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Economic Growth and Renewable Resource: Specialization of Clean Activities

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〈Abstract〉

This paper starts with a model of monopolistic competition and endogenous growth, and it adds pollution as an input to production. Then I adopt environmental quality as a renewable resource used in production. I show that increasing returns due to specialization of clean activities as inputs can help lead to sustainable growth with no harm to environmental quality. I also compare and evaluate alternative policy combinations (i.e. taxes + subsidies) that correct two distortions from pollution and monopolistic competition. Finally, I find that, if the productivity of environment in final good production is not sufficiently enough, the number of clean goods tends to increase with more environmental concerns.

Keywords : economic growth, monopolistic competition, pollution, renewable resource, market price system, optimal environmental policies, clean good expansion

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본 논문은 독점적 경쟁과 내생적 성장의 모형으로 시작하여 생산과정에 투입되는 생산요소로서 오염을 추가했다. 그리고 환경의 질이 생산에 사용되는 재생가능한 자원인 것으로 채택했다. 본 논문은 생산요소로서 청정 활동의 분화에 기인하는 수확체증이 환경의 질을 해치지 않고 지속가능한 성장을 이끄는 데 도움을 줄 수 있다는 것을 보였다. 또한 오염과 독점적 경쟁으로부터 발생되는 두 가지 왜곡들을 교정할 수 있는 서로 다른 여러 가지 정책조합(조세 + 보조금)을 비교하고 평가했다. 끝으로 본 논문은 최종재 생산에 있어 환경의 생산성이 충분치 않을 경우에 청정재의 수는 더 많은 환경적 우려와 함께 증가하는 경향이 있다는 것을 찾아냈다.

주제어 : 경제성장, 독점적 경쟁, 오염, 재생가능자원, 시장가격체계,
최적 환경정책, 청정재 확대

JEL Classification : H23, O13, O44, Q01, Q20, Q55

Ⅰ. Introduction

The Montreal Protocol in 1987 banned use of chemical products that damage the ozone layer, and the Kyoto protocol in 1997 regulated the emissions of greenhouse gases that cause global warming. In light of these environment-protecting measures, a policy maker may face a trade-off between economic growth and environmental preservation. Firms argue that environmental policy has negative effects on economic growth, because abatement activities increase production costs. However, environmental policy is needed for our society to ensure a better level of environmental quality which may help growth ultimately. Consequently, this problem arrives at a question. Which could play the key role for cleaner environment as well as better growth performance — government-oriented central planning or market-oriented pricing system?

Over the past decade, a number of papers have studied government-driven economic growth and environmental preservation in a central planning, using a model where technological changes are endogenous (see Bovenberg and Smulders, 1995; Greiner, 2005; Fullerton and S.-R. Kim, 2008).[1] Many of the

1) Bovenberg and Smulders(1995) use a two-sector endogenous growth model with pollution-augmenting technology. They find that the revenues from pollution taxes exceed the public expenditures on government R&D on an optimal growth path. Using the model of Bovenberg and Smulders(1995), Fullerton and Kim(2008) allow for distortionary taxes that finance public expenditures on abatement knowledge, and

previous papers have given much attention to the conditions under which endogenous growth in physical output is sustainable and compatible with a stable level of environmental quality.

Taking a different point of view, this paper explores market-driven economic growth and environmental protection in a pricing system, and it gives policy implications. In order to do so, I employs an endogenous growth model in which increasing returns are due to specialization of intermediate inputs (see Romer, 1987; Aghion and Howitt, 1998; Barro and Sala-i-Martin, 2003; Grimaud and Tournemaine, 2003; Morales, 2004; Sorger, 2006).[2] Then, adding pollution as an input to and environment as a positive externality for production, I derive and examine socially optimal growth path and market-induced growth path with price incentives (i.e. taxes and subsidies). Meanwhile, there exists some papers on growth and environment that follows the formulation of Romer(1987), but they do not study what the next two paragraphs present (see Grimaud, 1999; Grimaud and Ricci, 2004; Hart, 2004; Ricci, 2007).[3]

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they give growth and welfare implications. Greiner(2005) uses an endogenous growth model with public capital and pollution, and he analyzes the effects of fiscal policy on growth and welfare.

2) The presented papers do not study pollution, but the models of this kind would imply that pollution tax can spur specialization on intermediate goods, which environmental quality does not deteriorate. The specialization leads to increasing returns for economic growth. Consequently, the models of this kind can provide a tractable framework for the study of economic growth and environmental preservation in a market-oriented pricing system.

3) The papers presented above are relevant to this paper in that they use the formulation of Romer(1987), but each of them addresses its question in different objectives. Grimaud(1999) implements the optimal path obtained by Aghion and Howitt(1998) in

Therefore, for the market-induced growth path to be optimized socially, this paper compares and analyzes alternative policy combinations that can be taken in the market-oriented pricing system — (1) tax on pollution + subsidy to production of final goods, (2) tax on pollution + subsidy to purchase of clean inputs, and (3) tax on pollution + subsidy to invention of clean inputs. An environmental tax could generate 'double dividend', since it becomes not only a corrective instrument for internalizing negative environmental externality itself, but also a revenue-raising instrument for alleviating another distortion by revenue-recycling effect (see Bovenberg and de Mooij, 1994; Goulder, 1995; Oates, 1995; Fullerton and Metcalf, 2001; Metcalf, 2003). For this reason, the policy combinations that this paper considers could correct two distortions from pollution and monopolistic competition, and thus, they could improve economic growth and environmental quality. This paper evaluates the alternative policy combinations in practical use and easiness of policy application also.

Furthermore, this paper predicts how the growth performance and the production structure change in response to a more

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a decentralized economy. Grimaud and Ricci(2004) analyze economic growth and environment along two paradigms of variety expansion and quality improvements. Hart(2004) develops an innovative model in which researchers striving for monopoly profits drive growth, and he examines welfare implications and various distortions such as environmental externalities, monopoly power, business stealing and knowledge spillovers. Ricci(2007) explores the effect of environmental tax on the distribution of market shares when intermediate inputs are differentiated in pollution intensity.

ambitious environmental measure. In recent years, as more people have environmental concerns, governments are inclinable to set more stringent environmental policies. Thus, it could be worth to analyze economic changes according to environmental improvement.

The main results of this paper are as follows. In this dynamic model, increasing returns due to specialization of clean activities as inputs can help lead to sustainable economic growth with no harm to environmental quality. Fixed pollution use preserves natural environment at a constant level, and also, an expansion of a number of clean inputs overcomes an increase in relative scarcity of pollution in production of final goods at each point in time. In addition, for the market-induced growth path to be optimized socially, the policy combination of 'tax on pollution + subsidy to purchase of clean inputs' is better among all the alternative policy combinations in second-best as well as first-best. Thus, it could be useful in practice, even though it may be less easily applied than the policy combination of 'tax on pollution + subsidy to production of final goods.' Finally, if the productivity of environment has relatively more positive effect on final good production, then growth performance gets better with more environmental concern. Otherwise, it gets worse. In this case, the number of clean goods tends to increase. Less productivity improvement by an increase in environment quality pressures the invention of new clean input, and thus, the number of clean goods expands. According to the hypothesis of Porter(1991), additional constraints could trigger technological adjustments that could expand production possibilities when they are imposed on firms by environmental policy.

This paper is in the following order. In next section, I present the model and explain specialization of clean goods that generates economic growth. I solve for Pareto optimality in section III, and in turn, I derive a market equilibrium with two distortions from pollution and monopolistic competition in section IV. Section V compares and analyzes alternative policy combinations. Section VI explores economic changes in response to more stringent environmental measure. I conclude and summarize in section VII.

II. Model

1. Production

1) Final Output

Romer(1987) uses a production function for final goods with inputs of labor and differentiated intermediate goods. To adapt his model of the production of final goods Y_t at each time t, this paper uses pollution P_t as input instead of labor. Thus, intermediate goods $x_t(i)$ for each $i \in [1, N_t]$ in this version of the Romer model are clean inputs. Another addition to Romer's model is that the natural environment E_t affects production as discussed more below.4) Thus,

$$Y_t = Y(E_t, P_t, x_t) = A(E_t) P_t^{1-\alpha} \int_0^{N_t} x_t(i)^{\alpha} di \tag{1}$$

where $0 < \alpha < 1$. For the effect of natural environment on the production function, I assume that $A'(E_t) > 0$ and $A''(E_t) < 0$. Assume that $x_t(i)$ represents purchases of nondurable input i. We measure all prices in units of the homogeneous flow of goods, Y_t.

Technological progress takes the form of expansion in N_t, the number of specialized non-polluting goods available. To see the effect from an increase in N_t, suppose that the clean inputs can be measured in common physical units and that all are employed in the same quantity, $x_t(i) = x_t$ (which turns out to be true in equilibrium). The quantity of output from

4) The correlation between final goods (Y) and total productivity of natural environment ($A(E)$) is important for this analysis. Many theoretical papers use the formulation similar to this paper (see Bovenberg and Smulders, 1995, 1996; Michel, 1993; Rosendahl, 1996; Rubio and Aznar, 2000; Smulders, 1995; Smulders and Gradus, 1996). In addition, many empirical papers shows the evidence that natural environment positively affects the productivity of input and, so, output (see Alfsen, Brendemoen and Glomsrod, 1992; Ballard and Medema, 1993; Brendemoen and Vennemo, 1994; van Ewijk and van Wijnbergen, 1995; Margulis, 1992; Pearce and Warford, 1993). For examples, soil and air quality has effect on productivity in the agricultural sector, and air quality has effect on physical depreciation of equipment and productivity of labor as well as mental health.

(1) is then given by

$$Y_t = A(E_t)P_t^{1-\alpha}N_t x_t^{\alpha} = A(E_t)P_t^{1-\alpha}(N_t x_t)^{\alpha}N_t^{1-\alpha} \qquad (2)$$

Production exhibits constant returns to scale in P_t and $N_t x_t$, the total quantity of clean inputs. For given quantities P_t and $N_t x_t$, however, the term $N_t^{1-\alpha}$ indicates that Y_t increases with N_t over time. This effects, which captures a form of technology progress, reflects the benet from spreading a given total of intermediates, $N_t x_t$, over a wider range, N_t. The benefit arises because of the diminishing returns to each $x_t(i)$ individually.

An expansion of clean inputs, $N_t x_t(i)$, encounters diminishing returns if it occurs through an increase in $x_t(i)$ for given N_t. Diminishing returns do not arise, however, if the increase in $N_t x_t(i)$ takes the form of a rise in N_t for given $x_t(i)$. Thus, technological change in the form of continuing increases in N_t avoids the tendency for diminishing returns. This property of the production function provides the basis for endogenous growth.

The technical advance in an economy with this form of clean good expansion can be reinterpreted as the pollution -augmenting technology in Bovenberg and Smulders(1995). Reformulate the production function (2):

$$Y_t = A(E_t)(N_t P_t)^{1-\alpha}(N_t x_t)^{\alpha} \tag{3}$$

The term $N_t P_t$ can be interpreted as effective pollution and the number of clean inputs N_t can be interpreted as the pollution-augmenting techniques available in this economy at each point in time. However, in this context, the technique can be created by R & D in private sector, not by government expenditure as in Bovenberg and Smulders(1995).

2) Clean Inputs

Technology produces N_t varieties of clean inputs at each time t. A technological advance can increase the number of clean inputs N_t at each time, in the sense of an invention that allows a new kind of clean good to be produced. The cost to create a new type of clean input is fixed at η units of Y_t and does not change over time. The inventor of clean input i retains a perpetual monopoly right over the production and sale of the good $x_t(i)$. But, with the assumption of free entry into producing clean inputs, any inventor can pay the cost η (which is covered by the net present value of profits). Once the fixed cost η is paid to invent it, good i costs one unit of Y_t to produce at each time. In the case of the final good price being normalized to 1, the marginal cost to produce the clean good is normalized to 1 too. This is because it gives simplicity without loss of generality. In other words, either normalizing the marginal cost to 1 or setting it to be any positive constant c leads to the same results in this paper.

2. Renewable Resource

The natural environment is modeled as a renewable resource. The quality of the natural environment E_t is a public good that accumulates due to the regenerative capacity of nature, while it depreciates on account of pollution P_t. As in Bovenberg and Smulders(1995), I use the formulation of Tahvonen and Kuuluvainen(1991) where E_t evolves over time according to

$$\dot{E_t} = F(E_t, P_t) \tag{4}$$

which satisfies the condition:

$$F_E < 0, F_{EE} < 0, F_{PP} < 0, F_{EP} > 0, F(\overline{E}(P_t), P_t) = 0 \tag{5}$$

At each P_t exists a stable level of environmental quality $\overline{E_t}$. Nature E_t will reach a lower level if pollution is used at a higher level. An equilibrium of environmental quality $\overline{E_t}$ is stable, since it is assumed that $P_E(E_t, P_t) < 0$ around $\overline{E}(P_t)$. If environmental quality is high, nature can easily absorb pollution ($F_{EP} > 0$). The maximal steady-state quality of the environment is assumed to be finite ($\overline{E}^{\max} = \overline{E}(0) < \infty$).

3. Preferences

The household's preference is given by

$$\int_0^{\infty} U(c_t, E_t) \cdot e^{-\delta t} dt \tag{6}$$

where δ is the rate of time preference. Assume that the instantaneous utility function is strictly increasing and concave in c_t and satisfies the Inada condition:

$$U_c > 0,\ U_{cc} < 0,\ \lim_{c_t \to 0} U_c = \infty,\ \lim_{c_t \to \infty} U_c = 0 \tag{7}$$

III. Pareto Optimality

1. Optimal Allocation

The social planner uses $x_t(i) = x_t$ for all i to produce clean inputs in the efficient way, facing the economy's budget constraint

$$\eta \dot{N}_t = Y(E_t, P_t, x_t) - N_t x_t - c_t \tag{8}$$

where $\eta \dot{N}_t$ is the cost of inventing new clean inputs for next period, and $Y_t - N_t x_t$ is the net product of the economy at time t.[5] To maximize the utility of a representative household subject to the economy's budget constraint (8) and the ecological relationship (4), the social planner chooses time paths for the control variables (consumption c_t, clean inputs x_t, and pollution P_t) and the state variables (the number of clean inputs N_t and quality of natural environment E_t). The problem of the planner is, therefore, to[6]

$$
\begin{aligned}
&\max \int_0^\infty U(c_t, E_t) \cdot e^{-\delta t} dt \\
&s.t.\ \dot{N}_t = \frac{1}{\eta}\left(A(E_t) P_t^{1-\alpha} N_t x_t^\alpha - N_t x_t - c_t\right) \\
&\qquad \dot{E}_t = F(E_t, P_t) \\
&\qquad c_t \geq 0,\ N_0 = \overline{N_0}
\end{aligned}
$$

We can ignore the condition $c_t \geq 0$ by the Inada condition. Then we get optimal allocation rules in this economy as in Lemma 1.

5) Since this model preserves symmetry, each input is produced at the same amount in optimum.

6) The objective function of social planner depends only on consumption and environmental quality, i.e., consumers' surplus, since this dynamic model does not reflect producers' surplus. In this model, both of producers of final good and producers of clean goods have the normal profit of 0 in the long term. Producers of final good in perfect competition have the normal profit of 0 in the short term as well as the long term. Although producers of clean goods in monopolistic competition get a positive monopolistic profit in the short term, they have the normal profit of 0 in the long term.

Lemma 1 (Optimal Allocation Rules)

(1) *Optimal static allocation:*

$$\frac{\partial Y}{\partial x_t} = A(E_t)\alpha P_t^{1-\alpha} N_t x_t^{\alpha-1} = 1 \tag{9}$$

$$\frac{\partial Y}{\partial P_t} = A(E_t)(1-\alpha)P_t^{-\alpha} N_t x_t^{\alpha} = -\eta \cdot q_t F_P \tag{10}$$

(2) *Optimal dynamic allocation:*

$$r_s = \frac{1}{\eta}\left(A(E_t)P_t^{1-\alpha}x_t^{\alpha} - x_t\right) = \frac{1}{\eta q_t}\left(\frac{U_E}{U_c} + \frac{\partial Y}{\partial E_t} + F_{E+}\frac{\dot{q}_t}{q_t}\right) \tag{11}$$

$$r_s = \delta - \frac{\dot{U}_c}{U_c} \tag{12}$$

where q_t *is the shadow price of* E_t *relative to* N_t*.*

Proof. See Appendix A.1. ■

Lemma 1 states optimal allocation rules from the view of social planner with two corrections for monopolistic competition and pollution externalities. Eq. (9) implies that efficient use of clean inputs is set at the point where marginal product is equal to marginal cost. In order to correct the externality from pollution in Eq. (10), the social planner equates the marginal product of pollution to the R & D cost (η) multiplied by the marginal environmental damage ($q_t F_P$) which is the deterioration of the environmental quality. Especially, the economy has two distortions from monopolistic competition and pollution. So, if the economy were to eliminate one unit of pollution, then for optimal growth it should compensate one unit of new good. Therefore, the cost of one unit of pollution used should be the multiplication of one unit of R & D cost and marginal environmental damage. Eq. (11) is the arbitrage condition for N_t and E_t in which both of physical capital and environmental quality has the same interest rate r_t. The return on physical capital is equal to marginal product. The return on environmental quality consists of the sum of the marginal benefits of environmental quality in household's utility (U_E/U_c), production ($\partial Y/\partial E_t = A'(E_t)P_t^{1-\alpha}N_t x_t^{\alpha}$), ecological regeneration processes (P_E), and environmental capital gain ($\dot{q}_t/q_t$). Eq. (12) represents the Ramsey rule for optimal saving. The interest rate rewards foregone consumption, compensating for the rate of time preference and the change in the marginal value of consumption over time.

From the Ramsey rule (12), the behavior of the marginal utility of consumption at each time t is

$$r_s - \delta = -\frac{\dot{U}_c}{U_c} = -\frac{C_t U_{cc}}{U_c} \cdot \frac{\dot{C}_t}{C_t} - \frac{E_t U_{cE}}{U_c} \cdot \frac{\dot{E}_t}{E_t}$$

The environmental quality cannot grow forever at a constant positive rate (since it cannot exceed the virgin state) and, thus, the optimal amount of pollution is constant over time. This implies that $\dot{E}_t = \dot{P}_t = 0$ for all t, and the term $\left(\frac{E_t U_{cE}}{U_c} \cdot \frac{\dot{E}_t}{E_t}\right)$ is optimally zero. Also the social interest rate r_s is constant over time on a balanced growth path. Hence, for the balanced growth path to be optimal, the term $\left(-\frac{C_t U_{cc}}{U_c}\right)$ must be constant over time. Denote this term as θ. Then the intertemporal substitution elasticity $\left(-\frac{U_c}{C_t U_{cc}}\right)$ is equal to $1/\theta$. This requires that households' utility has a form of time-separable constant-relative-risk-aversion (CRRA). Proposition 1 states optimal balanced growth path in this economy.

Proposition 1 *(Optimal Balanced Growth Path)*

Let x_s, r_s and g_s be social optimal levels of a use of each clean input, an interest rate, and a growth rate. Suppose that the utility is a time-separable constant-relative-risk-aversion function of the form $c_t^{1-\theta} n(E_t)/(1-\theta)$. Then a unique optimal balanced growth path is identified by the following conditions:

$$x_s = A(E)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} P \tag{13}$$

$$r_s = \frac{1}{\eta} A(E)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \alpha^{\frac{1}{1-\alpha}} P \tag{14}$$

$$\dot{E}_t = \dot{P}_t = \dot{x}_s = \dot{r}_s = 0 \text{ and } \frac{\dot{N}_t}{N_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{q}_t}{q_t} = \frac{r_s - \delta}{\theta} \equiv g_s \tag{15}$$

Proof. See Appendix A.2. ■

Eqs. (13), (14) and (15) say that, in the socially optimal balanced growth path, the fixed use of pollution (P) preserves the natural environment (E) at a constant level, and consequently, the use of each clean input (x_s) and the social rate of return (r_s) are constant. Since the relative scarcity of pollution increases over time, the number of clean inputs as well as the shadow price (q_t) of natural environment grow at the same constant rate (g_s). This result implies that an expansion of a number of clean inputs overcomes an increase in relative scarcity of pollution in production of final goods at each point in time. To produce final goods, the level of pollution is held constant, but the amount of clean inputs ($N_t x_s$) increases over time instead. In this dynamic economy, the horizontal differentiation in clean inputs makes it possible for positive economic growth to be constant without harming environmental quality.

Ⅳ. Market Equilibriums

1. The Final Outputs Firms

Producers are competitive and therefore take the price $p_t(i)$ of the input i as given. Government levies a tax τ_t on use of pollution, and it gives a subsidy to purchase of clean inputs at rate σ_x, a subsidy to production of final goods at rate σ_Y and a subsidy to cost of R & D at rate σ_R. Taking N_t as given, these final output firms purchase inputs $x_t(i)$ from N_t different firms, each of which has a monopoly over the good $x_t(i)$ that it sells. Producers of final outputs maximize profits Π_t by their choice of P_t and $x_t(i)$ for any $t \in [0, \infty)$

$$\Pi_t = (1-\sigma_Y)A(E_t)P_t^{1-\alpha}\int_0^{N_t} x_t(i)^{\alpha} di - \int_0^{N_t}(1-\sigma_x)p_t(i)x_t(i)di - \tau_t P_t$$

in which the first term is the revenue from selling final goods, the second term is the cost for purchasing clean inputs, and the third term is the cost for using pollution at each time t.[7)][8)]

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7) Since the model is dynamic in this paper, the producers of final outputs should maximize the long-term profit $\int_0^{\infty} \Pi_t e^{-\int_0^t r_v dv} dt$ to choose $\{P_t, x_t(i)\}_{t=0}^{\infty}$. However,

Thus, the first-order conditions are

$$(1-\sigma_Y)A(E_t)\alpha P_t^{1-\alpha}x_t(i)^{\alpha-1}-(1-\sigma_x)p_t(i)=0$$

$$(1-\sigma_Y)A(E_t)(1-\alpha)P_t^{1-\alpha}\int_0^{N_t}x_t(i)^{\alpha}di-\tau_t=0$$

which yield

$$\frac{\partial Y}{\partial x_t(i)}=A(E_t)\alpha P_t^{1-\alpha}x_t(i)^{\alpha-1}=\frac{(1-\sigma_x)}{(1-\sigma_Y)}p_t(i) \tag{16}$$

$$\frac{\partial Y}{\partial P_t}=A(E_t)(1-\alpha)P_t^{1-\alpha}\int_0^{N_t}x_t(i)^{\alpha}di=\frac{\tau_t}{(1-\sigma_Y)} \tag{17}$$

for any $t\in[0,\infty)$.

Eqs. (16) and (17) state decentralized allocations of clean goods and pollution used at time t in the market economy. Without government subsidy and correction on two distortions from monopolistic competition and pollution use $(\tau_t=\sigma_x=\sigma_Y=0)$,

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this maximization of long-term profit becomes the above maximization of short-term profit by the choice P_t and $x_t(i)$ for all $t\in[0,\infty)$. In other words, either the maximization of long-term profit or the maximization of short-term profit leads to Eqs. (16) and (17) as the first-order conditions. Thus, this paper presents the maximization of short-term profit to avoid the complexity of more equations.

8) Note that the sale price of final good after subsidy is not $(1+\sigma_Y)$ but $(1-\sigma_Y)$ in the revenue function. A subsidy to production of final goods lowers the sale price of 1. In contrast, a tax raises the sale price of 1, which is in the case of $(1+\sigma_Y)$.

marginal product of each clean input is equal to its monopoly price, and marginal product of pollution is equal to zero. No government intervention on pollution means that final good producers overuse the amount of pollution, which, in turn, implies that the environmental quality deteriorates. For the decentralized allocations of clean inputs and pollution to be socially optimal, the government can sets a pollution tax and perhaps use other instruments. Our next task is to find the optimal levels of all policy instruments.

2. The Monopolists of Clean Inputs

In production of final goods, pollution can be restricted by a government tax. Then the final goods producers have incentives to employ more clean inputs and less pollution. This induces the producers of clean goods to undertake R & D intended to create more different types of clean inputs at each of time t. An example of the invention of a new type of clean input is a new scrubber, when clean inputs are interpreted as the different abatement technologies as expressed in Eq. (3). The present value of the returns from discovering the i-th clean good at time t is

$$V_t(i) = \int_t^{\infty} (p_t(i) - 1)x_t(i) \cdot e^{-\int_t^v r_w dw} dv \tag{18}$$

where $p_t(i)x_t(i)$ is the revenue from selling the clean good, $1 \cdot x_t(i)$ is the cost for producing it, and r_t is the interest rate at each time t. The fixed cost η for discovering a new good can be recovered only if $p_t(i)$ exceeds the marginal cost to produce the good, 1, for at least part of time after date t. The inverse demand for clean input i can be derived from Eq. (16) as

$$p_t(i) = A(E_t)\left(\frac{(1-\sigma_Y)\alpha}{1-\sigma_x}\right)P_t^{1-\alpha}x_t(i)^{\alpha-1} \tag{19}$$

Thus, each monopolist i sets the quantity of the good $x_t(i)$ at each date to maximize $V_t(i)$ in Eq. (18). The present-value Hamiltonian is

$$H = \left(A(E_t)\left(\frac{(1-\sigma_Y)\alpha}{1-\sigma_x}\right)P_t^{1-\alpha}x_t(i)^{\alpha-1} - 1\right)x_t(i) \cdot e^{-\int_t^v r_w dw}$$

Since no state variable is on the production side, and no intertemporal element is in demand function, the first-order condition is

$$\frac{\partial H}{\partial x_t(i)} = A(E_t)\left(\frac{(1-\sigma_Y)\alpha^2}{1-\sigma_x}\right)P_t^{1-\alpha}x_t(i)^{\alpha-1} - 1 = \alpha p_t(i) - 1 = 0$$

which yields

$$p_t(i) = \frac{1}{\alpha} \equiv p > 0 \tag{20}$$

Since α is a constant, the equilibrium price of each clean input i is the same for all i and constant over time. This monopoly price, $1/\alpha$, is greater than the marginal cost of production, 1. Thus the markup, $(1-\alpha)/\alpha$, covers the cost to create the new good. Substitute (20) into (19) to get

$$x_t(i) = A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{(1-\sigma_Y)\alpha^2}{1-\sigma_x} \right)^{\frac{1}{1-\alpha}} P_t \equiv x \tag{21}$$

As shown in Proposition 1, on the optimal balanced growth path, the amount of pollution P_t and the quality of nature E_t must be constant over time. I assume for the moment that the levels of pollution and environmental quality is constant over time, i.e. $P_t = P$ and $E_t = E$ for all t.[9] Hence each clean input $x_t(i)$ is also constant over time and the same for all i. Over time, the only thing that changes is the variety of clean inputs N_t. In the case where the amount of pollution is constant over time, the final goods firms have incentive to use more clean inputs over time. Substitute Eq. (21) into Eq. (18) to show that the monopolistic inventor's net present value of profits at time t is

9) That is actually an equilibrium, after the government installs the optimal pollution tax.

$$V_t(i) = (p-1)x\int_t^{\infty} e^{-\int_t^v r_w dw} dv \tag{22}$$

Assume that government gives subsidy to R & D at a rate σ_R. The free entry condition is $V_t(i) = \eta(1-\sigma_R)$ for all t. If the present value of profits is greater than the cost of inventing it, then infinite resources would flow into invention of a new clean input, which cannot hold in equilibrium. If the present value of profits is less than the cost of inventing it, then no resource would flow into invention of the good, so the number of clean inputs would be constant for all time. I focus on equilibria with increasing variety of clean inputs over time. In this case, the present value of profits equals the cost of inventing the clean good. Thus, Eq. (22) implies

$$\int_t^{\infty} e^{-\int_t^v r_w dw} dv = \frac{\eta(1-\sigma_R)}{(p-1)x}$$

The right side of the equation above is constant at each time t, which implies the interest rate must be constant at each time t (i.e. $r_t = r$ for all t). Since $\int_t^{\infty} e^{-r\cdot(v-t)} dv = \frac{1}{r}$, the interest rate is determined as

$$r = \frac{1}{\eta} A(E)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}}\right)^{\frac{1}{1-\alpha}} P \tag{23}$$

In Eq. (23), the market interest rate r becomes less than the social interest rate r_s without any subsidy to final good and clean good productions because each producer of a clean good provides his good at a monopoly price greater than the competitive price. So, for the dynamic allocations in this market economy to be socially optimal, the market interest rate need to be equal to the social one, because the optimal growth rate depends on the economy's interest rate. Thus the government could subsidize the use and invention of clean goods, indirectly supporting the optimal saving of households' assets. The market interest rate will be compared to the social for correcting the externalities optimally, and designing the government polices in the section 5.

3. Households

I assume many identical households. Each household is endowed with an amount of the initial stock of assets, $a_0 = \overline{a_0}$, at the initial time $t = 0$. Then the households' problem is to

$$\max_{(c_t,\, a_t)} \int_0^{\infty} U(c_t, E_t) \cdot e^{-\delta t} dt$$

$$s.t.\ \dot{a}_t = r a_t - c_t$$

$$c_t \geq 0$$

$$a_0 = \overline{a_0},\ \lim_{t \to \infty} a_t \cdot e^{-\int_0^t r_v dv} \geq 0$$

Assume that the credit market imposes the following constraint $\lim_{t\to\infty} a_t \cdot e^{-\int_0^t r_v dv \geq 0}$. We can ignore the condition $c_t \geq 0$ by the Inada condition. Then we get the equilibrium in this decentralized economy.

Proposition 2 (*Market Equilibrium*)

Let x, r and g be levels of each clean input, an interest rate, and a growth rate in the decentralized economy. Suppose that utility function is a time-separable constant-relative-risk-aversion function of the form $c_t^{1-\theta} n(E_t)/(1-\theta)$. Then a balanced growth path in the market economy is identified by the following conditions:

$$x = A(E)^{\frac{1}{1-\alpha}} \left(\frac{(1-\sigma_Y)\alpha^2}{1-\sigma_x} \right)^{\frac{1}{1-\alpha}} P \tag{24}$$

$$r = \frac{1}{\eta} A(E)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} P \tag{25}$$

$$\dot{E}_t = \dot{P}_t = 0 \text{ and } \frac{\dot{a}_t}{a_t} = \frac{\dot{N}_t}{N_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{q}_t}{q_t} = \frac{\dot{\tau}_t}{\tau_t} = \frac{r-\delta}{\theta} \equiv g \tag{26}$$

Proof. See Appendix A.3. ■

In Eqs. (24) and (25), the use of each clean inputs (x) and the market rate of return (r) become constant, if pollution is used at a constant level, and thus, natural environment is at a stable level. In Eq. (26), the positive pollution tax (τ_t) should grow at a rate (g) to hold the use of pollution constant, otherwise this decentralized economy cannot avoid environmental disasters.

Compared to the socially optimal growth path, the allocations of clean inputs and the market rate of return (and, in turn, growth rate) depend on the subsidy rates σ_Y, σ_x and σ_R in the equilibrium growth path of decentralized economy. Suppose that the government does not give all of these subsidies. Then, the allocations of clean inputs and the growth rate get lower in market equilibrium than in the social planner's solution, because monopolistic competition generates another form of distortion in addition to environmental externality. Therefore, if government gives either a subsidy to purchase of clean inputs, production of final goods or invention of new clean inputs with a revenue collected by pollution tax, the amount of each clean input used and the growth rate could increases and be achieved at the socially optimal level as well as the environmental externality can be corrected.

V. Government Corrections

The decentralized market economy suffers from the dual distortions of monopolistic competition and pollution. First, environmental quality has a public-good character, so that every agent in the economy uses and deteriorates it. If the government does not regulate use of pollution, then the final goods producers use too much pollution to avoid the diminishing returns in other inputs. The government should set the pollution tax, restrict pollution, or establish a market for tradable pollution permits.

Second, monopolistic competition is the other distortion in this economy. Each monopolist sets the price of a clean input higher than its marginal cost to cover the cost of R & D. To say it again, the clean inputs that are helpful for environmental quality are provided at higher prices than competitive prices due to monopoly mark-ups. This yields the final output at an inefficiently lower level in the market economy. To increase the output, the government can subsidize use of clean inputs or directly subsidize final goods, however. Also, the government subsidy to R & D could increase the output by making incentives to create new kinds of clean inputs.

Thus proposition 3 compares alternative policy combinations that correct the dual distortions of pollution and monopolistic competition.

Proposition 3 *(Alternative Government Corrections)*

(1) If government taxes pollution at the rate $\tau_t = -\eta \cdot q_t F_P$ *and subsidizes purchase of clean inputs at the rate* $\sigma_x = 1 - \alpha$*, then the market equilibrium is optimal, and*

the government's revenue exactly equals its spending. i.e.

$$x = x_s,\ r = r_s,\ g = g_s,\ \text{and}\ \int_0^{N_t} \sigma_x p_t(i) x_t(i) di = \tau_t P_t$$

(2) *If the government tax rate on pollution is* $\tau_t = -\eta \cdot q_t F_P$ *and the rate of subsidy to production of final goods is* $\sigma_Y = \frac{1-\alpha}{\alpha}$, *then the market equilibrium is optimal, but the government's revenue is less than its spending. i.e.*

$$x = x_s,\ r = r_s,\ g = g_s,\ \text{and}\ \frac{\sigma_Y Y_t}{\tau_t P_t} = \frac{1}{\alpha} > 1$$

(3) *If the government tax rate on pollution is* $\tau_t = -\eta q_t F_P$ *and the rate of subsidy to cost of invention is* $\sigma_R = 1 - \alpha^{\frac{1}{1-\alpha}}$, *then the market equilibrium is not optimal, but the government's revenue is more than its spending. i.e.*

$$x < x_s,\ r = r_s,\ g = g_s,\ \text{and}\ \frac{\sigma_R \eta \dot{N}_t}{\tau_t P_t} = \left(1 - \alpha^{\frac{1}{1-\alpha}}\right)\frac{g_s}{r_s} < 1$$

Proof. See Appendix A.4. ■

From Eq. (10), in order to correct the pollution externality, the government impose such a pollution tax that it equates the marginal product of pollution to the product of the R & D cost and the marginal environmental damage which is the deterioration of the environmental quality. In addition to pollution externality, the economy has another externality from monopolistic competition. Pollution and clean inputs as two different types of production factors are related each other in final good production technology. So, if the government restricts one unit of pollution used, then it should create one more unit of new good for final good production and optimal growth. So, the property of pollution tax is that the tax must include the invention cost of one new type of clean good. Therefore, the cost of one unit of pollution used should be the times of one unit of R & D cost and marginal environmental damage.

In the case of tax on pollution and subsidy to purchase of clean inputs (Proposition 3, (1)), government needs no additional, possibly distortionary, taxes to finance government spending. The revenue from the pollution tax exactly covers the spending to correct the inefficiency from monopoly. This correction makes the economy socially optimal because the levels of use of clean inputs, interest rate and, in turns, growth rate equal to the social planner's levels of them in this decentralized economy. In the case of tax on pollution and subsidy to production of final goods (Proposition 3, (2)), the economy can achieve optimal growth because the market economy have the socially efficient levels of use of clean goods and interest rate. But, for the optimal growth to be guaranteed, government may use additional, possibly distortionary, taxes to finance spending for inefficiency from monopoly. If government places a tax on pollution and subsidy to R & D cost (Proposition 3, (3)), environment tax can be used enough to finance government spending. But, the economy may not be Pareto optimal, because static allocation is inefficient, i.e., the level of clean goods used in final good production is less than socially optimal level. Subsidy to the cost to invention of new types of clean goods alters the inefficient interest rate and balanced growth path while it cannot change static allocation of clean inputs. But, there may be some more improvement in the use of clean inputs if the remaining revenue from environment tax supports on the subsidy to purchase of clean inputs by the output production technology.

All of three policy combinations — (1) tax on pollution + subsidy to purchase of clean inputs, (2) tax on pollution + subsidy to production of final goods, (3) tax on pollution + subsidy to R & D cost — have similarity of policy in the double dividend hypothesis of environmental tax. In other words, tax on pollution as corrective instrument corrects the negative environmental externality from pollution, and it as revenue-raising instrument has revenue-recycling effect that alleviates a different type of distortion from monopolistic competition.

The three policy combinations become different, however, if practical limitations are considered. First of all, government faces second-best circumstance in which it should make certain expenditures from certain revenues in reality. If this budget constraint of government is considered, (1) tax on pollution + subsidy to purchase of clean inputs becomes the most efficient policy combination. This is because the revenue from pollution tax and the expenditure on subsidy to clean good purchase match exactly, and market equilibrium as well as growth are optimal. Compared to this combination, (2) tax on pollution + subsidy to production of final goods is inefficient because of more spending than tax revenue, although it achieves optimal market equilibrium and growth. In addition, (3) tax on pollution + subsidy to R & D cost could be inefficient due to not optimal market equilibrium, even if spending on subsidy is less than tax revenue. Therefore, another policy will be added to this combination to raise efficiency.

Meanwhile, (2) tax on pollution + subsidy to production of final goods has the simplest way in easiness of policy application. This is because the government that spends on subsidy identifies final goods more easily than intermediate goods, and it needs not do the evaluations on clean good inventions one by one.

VI. Environmental Policy

In this section, I study steady-state equilibrium. Suppose government sets more ambitious environmental policy in response to a change in preferences towards more environmental concern. This preference shock leads to a reduction in pollution, which, in turn, changes other endogenous variables in the economy. A cut in the use of pollution affects real returns and growth, as well as the expansion of clean input.

First, pollution abatement affects the production of final good in two different ways. Less use of pollution directly reduces the productivity of clean inputs. But, it can indirectly alter the productivity of clean inputs because less pollution improves the environmental quality E in $A(E)$. Set Eq. (4) equal to zero, and log-linearize to get $F_E E\hat{E} + F_P P\hat{P} = 0$, which implies $\hat{E} = -\left(\frac{F_P P}{F_E E}\right)\hat{P}$. Log-linearizing $A(E)$ and use this equation. Then, we have

$$\hat{A}(E) = -\lambda\left(\frac{F_P P}{F_E E}\right)\hat{P} \qquad (27)$$

where a hat denotes a relative change $(\hat{A} \equiv dA/A)$ and $\lambda \equiv \frac{dA(E)}{dE} \cdot \frac{E}{A(E)} = \frac{\hat{A}}{\hat{E}}$ is the positive elasticity of $A(E)$ with respect to E. In Eq. (27), the term $-\lambda\left(\frac{F_P P}{F_E E}\right)$ is negative, since F_P and F_E is negative (in turn, the term $\left(\frac{F_P P}{F_E E}\right)$ is positive). Thus, the productivity of natural environment ($A(E)$) increases as more environmental concern decreases the use of pollution (P).

The policy shock also changes the amount of each clean input and the number of clean inputs devoted to production of final outputs. In particular, to explore the long-run impact on expansion of clean inputs, I calculate the ratio of the specialization to output production, $\frac{N}{Y}$. The following proposition says the long-run impact of pollution reduction.

Proposition 4 (Long-run Equilibrium Impact)
With a change in preferences that favors the environment E,

(1) the effect on the optimal use of each clean input x_s, the real return r_s, and the growth rate g_s are given by

$$\widehat{x_s} = \widehat{r_s} = -\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P}$$

$$\widehat{g_s} = \left(1 + \frac{\delta}{\theta g_s}\right)\widehat{r_s} = -\left(1 + \frac{\delta}{\theta g_s}\right)\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P}$$

(2) the effect on the specialization of clean inputs N/Y is

given by

$$\hat{N}-\hat{Y}=\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right)-1\right)\hat{P}$$

Proof. See Appendix A.5. ■

Proposition 4 implies that, in response to more ambitious environmental measure, growth performance and specialization in clean goods can vary because the production of final goods depends on the total productivity of natural environment. Suppose the output elasticity of natural environment (λ) is relatively large, but the output elasticity of pollution ($1-\alpha$) is relatively small (i.e. $\frac{\lambda}{1-\alpha}>\frac{F_E E}{F_P P}$).[10] In other words, the productivity of natural environment has relatively more positive effect on production of final goods. Then, a reduction in pollution increases the use of each clean input, the real return and the growth rate. The number of clean inputs relative to final good decreases, however. This is because, even if the economy imposes more ambitious environmental measure, more productivity improvement by an increase in natural environment can relieve the pressure on invention of

10) The output elasticity of natural environment is equal to the elasticity of $A(E)$ with respect to E (i.e. $\frac{dY}{dE}\cdot\frac{E}{Y}=\left(\frac{dY}{dA}\cdot\frac{A}{Y}\right)\cdot\left(\frac{dA}{dE}\cdot\frac{E}{A}\right)=\lambda$), since $\frac{dY}{dA}\cdot\frac{A}{Y}=1$ and $\frac{dA}{dE}\cdot\frac{E}{A}=\lambda$.

clean inputs for economic growth with environmental preservation.

In contrast, suppose the output elasticity of natural environment (λ) is relatively small or zero, but the output elasticity of pollution ($1-\alpha$) is relatively large (i.e. $\frac{\lambda}{1-\alpha} < \frac{F_E E}{F_P P}$). In other words, the productivity of natural environment has relatively less positive effect or no effect on production of final goods. Then, a cut in use of pollution decreases the use of each clean input, the real return and the growth rate. On the contrary to this, the number of clean inputs relative to final good increases. Corresponding to more environmental concern, less productivity improvement by an increase in natural environment pressures the invention of new clean input, and thus, the number of clean goods expands. The tighter environmental measure creates new kinds of inputs and changes the production structure of final outputs.

VII. Conclusion

This paper solves the trade-off between economic growth and environmental preservation in a market-oriented pricing system. The specialization of clean activities as inputs (abatement technologies) to production makes it possible for positive economic growth to be constant with no harm to environmental quality. This paper also compare and evaluate alternative policy combinations in price incentives (i.e. taxes + subsidies) that correct two distortions from pollution and monopolistic competition. The policy combination of 'tax on pollution + subsidy to purchase of clean inputs' seem to be better when all the policy combinations are evaluated on practical use and easiness of policy application. Finally, this paper finds that, if the productivity of environment in final good production is sufficiently enough, the growth performance gets better with more environmental concerns. Otherwise, the growth performance gets worse, and thus, the number of clean goods tends to increase.

Furthermore, the model of this paper could be extended as follows. In this model, the clean inputs are assumed to be monopolized with perpetual monopoly rights over production and sale. The assumption of perpetual monopoly right should be relaxed, however, if competitors who can learn about new clean inputs imitate them or create close substitutes. In this way, the clean inputs are used even more than before, since they lose monopoly's mark-up and become more competitive. Consequently, the more use of clean inputs could lead to better economic growth with the same level of environmental quality as before.

In addition, the paper assumes that production of clean goods does not depend on pollution and productivity of environment like production of final goods, and only clean inputs can be specialized. In general, pollution can be used in and productivity of environment can affect the production of clean inputs, however. Dirty inputs can be specialized and the number of them increases over time also. Therefore, if they are considered in the model of this paper, these features could provide some other implications on economic growth and environmental preservation. All of these things put limitations on and give future works to this paper.

⟨Appendix⟩

A.1. Proof of Lemma 1

Proof. Write the current-value Hamiltonian:

$$H(c_t, x_t, P_t, N_t, E_t)$$
$$= U(c_t, E_t) + \mu_{1t}\frac{1}{\eta}\left(A(E_t)P_t^{1-\alpha}N_t x_t^{\alpha} - N_t x_t - c_t\right) + \mu_{2t}F(E_t, P_t)$$

Then, we have the first order conditions for optimal infinite time paths:

$$\frac{\partial H}{\partial c_t} = U_c - \mu_{1t}\frac{1}{\eta} = 0 \Rightarrow \mu_{1t} = \eta U_c \tag{28}$$

$$\frac{\partial H}{\partial x_t} = \mu_{1t}\frac{1}{\eta}\left(A(E_t)\alpha P_t^{1-\alpha}N_t x_t^{\alpha-1} - N_t\right) = 0$$
$$\Rightarrow \frac{\partial Y_t}{\partial x_t} = A(E_t)\alpha P_t^{1-\alpha}x_t^{\alpha-1} = 1 \tag{29}$$

$$\frac{\partial H}{\partial P_t} = \mu_{1t}\frac{1}{\eta}A(E_t)(1-\alpha)P_t^{-\alpha}N_t x_t^{\alpha} + \mu_{2t}F_P = 0$$
$$\Rightarrow \frac{\partial Y_t}{\partial P_t} = A(E_t)(1-\alpha)P_t^{-\alpha}x_t^{\alpha}N_t = -\eta\frac{\mu_{2t}}{\mu_{1t}}F_P \tag{30}$$

$$\frac{\partial H}{\partial N_t}=\mu_{1t}\frac{1}{\eta}\left(A(E_t)P_t^{1-\alpha}x_t^{\alpha}-x_t\right)=\delta\mu_{1t}-\dot{\mu}_{1t} \tag{31}$$

$$\frac{\partial H}{\partial E_t}=U_E+\mu_{1t}\frac{1}{\eta}A'(E_t)P_t^{1-\alpha}N_t x_t^{\alpha}+\mu_{2t}F_E=\delta\mu_{2t}-\dot{\mu}_{2t} \tag{32}$$

Eq. (29) yields

$$x_t=A(E_t)^{\frac{1}{1-\alpha}}\alpha^{\frac{1}{1-\alpha}}P_t\equiv x_s$$

where x_s is the amount of intermediate good that the social planner chooses. From Eq. (30), we have the equation standing for correction on the pollution externality

$$\frac{\partial Y_t}{\partial P_t}=A(E_t)(1-\alpha)P_t^{-\alpha}x_t^{\alpha}N_t=-\eta q_t F_P$$

where $q_t\equiv\frac{\mu_{2t}}{\mu_{1t}}$ is the shadow price of E_t relative to that of N_t. Eq. (28) and (31) give the Ramsey rule of optimal saving

$$r_s=\delta-\frac{\dot{U}_c}{U_c} \tag{33}$$

where $r_s=\frac{1}{\eta}\left(A(E_t)P_t^{1-\alpha}x_t^{\alpha}-x_t\right)=\frac{1}{\eta}A(E_t)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\alpha^{\frac{1}{1-\alpha}}P_t$ is the implicit interest rate that the social planner chooses. Eq. (32) and the Ramsey rule (33) leads to the return on the environmental quality which yields

$$r_s = \frac{1}{\eta q_t}\left(\frac{U_E}{U_c} + A'(E_t)P_t^{1-\alpha}N_t x_t^{\alpha}\right) + F_E + \frac{\dot{q}_t}{q_t}$$

Then the arbitrage condition for N_t and E_t is derived as

$$\begin{aligned} r_s &= \frac{1}{\eta}\left(A(N)Z^{1-\alpha}x^{\alpha} - x\right) \\ &= \frac{1}{\eta q_t}\left(\frac{U_E}{U_c} + \frac{\partial Y}{\partial E_t}\left[= A'(E_t)P_t^{1-\alpha}N_t x_t^{\alpha}\right]\right) + F_E + \frac{\dot{q}_t}{q_t} \end{aligned}$$

■

A.2 Proof of Proposition 1

Proof. A proof of equilibrium shows that it is not possible for other equilibria to exist.[11] Note that E_t cannot grow forever at a constant positive rate since it cannot exceed the virgin state. Thus $\dot{E}_t = \dot{P}_t = 0$. Also, for growth to be balanced, $\frac{U_{cc}c}{U_c} \equiv -\theta$ has to be constant over time. Hence,

11) See chapter 4 in Barro and Sala-i-Martin(1998) for details.

$$r_s = \delta - \frac{U_{cc}c_t}{U_c}\frac{\dot{c}_t}{c_t} - \frac{U_{cE}E_t}{U_c}\frac{\dot{E}_t}{E_t} = \delta + \theta \cdot \frac{\dot{c}_t}{c_t}$$

Hence we have the optimal growth rate

$$\frac{\dot{c}_t}{c_t} = \frac{r_s - \delta}{\theta} \equiv g_s \tag{34}$$

where g_s is the growth rate that the social planner can get. This model doesn't exhibit transitional dynamics. So, the growth rate of the number of clean inputs is determined by that of consumption. Integrate Eq. (34) to get the consumption path

$$c_t = c_0 \cdot e^{(r_s - \delta)t/\theta} \tag{35}$$

Substitute Eq. (35) into the economy budget constraint

$$\dot{N}_t = \frac{1}{\eta}\left(A(E_t)P_t^{1-\alpha}N_t x_t^{\alpha} - N_t x_t - c_t\right)$$
$$= r_s N_t - \frac{1}{\eta}c_t = r_s N_t - \frac{1}{\eta}c_0 \cdot e^{(r_s - \delta)t/\theta}$$

or

$$\dot{N}_t - r_s N_t = -\frac{1}{\eta}c_0 \cdot e^{(r_s - \delta)t/\theta}$$

Multiply the above equation by $e^{-r_s t}$ and integrate it. Then, we have

$$\int_0^t e^{-r_s v}(\dot{N}_v - r_s N_v)dv = \int_0^t -\frac{1}{\eta}c_0 \cdot e^{((r_s-\delta)/\theta - r_s)v}dv$$
$$e^{-r_s t}N_t - N_0 = -\frac{1}{\eta}\frac{c_0}{((r_s-\delta)/\theta - r_s)}\left(e^{((r_s-\delta)/\theta - r_s)t} - 1\right)$$

or

$$N_t = \left(N_0 - \frac{1}{\eta}\frac{c_0}{\varphi}\right)e^{r_s t} + \frac{1}{\eta}\frac{c_0}{\varphi}e^{(r_s-\delta)t/\theta} \qquad (36)$$

where $\varphi \equiv r_s - (r_s-\delta)/\theta = r_s - g_s > 0$. By the transversality condition,

$$\lim_{n\to\infty} N_t \cdot e^{-r_s t} = \lim_{n\to\infty}\left(N_0 - \frac{1}{\eta}\frac{c_0}{\varphi}\right) + \frac{1}{\eta}\frac{c_0}{\varphi}e^{-\varphi t} = 0$$

Since $\frac{1}{\eta}\frac{c_0}{\varphi} > 0$, then $\lim_{t\to\infty}\frac{1}{\eta}\frac{c_0}{\varphi}e^{-\varphi t} = 0$. The term $N_0 - \frac{1}{\eta}\frac{c_0}{\varphi}$ must be zero, which implies that the given number N_0 determines the consumption c_0 at time $t=0$. Hence, using Eq. (35) Eq. (36) is expressed as

$$N_t = \frac{1}{\eta}\frac{c_0}{\varphi}e^{(r_s-\delta)t/\theta} = \frac{1}{\eta\varphi}c_t$$

So consumption and the number of clean inputs grow at the same rate g_s. From the production function of final goods (1) and Eq. (13)

$$Y_t = A(E_t)P_t^{1-\alpha}N_t x_t^{\alpha} = A(E_t)^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}P_t N_t$$

Hence, final good and the number of clean inputs grow at the same rate g_s. Taking and differentiating Eq. (10) give

$$\frac{\dot{N}_t}{N_t} = \frac{\dot{q}_t}{q_t}$$

Finally, the optimal growth rate and path are derived as follow:

$$\dot{E}_t = \dot{P}_t = 0 \text{ and } \frac{\dot{N}_t}{N_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{q}_t}{q_t} = \frac{r_s - \delta}{\theta} \equiv g_s$$

■

A.3 Proof of Proposition 2

Proof. The current-value Hamiltonian is

$$H(c_t, a_t) = U(c_t, E_t) + \mu_t(ra_t - c_t)$$

where μ_t is the current-value shadow price of income. The first order conditions of this maximization are

$$\frac{\partial H}{\partial c_t} = U_c - \mu_t = 0 \Rightarrow \mu_t = U_c \tag{37}$$

$$\frac{\partial H}{\partial a_t} = \mu_t r = \delta \mu_t - \dot{\mu}_t \tag{38}$$

Take log and differentiate Eq. (37) with respect to t and use Eq. (38) to get Ramsey rule for household in market equilibrium

$$r = \delta - \frac{\dot{U}_c}{U_c}$$

Integrate Eq. (38) with respect to t, to get

$$\int_0^t \frac{\dot{\mu}_v}{\mu_v} dv = -\int_0^t (r_v - \delta) dv$$

$$\mu_t = \mu_0 \cdot e^{-\int_0^t (r_v - \delta) dv}$$

Then, the transversality condition is

$$\lim_{t \to \infty} e^{-\delta t} \mu_t a_t = \lim_{t \to \infty} \mu_0 e^{-\int_0^t r_v dv} a_t = \lim_{t \to \infty} e^{-\int_0^t r_v dv} a_t = 0$$

since $\mu_0 = U_c(c_0) > 0$. Hence, the credit market condition is satisfied. Ramsey rule says

$$r = \delta - \frac{U_{cc}c}{U_c}\frac{\dot{c}_t}{c_t}$$

or

$$\frac{\dot{c}_t}{c_t} = \frac{r-\delta}{\theta} \equiv g \qquad (39)$$

where g is the growth rate of consumption in market solution. Substitute the interest rate of asset market (23) into (39)

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}\left(\frac{1}{\eta}A(E_t)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}}\right)^{\frac{1}{1-\alpha}}P_t - \delta\right) \qquad (40)$$

To show that the consumption c_t and the number of clean inputs N_t grow at the same rate is similar to that in Proof of Proposition 1. From the production function (1)

$$Y_t = A(E_t)P_t^{1-\alpha}x_t^{\alpha}N_t = A(E_t)^{\frac{1}{1-\alpha}}\left(\frac{(1-\sigma_Y)\alpha^2}{1-\sigma_x}\right)^{\frac{a}{1-\alpha}}P_tN_t \qquad (41)$$

the total output Y_t and the number of clean inputs N_t grow at the same rate. Take log and differentiate Eq. (17) Then, we have

$$\frac{\dot{N}_t}{N_t} = \frac{\dot{\tau}_t}{\tau_t}$$

For fixed pollution P_t, the growth rate of c_t, N_t, Y_t is

$$\frac{\dot{N}_t}{N_t} = \frac{\dot{c}_t}{c_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{\tau}_t}{\tau_t}$$

In this model, households are identical. So, the following procedure are not necessary. But, here I just want to show. Since total households' assets equals the market value of the firms, $\eta\dot{N}$ in closed economy, the households' aggregate income is

$$r\eta N_t = \left(\frac{1-\alpha}{\alpha}\right)x_t N_t = px_t N_t - x_t N_t = Y_t - x_t N_t$$

where $Y_t - x_t N_t$ is the economy's net product. The economy's budget constraint is

$$C_t = r\eta N_t - \eta \dot{N}_t = Y_t - x_t N_t - \eta \dot{N}_t$$

$$= Y_t - x_t N_t - \eta g N_t \quad \left(\text{since } \eta \dot{N}_t = \eta g N_t\right)$$

$$= \left(A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{(1-\sigma_Y)\alpha^2}{1-\sigma_x} \right)^{\frac{a}{1-\alpha}} P_t N_t \right)$$

$$- \left(A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)} \right)^{\frac{1}{1-\alpha}} P_t \right) N_t$$

$$- \frac{1}{\theta} \left(A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} P_t - \eta\delta \right) N_t$$

$$= \frac{1}{\theta} \left(A(E_t)^{\frac{1}{1-\alpha}} \alpha^{2\alpha 1-\alpha} \Phi P_t + \eta\delta \right) N_t$$

where

$$\Phi \equiv \theta \left(\frac{1-\sigma_Y}{1-\sigma_x} \right)^{\frac{a}{1-\alpha}} - \theta\alpha^2 \left(\frac{1-\sigma_Y}{1-\sigma_x} \right)^{\frac{1}{1-\alpha}}$$

$$- (1-\alpha)\alpha \left(\frac{(1-\sigma_Y)}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}}$$

From the transversality condition, $r > g$,

$$r > \frac{r-\delta}{\theta}$$

$$(1-\theta)\frac{1}{\eta} A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha} \right) \left(\frac{(1-\sigma_Y)\alpha^2}{(1-\sigma_x)(1-\sigma_R)^{1-\alpha}} \right)^{\frac{1}{1-\alpha}} P_t < \delta$$

$$A(E_t)^{\frac{1}{1-\alpha}} \alpha^{\frac{2\alpha}{1-\alpha}} \Psi P_t < \eta\delta$$

where $\Psi \equiv (1-\theta)\alpha(1-\alpha)\left((1-\sigma_Y)(1-\sigma_x)(1-\sigma_R)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$. Since $\Phi < \Psi$, then the level of consumption C_t is positive. Thus the aggregate consumption C_t and the number of clean inputs grow at the same rate. ■

A.4 Proof of Proposition 3

Proof. From Proposition 2, first, if government designs the policy of $\sigma_x = 1-\alpha$, $\sigma_Y = 0$, $\sigma_R = 0$ and $\tau_t = -\eta \cdot q_t F_P$, then $x = A(E_t)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} P_t = x_s$, $r = \frac{1}{\eta} A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{1}{1-\alpha}} P_t = r_s$ and, using Eq. (9) and (10), the ratio of its spending to revenue is

$$\frac{\int_o^M \sigma_x p_t(i) x_t(i) di}{\tau_t P_t} = \frac{(1-\alpha)\int_o^{N_t} \frac{1}{\alpha} x_t(i) di}{-\eta \cdot q_t F_P} = \frac{\frac{1-\alpha}{\alpha} N_t x_s}{A(E_t)(1-\alpha) P_t^{-\alpha} N_t x_s^{\alpha} I}$$

$$= \frac{1}{A(E_t)\alpha P_t^{1-\alpha} x_s^{\alpha-1}} = 1$$

Second, if government designs the policy of $\sigma_x = 0$, $\sigma_Y = \frac{1-\alpha}{\alpha}$, $\sigma_r = 0$ and $\tau_t = -\eta \cdot q_t F_P$, then $x = A(E_t)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} P_t = x_s$, $r = \frac{1}{\eta} A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{1}{1-\alpha}} P_t = r_s$ and, using Eq. (10), the ratio of its spending to revenue is

$$\frac{\sigma_Y Y_t}{\tau_t P_t} = \frac{\frac{1-\alpha}{\alpha} Y_t}{-\eta \cdot q_t F_P} = \frac{\frac{1-\alpha}{\alpha} A(E_t) P_t^{1-\alpha} N_t x_s^{\alpha}}{A(E)(1-\alpha) P_t^{-\alpha} N_t x_s^{\alpha} P_t}$$

$$= \frac{\frac{1-\alpha}{\alpha} A(E_t) P_t^{1-\alpha} N_t x_s^{\alpha}}{A(E_t)(1-\alpha) P_t^{1-\alpha} N_t x_s^{\alpha}} = \frac{1}{\alpha} > 1$$

since $0 < \alpha < 1$. Finally, if government designs the policy of $\sigma_x = 0$, $\sigma_Y = 0$, $\sigma_r = 1 - \alpha^{\frac{1}{1-\alpha}}$ and $\tau_t = -\eta \cdot q_t F_P$ then $x = A(E_t)^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} P_t = x_s$, $r = \frac{1}{\eta} A(E_t)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\right) \alpha^{\frac{1}{1-\alpha}} P_t = r_s$ and, using Eq. (9) and (10), the ratio of its spending to revenue is

$$\frac{\sigma_R \eta \dot{N}_t}{\tau_t P_t} = \frac{\left(1-\alpha^{\frac{1}{1-\alpha}}\right) \eta \dot{N}_t}{-\eta \cdot q_t F_P} = \frac{\left(1-\alpha^{\frac{1}{1-\alpha}}\right)(p-1) x_s \frac{1}{r} \dot{N}_t}{A(E_t)(1-\alpha) P_t^{-\alpha} N_t x_s^{\alpha} P_t}$$

$$= \frac{\left(1-\alpha^{\frac{1}{1-\alpha}}\right)\left(\frac{1-\alpha}{\alpha}\right) x_s \frac{1}{r} \dot{N}_t}{A(E_t)(1-\alpha) P_t^{1-\alpha} N_t x_s^{\alpha}} = \frac{\left(1-\alpha^{\frac{1}{1-\alpha}}\right) \frac{1}{r} \dot{M}}{A(N) \alpha Z^{-\alpha} M x_s^{\alpha-1}}$$

$$= \left(1-\alpha^{\frac{1}{1-\alpha}}\right) \frac{1}{r_s} \frac{\dot{N}_t}{N_t} = \left(1-\alpha^{\frac{1}{1-\alpha}}\right) \frac{g_s}{r_s} < 1$$

since $0 < 1 - \alpha^{\frac{1}{1-\alpha}}$ and $r_s > g$. ■

A.5 Proof of Proposition 4

Proof. Log-linearize Eq. (13). Then, by Eq. (27)

$$\widehat{x_s} = \left(\frac{1}{1-\alpha}\right)\hat{A}(E) + \hat{P} = -\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P}$$

Log-linearize Eq. (14). Then

$$\widehat{r_s} = \left(\frac{1}{1-\alpha}\right)\hat{A}(E) + \hat{P} = -\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P} \tag{42}$$

From the steady-state version of the Ramsey rule (12),

$$r_s = \delta + \theta g_s \tag{43}$$

Log-linearizing the equation above to get

$$\widehat{g_s} = \widehat{r_s} + \left(\frac{\delta}{\theta g_s}\right)\widehat{r_s}$$

Substituting Eq. (42) into (43), to see the effect on growth

$$\widehat{g_s} = \left(1 + \frac{\delta}{\theta g_s}\right)\widehat{r_s} = -\left(1 + \frac{\delta}{\theta g_s}\right)\left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P}$$

The ration of the number of clean inputs to nal outputs is

$$\frac{N}{Y} = \frac{N}{A(E)P^{1-\alpha}Nx^{\alpha}} = \frac{1}{A(E)P^{1-\alpha}x^{\alpha}} \tag{44}$$

Log-linearize Eq.(44) to get the expression of the effect on the number of clean inputs

$$\begin{aligned}
\hat{N} - \hat{Y} &= -\hat{A}(E) - (1-\alpha)\hat{P} - \alpha\left(\frac{1}{1-\alpha}\hat{A}(E) + \hat{P}\right) \\
&= -\hat{A}(E) - (1-\alpha)\hat{P} - \left(\frac{\alpha}{1-\alpha}\right)\hat{A}(E) - \alpha\hat{P} \\
&= -\hat{A}(E) - \hat{P} - \left(\frac{\alpha}{1-\alpha}\right)\hat{A}(E) \\
&= -\left(\left(\frac{1}{1-\alpha}\right)\hat{A}(E) + \hat{P}\right) = \left(\left(\frac{\lambda}{1-\alpha}\right)\left(\frac{F_P P}{F_E E}\right) - 1\right)\hat{P}
\end{aligned}$$

■

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