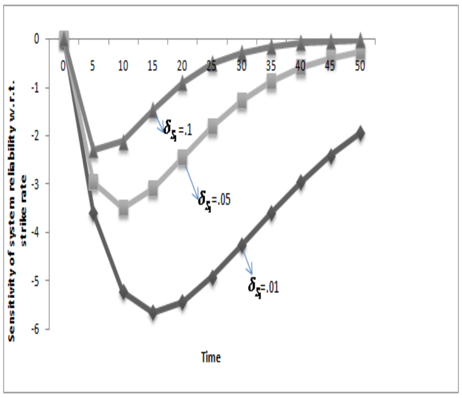


Figure9.5 Cost Vs Time



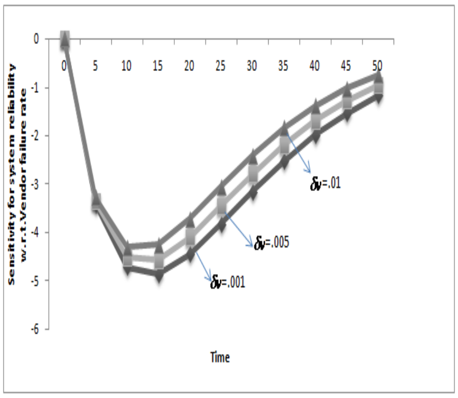


Figure 9.6 Sensitivity of system reliability . r. t. δ_V

Figure 9.7 Sensitivity of system reliability w. r. t. δ_{S_1}

10. CONCLUSION

In this paper, the operational readiness of Viscose Staple Fibre system is discussed using mathematical modeling approach. Also the comparative study of the reliability with time, MTTF analysis with Vendor, CS_2 Plant, workers strike, catastrophic and Acid Plant failure rate and variation of costs with respect to time is presented. The proposed method has the advantages of modeling and analyzing system reliability in a more flexible and more intelligent manner.

This analysis may help managerial staff in the following ways.

a. Managing resources, Vendors.
b. Taking decisions timely.
c. Planning preventive maintenance policies.
d. Planning strategies of production.

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