ARCHITECTURAL RESEARCH, Vol. 14, No. 1(March 2012). pp. 27-33 ISSN 1229-6163

Substructuring and Decoupling of Discrete Systems from Continuous System

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http://dx.doi.org/10.5659/AIKAR.2012.14.1.27

Abstract This study proposes analytical methods to establish the eigenfunction of continuous system due to substructuring and decoupling of discrete subsystems. The dynamic characteristics of updated continuous system are evaluated by the constraint effect of consistent deformation at the interfaces between two systems. Beginning with the dynamic equation for constrained discrete system, this work estimates the modal eigenmode function for the continuous system due to the addition or deletion of discrete systems. Numerical applications illustrate the validity and applicability of the proposed method.

Keywords : Decoupling, Substructuring, Continuous System, Constraint, Eigenfunction

1. INTRODUCTION

Modal synthesis technique predicts the dynamic behavior of a coupled system based on the consistency of modal displacements in the interfaces among uncoupled subsystems. A complex structure is combined by analytical models of several substructures and then analyzed by proper approaches. While subcomponent modeling of built-up structures has become commonplace using finite elements, direct coupling of experimental and analytical models is rare because of the difficulties encountered.

The process of removing one substructure from another is called substructure uncoupling or substructure decoupling and is carried out by the applicability of the substructuring method. If masses are installed on structure while making experiments for describing its dynamic characteristics, it is necessary to get rid of their effect from the experimental data for more accurate analysis. The masses should be regarded as substructures and the decoupling process of masses from the entire system must be performed for the analysis of pure structure itself. A number of researchers have used substructure uncoupling to remove rigid masses from a structure, have accounted for the static flexibility in the joint between a fixture and the substructure in order to approximate the residual effect of out of band modes, and have removed a flexible substructure from a master system using FRF (Frequency Response Function) based substructuring (FBS).

Decoupling problem can be seen as the reverse of the substructuring problem. Starting from the known dynamic behavior of the coupled system and from information about the remaining part of the structural system, D'Ambrogio and Fregolent (2010) identified the dynamic behavior of a structural subsystem. Matthew et. al (2010) presented a method that removes the effects of a flexible fixture from an experimentally obtained modal model on the modal basis of the substructure to accurately estimate the modal parameters of the built-up system. Based on reconstruction of the interface forces acting between the unknown subsystem and its neighbor, Sjövall and Abrahamsson (2008) presented a theoretical method regarding frequency domain load identification. Voormeeren et. al (2010) presented a method to quantify the uncertainty of the coupled system's FRFs based on the uncertainties of the subsystem FRFs. Ryberg and Mir (2007) developed an experimental model with forward prediction capabilities for passenger vehicle axle whine performance based on FBS techniques to predict the dynamic behavior of complex structures based on the dynamic properties of each component of the structure. D'Ambrogio and Sestieri (2004) analyzes the possibility of assembling together different substructures' models using expansion techniques to provide the information on the rotational DOFs as well as appropriate modeling of joints and combining modal models and FE models. Sjövall et. al (2006) presented a formulation in terms of the state-space parameterization to represent transfer function constraints. Rodriguez et al. (2009)

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This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MEST) (No. 2011-0012164).

presented damage submatrices method (DSM) that localizes and assess degradation of stiffness at any structural element in a building. And they presented an approach to expand the condensed stiffness matrix of the damaged structure to global coordinates and to identify damage. Ozgen and Kim (2007) developed the analytical methods to expand the experimental damping matrix to the size of the analytical model. Based on the dual and primal assembly of substructures, de Klerk et. al (2008) provided a framework for the various classes of methods and a mathematical description of substructured problems. Schmitz and Duncan (2005) described the FRF or receptance of the tool and machine-spindle-holder substructure coupled through translational and rotational springs and dampers using receptance substructure analysis method. The modal synthesis of a structure that is decomposed into substructures is a Rayleigh-Ritz approximation of the global eigenvalue problem using coupling modes describing the interfacial displacements. Based on special extension operators from the boundary of each subdomain to the whole interface, Bourquin and Namar (1998) introduced less expensive coupling modes. Retaining the lower-frequency normal modes of the substructures, neglecting a frequency truncation criterion and considering the lower-frequency normal modes and residual flexibility, Zou et. al (2002) provided a modal synthesis method of lateral vibration analysis for rotor-bearing system.

This work presents analytical methods to estimate the modal characteristics of continuous system due to substructuring and deleting of discrete subsystems. The analysis is based on the evaluation of the constraint effect at the interfacial positions between adjacent discrete system and continuous system. The constraint effect includes the variation in the internal displacements or forces of the initial system due to the constraints. The validity of the proposed methods is illustrated in several applications.

2. FORMULATION

2.1 Discrete system

The substructuring and decoupling of discrete subsystems are performed by evaluating the interaction at the interfaces between adjacent systems. The interactive constraint indicates the consistent deformation among the subsystems and the modal responses of the resulting systems can be estimated based on the constrained modal effect. This section simply introduces the mathematical form to describe the constraint effect from the generalized inverse method (1992) on discrete dynamic system.

The dynamic response of a system that is assumed to be linear and approximately discretized for n degrees of freedom can be described by the equations of motion

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{f}(t) \tag{1}$$

where $\mathbf{q} = [q_1 \quad q_2 \quad \cdots \quad q_n]^T$, $\mathbf{M}$ denotes an $n \times n$ initial mass matrix, and $\mathbf{C} \in R^{n \times n}$ and $\mathbf{K} \in R^{n \times n}$ are the damping and stiffness matrices, respectively. The dynamic equation can be expressed in matrix form of

$$\mathbf{M}\ddot{\mathbf{q}} = \mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}, t) \tag{2}$$

where $\mathbf{F}(\mathbf{q}, \dot{\mathbf{q}}, t) = -\mathbf{C}\dot{\mathbf{q}} - \mathbf{K}\mathbf{q} + \mathbf{f}(t)$. Assume that the system is constrained by $m(n > m)$ acceleration-based constraint equations expressed as

$$\mathbf{A}(\mathbf{q}, \dot{\mathbf{q}}, t)\ddot{\mathbf{q}} = \mathbf{b}(\mathbf{q}, \dot{\mathbf{q}}, t) \tag{3}$$

where A is an $m \times n$ matrix, $\ddot{\mathbf{q}}$ is the actual acceleration, and $\mathbf{b}$ is an $m \times 1$ vector.

The generalized inverse method derived the dynamic equation for constrained dynamic system by minimizing the Gauss function with respect to the actual acceleration vector :

$$\ddot{\mathbf{q}} = \mathbf{a} + \mathbf{M}^{-1/2}\left(\mathbf{A}\mathbf{M}^{-1/2}\right)^{+}(\mathbf{b} - \mathbf{A}\mathbf{a}) \tag{4}$$

where $\mathbf{a} = \mathbf{M}^{-1}\mathbf{F}$.

The synthesis of subsystems requires the action of constraint forces in the satisfaction of compatibility conditions between adjacent subsystems. The constraint equations of compatibility conditions between adjacent subsystems are expressed by

$$\mathbf{A}(\mathbf{q}, \dot{\mathbf{q}}, t)\ddot{\mathbf{q}} = \mathbf{0} \tag{5}$$

and the dynamic equation is modified as

$$\ddot{\mathbf{q}} = \mathbf{a} - \mathbf{M}^{-1/2}\left(\mathbf{A}\mathbf{M}^{-1/2}\right)^{+}\mathbf{A}\mathbf{a} \tag{6}$$

The second term in the right-hand side of Eqn. (6) indicates the variation in the acceleration to be necessary for satisfying constraint conditions and the multiplication of the second term by mass matrix leads to the constraint force vector. Conversely, the decoupling of subsystem from an entire system requires the removal of constraint forces between an entire system and removed subsystems. _Assuming the entire system of $(n+m)$ DOFs, the removed subsystem of m DOFs and the residual system of n DOFs, the dynamic equation of the residual system after deleting the subsystem can be written as

$$\ddot{\mathbf{q}}_d = \mathbf{a}_d + \mathbf{M}_d^{-1/2}\left(\mathbf{A}_d\mathbf{M}_d^{-1/2}\right)^{+}\mathbf{A}_d\mathbf{a}_d \tag{7}$$

where $\ddot{\mathbf{q}}_d = \left[\ddot{\mathbf{q}}_e{}^T \quad \ddot{\mathbf{q}}_r{}^T\right]^T$, $\mathbf{M}_d = \begin{bmatrix} \mathbf{M}_e & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_r \end{bmatrix}$, and $\mathbf{a}_d = \mathbf{M}_d^{-1}\mathbf{F}_d$. The subscripts e, r and d represent the entire system, removed subsystem and decomposed residual system, respectively. The substructuring and decoupling of subsystems are summarized as addition and removal of constraint effect between subsystems, respectively. The following example describes the mode shapes of residual system to remove subsystems from an entire discrete system.

Example 1)

The mass of accelerometers installed on a system affects the measured data and their effect should be removed for obtaining pure data. It needs a decoupling process to remove the effect of the accelerometers from the entire system. This application considers an undamped dynamic system of 7 DOFs in Fig. 1. The numerical values utilized for this example were selected as shown in Table 1.

Table 1. Dynamic property

Mass(kg)							
m_1	m_2	m_3	m_4	m_5	m_6	m_a	m_a
15	8	10	9	12	13	9	0.4
Stiffness(N/m)							
k_1	k_2	k_4	k_4	k_5	k_6	k_7	
4000	12000	8000	7000	9000	5000	6000	

In the table 1, m_a denotes the additional mass attached at the 3rd and 7th mass locations.

Table 2 represents the first five natural frequencies of the initial beam and the residual system to remove two masses from an entire system, and the corresponding mode shapes, respectively. Figure 2 represents that the slight difference in the modal properties between the residual system and the initial system is due to the affect of the removed masses while decoupling. It is observed that the modal parameters of the residual system can be properly described by the proposed decoupling method.

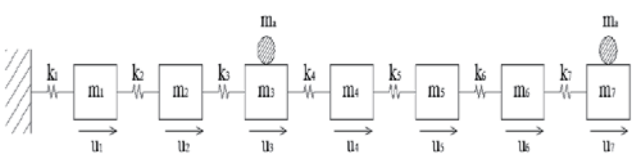

Figure 1. A dynamic system of 7 DOFs to install additional masses

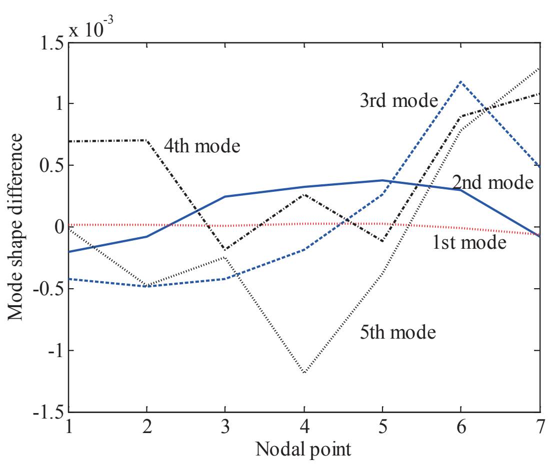

Figure 2. Difference in normalized mode shape vectors between two states

Table 2. Comparison of natural frequencies of the initial and residual systems $(rad./\sec.)$

	1st	2nd	3rd	4th	5th
Initial beam	5.15	14.54	23.29	34.16	38.65
Residual beam	5.14	14.53	23.28	34.13	38.63

2.2 Continuous system

A complicated system is likely to have parts that are best modeled with continuous equations. Modal synthesis model is well motivated as the linear partial differential equation for a vibrating system, with appropriate boundary conditions, has as solutions a superposition of vibration modes. The eigenfunction of continuous system is smooth curve to be differentiable with respect to the coordinate x.

Modes of continuous systems are computed by the solution of the linear partial differential equation found for the beam. The differential equation of motion for transverse free vibration of flexural beam in Fig. 3 can be written as

$$\frac{\partial^2}{\partial x^2}\left(E\,\frac{\partial^2 v}{\partial x^2}\right)+\rho A\frac{\partial^2 v}{\partial t^2}=0 \tag{8}$$

where $v(x,t)$ denotes the displacement in the y direction, E and I are Young's modulus and moment of inertia, and ρ and A represent the unit mass per volume and sectional area. For a uniform beam, Eqn. (8) can be expressed as

$$c^2\frac{\partial^4 v}{\partial x^4}+\frac{\partial^2 v}{\partial t^2}=0 \tag{9}$$

where $c=\sqrt{\dfrac{E}{\rho A}}$.

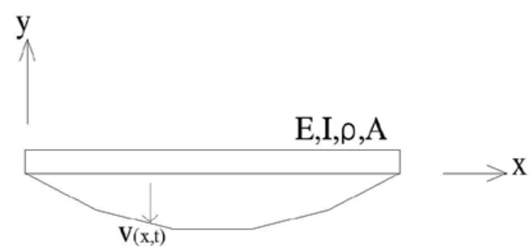

Figure 3. A simple beam of arbitrary boundary conditions

The free vibration solution can be found using the method of separation of variables as

$$v(x,t)=\phi(x)T(t) \tag{10}$$

Using Eqn. (10) into Eqn. (9) and arranging yields

$$\frac{c^2}{\phi(x)}\frac{d^4\phi(x)}{dx^4}=-\frac{1}{T(t)}\frac{d^2T(t)}{dt^2}=a=\omega^2 \tag{11}$$

where $a=\omega^2$ can be shown to be a positive constant. Equation (11) can be rewritten as two equations:

$$\frac{d^4\phi(x)}{dx^4}-\beta^4\phi(x)=0 \tag{12a}$$

$$\frac{d^2T(t)}{dt^2}+\omega^2T(t)=0 \tag{12b}$$

where $\beta^4=\dfrac{\omega^2}{c^2}=\dfrac{\rho A\omega^2}{E}$.

The solution of Eqn. (12b) is given by

$$T(t)=A\cos\omega t+B\sin\omega t \tag{13}$$

where A and B are constants that can be found from the initial conditions. The solution of Eqn. (12a) is assumed to be of exponential form as

$$\phi(x) = Ce^{sx} \tag{14}$$

where C and s are constants. Substitution of Eqn. (14) into Eqn. (12a) results in the auxiliary equation

$$s^4 - \beta^4 = 0 \tag{15}$$

The roots of Eqn. (15) are given by

$$s_{1,2} = \pm\beta, \qquad s_{3,4} = \pm i\beta \tag{16}$$

Thus, the solution of Eqn. (12a) using trigonometric property can be expressed as

$$\phi(x) = C_1 \cos\beta x + C_2 \sin\beta x + C_3 \cosh\beta x + C_4 \sinh\beta x \tag{17}$$

where C_1, C_2, C_3 and C_4 are constants to be determined by boundary conditions. The natural frequencies of the beam can be determined from

$$\omega = \beta^2 \sqrt{\frac{EI}{\rho A}} \tag{18}$$

Assuming that a known mass is added on a position x_1 from a left end support of a simply supported beam, the eigenfunction should be changed by the mechanical interaction at the interface x_1 between the beam and the mass m_1. The interaction requires the coincidence of modal displacement at the connection point between two systems. It can be mentioned as a constraint.

The dynamic equation of the added mass at x_1 can be expressed as

$$m_1 \ddot{v}(x = x_1) = 0 \tag{19a}$$

or

$$m_1 \bar{\phi}_1 \ddot{\bar{T}}_1 = 0 \tag{19b}$$

where $\bar{\phi}_1$ means the mode shape at the location x_1 and $\bar{T}_1(t)$ represents the generalized response function of time t. And the dynamic equation of the beam at x_1 from Eqn. (8) can be written by

$$\beta^4 EI\phi_1 T_1 + \rho A \phi_1 \ddot{T}_1 = 0 \tag{20}$$

The constraint condition between two systems of the continuous system and the discrete system can be also written as

$$\phi_1 T_1 = \bar{\phi}_1 \bar{T}_1 \tag{21}$$

where $\bar{\phi}_1(x_1) = \phi_1(x_1)$ because of the geometric constraint. Utilizing the dynamic equation of Eqns. (19b) and (20) and the constraint equation of Eqn. (21) into Eqn. (6), it can be written as

$$\phi_1 \ddot{T}_1 = \mathbf{G}_1\left[\mathbf{I} - \mathbf{M}_1^{-1/2}\left(\mathbf{A}_1\mathbf{M}_1^{-1/2}\right)^+ \mathbf{A}_1\right]\mathbf{M}_1^{-1}\mathbf{F}_1 \tag{22}$$

where $\mathbf{G}_1 = \begin{bmatrix}1 & 0\end{bmatrix}$, $\mathbf{M}_1 = \begin{bmatrix}\rho A & 0 \\ 0 & m_1\end{bmatrix}$, $\mathbf{A}_1 = \begin{bmatrix}1 & -1\end{bmatrix}$, $\mathbf{F}_1 = \begin{bmatrix}-\beta^4 EI\phi_1 T_1 \\ 0\end{bmatrix}$. The second term in the right-hand side of Eqn. (22) represents the variation of dynamic variation due to the added mass.

The coefficient $\mathbf{G}_1\left[\mathbf{I} - \mathbf{M}_1^{-1/2}\left(\mathbf{A}_1\mathbf{M}_1^{-1/2}\right)^+ \mathbf{A}_1\right]$ indicates the multiplication factor of the initial dynamic equation after the addition of the mass at the location x_1. The derived equation can be expanded to the addition of m rigid masses along the beam in Fig. 4 and it is written as

$$\begin{bmatrix}\phi_1\ddot{T}_1 \\ \phi_2\ddot{T}_2 \\ \vdots \\ \phi_m\ddot{T}_m\end{bmatrix} = \mathbf{G}\left[\mathbf{I} - \mathbf{M}^{-1/2}\left(\mathbf{A}\mathbf{M}^{-1/2}\right)^+ \mathbf{A}\right]\mathbf{M}^{-1}\mathbf{F} \tag{23}$$

where $\mathbf{I}$ represents the $2m \times 2m$ identity matrix, $\mathbf{M}$ is an $2m \times 2m$ mass matrix, $\mathbf{A}$ is a Boolean matrix to represent the interfaces between two systems,

$$\mathbf{G} = \begin{bmatrix}1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & 0\end{bmatrix}_{m\times 2m}.$$

The newly updated eigenfunction estimates the modal displacement curve for the beam with added masses by taking least square fitting method. The modal variations deviated from the initial modal displacement mean the effect due to the added masses.

The decoupling of subsystems from an entire system is similar to the substructuring process. However, taking into account that the decoupling is the opposite process of the substructuring, the minus sign of the second term in the right-hand side of Eqn. (23) should be changed to adverse sign for decoupling of subsystems. And the decoupling process indicates the relation between the entire system and the removed subsystem. The decoupling of the m masses from an entire system also can be expressed as

$$\begin{bmatrix}\phi_1\ddot{T}_1 \\ \phi_2\ddot{T}_2 \\ \vdots \\ \phi_m\ddot{T}_m\end{bmatrix} = \mathbf{G}\left[\mathbf{I} + \mathbf{M}^{-1/2}\left(\mathbf{A}\mathbf{M}^{-1/2}\right)^+ \mathbf{A}\right]\mathbf{M}^{-1}\mathbf{F} \tag{24}$$

The second term in the right-hand side of Eqn. (24) represents the variations of the dynamic response caused by the removal of the masses. The resulting eigenmodes are estimated by multiplying the coefficients of Eqn. (24) and the eigenfunction of entire system.

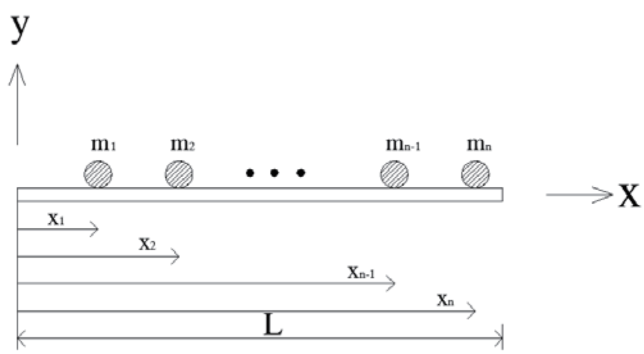

Figure 4. Substructuring and decoupling of subsystems

Example 2)

Consider a simply supported beam in Fig. 5 modeled as a continuous system. Assume that seven masses were installed on seven different locations indicated in Table 3. The beam has elastic modulus of 200GPa, cross-sectional area of $300\text{mm} \times 400\text{mm}$, length of 6000mm and unit mass per volume of 8000kg/m^3. Utilizing the proper boundary conditions into Eqn. (17) and arranging the result, the eigenfunction of the beam corresponding to the first mode can be derived as

$$v(x) = \sin \beta x \tag{25}$$

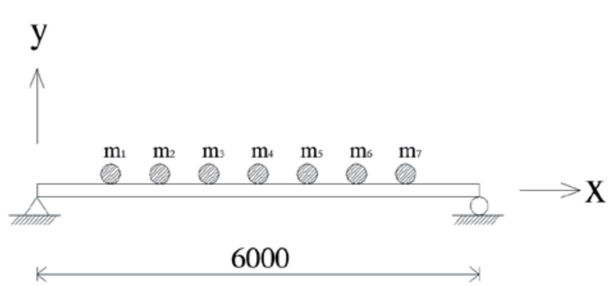

Figure 5. A simply supported beam as a continuous system (unit:mm)

The eigenmode of the entire system to be composed of the beam and discrete masses is estimated based on the proposed method. Utilizing the compatibility conditions at the interfacial positions between the beam and the additional masses and their dynamic equations into Eqn. (23), the mode shapes at the measured nodes are estimated. Taking the linear square fitting method based on the seven modal data and the boundary conditions, the eigenmode function of the beam to install the masses is estimated.

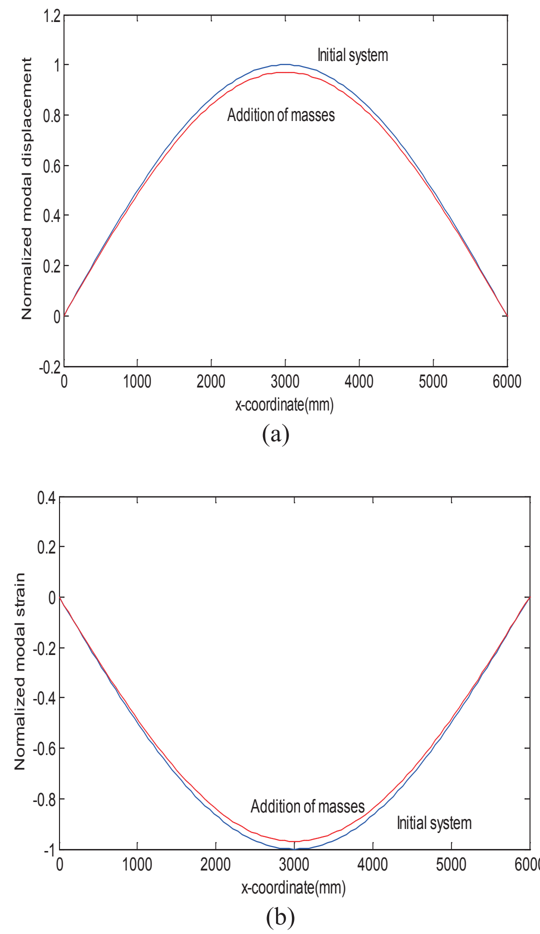

Figure 6. Comparison of mode shape curve before and after the addition of masses; (a) modal displacement, (b) modal strain. The solid and dashed lines represent the initial beam and the beam modified by masses, respectively.

Figure 6 represents the modal displacement curve and strain curve of the system corresponding to the first natural frequency. Both curves were normalized with respect to the maximum modal displacement and the maximum modal strain of the initial system. The difference between two states represents the variation that the masses affect the eigenmode of the beam. It is observed that the proposed method can properly describe the mode shape of the synthesized system.

Example 3)

As an opposite of Example 2, this application estimates the mode shape of residual system to remove the masses from an entire beam system as shown in Fig. 7. The residual system coincides with the initial system without the additional masses of example 2. Substituting the dynamic equations of the entire system and removed subsystems, and the compatibility conditions between two systems into Eqn. (24), the modal displacements of the residual system at the interfacial nodes are obtained and its eigenfunction is statistically derived. Figure 8 compares the modal displacement and strain corresponding to the first natural frequency of the initial and residual systems. Both modal curves were normalized with respect to the maximum modal displacement value of the initial system. It is expected that the difference in the modal responses comes from the removal of masses.

Table 3. Masses and their locations installed on the beam

Location (mm)	900	1440	2220	3060	3840	4620	5400
Masses (kg)	30	30	30	30	30	30	30

This method is offering the substructuring and decoupling methods of substructures in the continuous system unlike the existing methods to consider the discrete system. The substructuring and decoupling of substructures are performed by the compatibility conditions between them. The proposed method can be utilized in the dynamic analysis to combine small substructures into a huge structure and to tear small substructures from a complex structure.

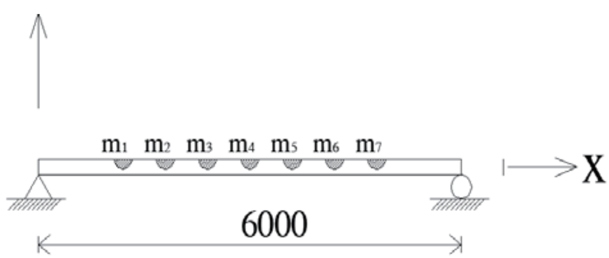

Figure 7. Decoupled residual system(unit:mm)

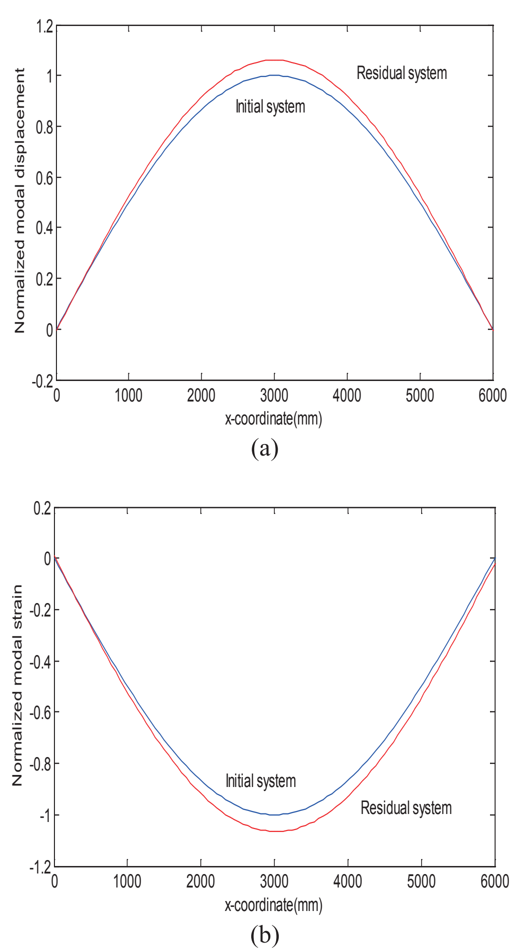

Figure 8. Comparison of mode shape curve before and after removing masses: (a) mode shape, (b) strain mode shape. The solid and dashed lines represent the initial beam and the beam modified by masses, respectively.

3. CONCLUSIONS

This work proposed analytical methods to estimate modal response of continuous system by substructuring and decoupling of subsystems using compatibility conditions at the interfaces between subsystems. The proposed method based on the beam model of continuous system does not indicate accurate damage location but its vicinity to complete a fundamental mode. The damage of single damaged beam locates in the vicinity of beginning or ending point to complete a fundamental mode that the squared modal displacement difference minimizes. The damages of multiple damaged beam position in the vicinity of both ending points to complete a fundamental mode. The validity of the proposed method was illustrated in several applications. The method can be widely utilized in the dynamic synthesis of substructures and the dynamic tearing of substructures.

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(Date of Submission : 2011.8.29)