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Performance Comparison of Orthogonal and Non-orthogonal AF Protocols in Cooperative Relay Systems

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Abstract

For a single relay channel, we compare the capacity of two different amplify-and-forward (AF) protocols, which are orthogonal AF (OAF) and non-orthogonal AF (NAF). The NAF protocol has been proposed to overcome a significant loss of performance of OAF in the high spectral efficiency region, and it was also theoretically proved that NAF performs better than OAF in terms of the diversity-multiplexing tradeoff. However, existing results have been evaluated at the asymptotically high signal to noise ratio (SNR), thus the power allocation problem between the source and the relay was neglected. We examine which protocol has better performance in a practical system operating at a finite SNR. We also study where a relay should be located if we consider the power allocation problem. A notable conclusion is that the capacity performance depends on both SNR and power allocation ratio, which indicates OAF may perform better than NAF in a certain environment.

Keywords: Cooperative relay systems, amplify-and-forward, capacity analysis, orthogonal and nonorthogonal relay protocols

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1. Introduction

Multiple antenna techniques in [1][2][3] have practically limited gains for the systems with small size equipment even if there is theoretically large performance gain compared to single antenna systems. As an alternative, a relay channel model using antenna at separate nodes has been studied and many low complexity protocols have been developed [4][5][6][7]. We usually call it cooperative relay systems.

Laneman et al. proposed two TDMA-based efficient protocols for cooperative relay systems, i.e., amplify-and-forward (AF) and decode-and-forward (DF) in [7]. Here a TDMA-based protocol means that the source and relay use a separate time slot or orthogonal channel to transmit a symbol. Assuming the relay uses AF protocol, this is called orthogonal AF (OAF). Nabar et al. extend the results further by proposing two additional protocols to increase a spectral efficiency in [8]. If the source transmits a new symbol when the relay forwards the received symbol, this protocol does not use the orthogonal time slot so we call non-orthogonal AF (NAF). OAF and NAF corresponds to protocol II and protocol I in [8]. Authors of [9] analyze the achievable diversity-multiplexing (D-M) tradeoff of cooperative relay systems for delay limited coherent channel. In [9], the optimal D-M tradeoff for NAF is given by $d_{NAF}^*(r) = (1-r) + (1-2r)^+$ whereas that for OAF is given by $d_{OAF}^*(r) = (2-4r)^+$ as shown in Fig. 1. This shows that NAF is always better than OAF in terms of multiplexing and diversity gain.



Fig. 1. Optimal diversity-multiplexing tradeoff for a single relay AF protocol.

However, diversity and multiplexing gain curves are obtained at asymptotically high SNR regime, therefore their analysis does not consider a power allocation problem. That is, D-M tradeoff formulation gives us only a partial information of the overall performance even if it is a theoretically useful tool. In this respect, Narasimhan generalized the D-M tradeoff analysis

to finite signal to noise ratio (SNR) regime and revealed that the D-M curves at the finite SNR region become lower than asymptotical results in [10][11]. Therefore it is expected that NAF may not always perform better than OAF. There exists literature on the power allocation for either OAF or NAF [12][13][14][15][16][17]. However, the comparison between the two protocols in terms of power allocation has not been reported. This is the motivation of this paper.

This paper first compares the average throughput between OAF and NAF. Though NAF has a larger throughput than OAF when the signal power is high, we show that OAF can be better than NAF below a specific SNR. To do this analysis, we need to find the probability density function of the composite function of exponential random variables. As will be discussed, it will be difficult to compute the probability density function (pdf), and even if this approach is possible, the results may not be useful to give a valuable intuition for systems. We first obtain an upperbound on a concave log function by using the Jansen's inequality. Unfortunately, it is also difficult to find the pdf of this upperbound directly. However we can get another useful upperbound for exponentially distributed random variables by numerical method, which simplify the analysis. Using this upperbound, we derive a sufficient condition for which OAF will be better than NAF. In the second part of paper, we investigate where a relay should be located for OAF to be better by using the sufficient condition. To do this investigation, we include the path loss factor to the channel model. Through numerical method, we also derive the region in which OAF is better than NAF.

The rest of paper is organized as follows. In Section 2, we revisit a two-hop system model of wireless cooperative relay networks. Section 3 discusses on the achievable rate difference between OAF and NAF. We then consider how to determine a relay position with a path loss channel model in Section 4. Numerical results will be presented in Section 5. Section 6 provides the concluding remarks.

2. System Model

Consider a two-hop system transmitting from a source to a destination with the help of a cooperating relay. Each node such as source, relay, destination has a single antenna, and they are in the mode of half duplexing, i.e., cannot transmit and receive simultaneously. This scheme is composed of two phases for transmission.

In Phase I, the source broadcasts the modulated symbol to the relay and the destination. Phase II can be categorized into AF and DF according to forwarding strategies. We consider only the AF strategy in this paper, but the same procedure can be applied to DF. Another categorization can be whether the source participates or not in phase II, that is, OAF and NAF. The baseband equivalent signal model can be summarized for Phase I

$$y_{d,1} = \sqrt{E_s} h_{sd} x_{s,1} + n_{d,1} \tag{1}$$

$$y_r = \sqrt{E_s} h_{sr} x_{s,1} + n_r \tag{2}$$

where $x_{s,1}$ is the transmitted symbol from source node with normalized unit energy, and $y_{d,1}$ and y_r denote the received signal at the destination and the relay from the source in Phase I. E_s is the average transmit power, and h_{sd} and h_{sr} are the channel coefficients from the source to the destination and the relay, which are independent and complex Gaussian distributed with zero mean and unit variance (Rayleigh fading). $n_{d,1}$ and n_r are additive complex Gaussian noise with variance N_0 .

In Phase II, depending on the participation of the source, we have

$$y_{d,2} = \begin{cases} \sqrt{E_r} h_{rd}(\beta y_r) + n_{d,2} & \text{for OAF} \\ \sqrt{E_r} h_{rd}(\beta y_r) + \sqrt{E_s} h_{sd} x_{s,2} + n_{d,2} & \text{for NAF} \end{cases}$$

where β is the amplification factor at the relay. The system need to satisfy the power constraint (with high probability) at the relay, so we set the amplification factor as

$$\beta = \frac{1}{\sqrt{E_s |h_{sr}|^2 + N_0}}.$$
(3)

These equations for both phases can be put together by following matrix notation

$$\mathbf{y} = \mathbf{H}_l \mathbf{x}_l + \mathbf{n} \tag{4}$$

where \mathbf{H}_l and \mathbf{x}_l will be different according to $l \in \{\text{OAF}, \text{NAF}\}$. For fair comparison, the source power should be divided by two in the case of NAF due to the participation of source in both phases. Considering this fact, the corresponding \mathbf{H}_l matrix and noise \mathbf{n} vector for each protocol are given by

$$\mathbf{H}_{OAF} = \begin{bmatrix} \sqrt{E_s} h_{sd} \\ \beta \sqrt{E_s E_r} h_{sr} h_{rd} \end{bmatrix}$$
(5)

$$\mathbf{H}_{NAF} = \begin{vmatrix} \sqrt{\frac{E_s}{2}} h_{sd} & 0\\ \beta \sqrt{\frac{E_s E_r}{2}} h_{sr} h_{rd} & \sqrt{\frac{E_s}{2}} h_{sd} \end{vmatrix}$$
(6)

$$\mathbf{n} = \begin{bmatrix} n_{d,1} \\ \beta \sqrt{E_r} h_{rd} n_r + n_{d,2} \end{bmatrix}$$
(7)

where $x_{\text{OAF}} = x_{s,1}$ is a scalar and $\mathbf{x}_{\text{NAF}} = \begin{bmatrix} x_{s,1}, x_{s,2} \end{bmatrix}^T$ is a column with two components and $\mathbf{y} = \begin{bmatrix} y_{d,1}, y_{d,2} \end{bmatrix}^T$. Note that the noise vector **n** is the same for two protocols, OAF and NAF.

3. On Average Achievable Rate

In this section, we investigate the achievable rate difference between OAF and NAF. By using the general MIMO channel capacity formula in [1] as the achievable rate and the fact that two time slots are used, the following equations denote the achievable rate for a fixed channel realization

$$I = \frac{1}{2} \log_2 \left[\det \left(\mathbf{I} + \mathbf{H}_l \mathbf{K}^{\mathbf{x}_l} \mathbf{H}_l^* (\mathbf{K}^{\mathbf{n}})^{-1} \right) \right]$$
(8)

where \mathbf{H}_{l}^{*} denotes the conjugate transpose of matrix. As input symbols are assumed to be normalized and independent, the covariance matrix of the transmit symbol and noise are given as follows

$$\mathbf{K}^{\mathbf{x}_{l}} = \mathbf{I}, \quad \mathbf{K}^{\mathbf{n}} = \begin{bmatrix} N_{0} & 0\\ 0 & \left(\beta^{2} E_{r} \left| h_{rd} \right|^{2} + 1 \right) N_{0} \end{bmatrix}.$$
(9)

By substituting previous channel matrices \mathbf{H}_{OAF} and \mathbf{H}_{NAF} and the covariance matrices of the transmit symbol and noise into the capacity formula, we have

$$I_{\text{OAF}} = \frac{1}{2} \log_2 \left(1 + \frac{E_s \left| h_{sd} \right|^2}{N_0} + \frac{\beta^2 E_s E_r \left| h_{sr} \right|^2 \left| h_{rd} \right|^2}{N_0 + \beta^2 E_r \left| h_{rd} \right|^2 N_0} \right)$$
(10)

$$I_{\text{NAF}} = \frac{1}{2} \log_2 \left(1 + \frac{E_s \left| h_{sd} \right|^2}{2N_0} + \frac{2\beta^2 E_s E_r \left| h_{sr} \right|^2 \left| h_{rd} \right|^2 + E_s^2 \left| h_{sd} \right|^4 / N_0 + 2E_s \left| h_{sd} \right|^2}{4 \left(N_0 + \beta^2 E_r \left| h_{rd} \right|^2 N_0 \right)} \right).$$
(11)

To simplify the equation, we use the change of variables as following

$$u = \frac{E_s |h_{sr}|^2}{N_0} \qquad v = \frac{E_r |h_{rd}|^2}{N_0} \qquad w = \frac{E_s |h_{sd}|^2}{N_0}.$$
 (12)

Finally we have the following expression

$$I_{\text{OAF}} = \frac{1}{2} \log_2 \left(1 + w + \frac{uv}{u + v + 1} \right)$$
(13)

$$I_{\text{NAF}} = \frac{1}{2} \log_2 \left(1 + \frac{w}{2} + \frac{w(w+2)(u+1) + 2uv}{4(u+v+1)} \right).$$
(14)

To know whether NAF is better than OAF or not, we only have to consider the difference. After a direct calculation

$$I_{\text{NAF}} - I_{\text{OAF}} = \frac{1}{2} \log_2 \left(1 + \frac{w^2 u + w^2 - 2uv - 2vw}{4(u + v + w + uv + vw + wu + 1)} \right)$$
(15)

where, if we know the pdf of inner term of log function, we can get the exact average capacity difference by direct integration. However, this can be difficult, so we will upper bound this by using the Jensen's inequality and a numerical method.

We assume the total available energy in the entire network is limited by E_T and a partial portion of E_T is used at the source and relay according to the ratio $\alpha = E_s/E_T$. Each noise variance at the relay and destination is the same as N_0 . We set $E_s = \alpha E_T$, $E_r = (1 - \alpha)E_T$ where $0 < \alpha < 1$. Define the network SNR $\rho = E_T/N_0$, then we can denote

$$u = \alpha \rho x, \quad v = (1 - \alpha) \rho y, \quad w = \alpha \rho z$$
 (16)

where all x, y, z are the random variables as the square of absolute channel coefficients. These are exponentially distributed for Rayleigh fading channel and has the pdf $f_T(t) = e^{-t}$ where $T \in \{X, Y, Z\}$.

To average the difference of achievable rate over all possible x, y, z values, we should perform a triple-folded integration

$$E\left[I_{\text{NAF}} - I_{\text{OAF}}\right] = \iiint_{x,y,z} \left(I_{\text{NAF}} - I_{\text{OAF}}\right) f_X(x) f_Y(y) f_Z(z) dx dy dz .$$
(17)

Since this will be difficult to obtain directly, for the moment, we set the result as a function over α and ρ , $f(\alpha, \rho)$, which is calculated by averaging out the random variables. If we set the denominator to D and numerator to N of the second term in the log function of (15), then by the Jensen's inequality

$$f(\alpha, \rho) \leq \frac{1}{2} \log_2 \left(1 + E\left[\frac{N}{D}\right] \right) = g(\alpha, \rho).$$
(18)

Unfortunately, it is also very complicated to calculate the inner expectation term of log function. At this time, we want to use separate expectations as an approximation. Therefore we decide to resort to numerical method to find the relation between E[N/D] and E[N]/E[D].



Fig. 2. Difference between E[N/D] and E[N]/E[D] at each point of α and ρ obtained by numerical method.

From Fig. 2, it is observed that $E[N/D] \leq E[N]/E[D]$ in most of the region of α and ρ . Using this result

$$g(\alpha, \rho) \leq \frac{1}{2} \log_2 \left(1 + \frac{E[N]}{E[D]} \right) = h(\alpha, \rho) .$$
⁽¹⁹⁾

Therefore we get the following relation

$$f(\alpha, \rho) \leq h(\alpha, \rho) \,. \tag{20}$$

As mentioned before, NAF will have eventually better performance than OAF for a fixed α as ρ increase, that is, $f(\alpha, \rho)$ becomes larger than zero as ρ increases. This means that the region satisfying $f(\alpha, \rho) \le 0$ is to the left of the curve $f(\alpha, \rho) = 0$ in the (ρ, α) plane. Apparently the $h(\alpha, \rho) = 0$ means $f(\alpha, \rho) \le 0$. Therefore the $h(\alpha, \rho)$ curve should be located below the $f(\alpha, \rho) = 0$ curve, i.e., $h(\alpha, \rho) = 0$ curve is lower bound of the exact boundary curve, $f(\alpha, \rho) = 0$.

Due to the property of the log function, the following equivalence satisfies

$$h(\alpha, \rho) = 0 \iff E[N] = 0 \tag{21}$$

where the expectation of numerator can be denoted by the substitution of variables

$$E[N] = \mathop{E}_{x,y,z} \left[\alpha^{2} \rho^{2} (\alpha \rho x + 1) z^{2} - 2\alpha (1 - \alpha) \rho^{2} (xy + yz) \right].$$
(22)

By using the results on the moments of exponential random variables, E[x] = E[y] = E[z] = 1, $E[z^2] = 2$, we can get the simple relation between α and ρ as

$$E[N] = 0 \iff \rho \alpha^2 + 3\alpha - 2 = 0$$

$$\alpha = \frac{-3 + \sqrt{9 + 8\rho}}{2\rho}.$$
 (23)

Note that we obtain a sufficient condition in which OAF is better than NAF from upper bound such as (20). Namely, if α is lower than the above solution at a specific ρ value, then it is desirable to use the OAF relay protocol. To complete the analysis thoroughly, we may need a necessary condition which can be obtained by either lower bound or exact value of $f(\alpha, \rho)$ if possible.

4. On Relay Position

Now we consider the problem of relay position. Signal power decays in the wireless environment as the transmitter-receiver distance increases. Therefore the power decay factor can be included in the previous channel model.

$$y_{j,t} = \frac{h_{ij}}{\sqrt{d^{\lambda}}} x_{i,t} + n_{j,t}$$
(24)

where λ denote the path loss exponent and each subscript means transmitter *i*, receiver *j* and the time slot *t*. Note that the previous numerical inequality also holds for almost every α and ρ except extremely low α , even if we put path loss terms. Therefore we replace the u, v, w variables by

$$u = \frac{\alpha \rho x}{d_x^{\lambda}}, \quad v = \frac{(1-\alpha)\rho x}{d_y^{\lambda}}, \quad u = \frac{\alpha \rho z}{d_z^{\lambda}}$$
(25)

where x, y, z corresponding to the squared channel coefficients remain unchanged which are exponentially distributed. d_x and d_z are the distances from the source to the relay and the destination, respectively, d_y is the distance from the relay to the destination. By inserting new variables u, v, w into E[N], we get

$$E[N] = 2\alpha\rho^2 \left\{ \frac{\rho\alpha^2}{d_z^{2\lambda}d_x^{\lambda}} + \left(\frac{1}{d_z^{2\lambda}} + \frac{1}{d_x^{\lambda}d_y^{\lambda}} + \frac{1}{d_y^{\lambda}d_z^{\lambda}}\right)\alpha - \frac{1}{d_x^{\lambda}d_y^{\lambda}} - \frac{1}{d_y^{\lambda}d_z^{\lambda}} \right\}$$

From the fact that $E[N] \le 0$ is a sufficient condition for OAF, we have

$$E[N] \le 0 \iff d_y^{\lambda}(\alpha^2 \rho + \alpha d_x^{\lambda}) - (1 - \alpha)d_z^{\lambda}(d_z^{\lambda} + d_x^{\lambda}) \le 0$$

From now on, we assume that d_z is normalized to a unit distance for the simplicity of analysis. Therefore we put the source and the destination at (-0.5,0) and (0.5,0) in the cartesian coordinate X-Y plane. If a relay is located at (x, y) and we use the 2-norm as a distance measure, we can set

$$d_x^{\lambda} = \left(\left(\mathbf{x} + 0.5 \right)^2 + \mathbf{y}^2 \right)^{\lambda/2}$$
(26)

$$d_{y}^{\lambda} = \left(\left(\mathbf{x} - 0.5 \right)^{2} + \mathbf{y}^{2} \right)^{\lambda/2}.$$
 (27)

For easier demonstration, we set the path loss exponent to $\lambda = 2$. We can then get the sufficient condition in which OAF is better than NAF in terms of x and y. Let $x + 0.5 = x_s$ and $x - 0.5 = x_d$. We then have

$$(\mathbf{x}_{d}^{2} + \mathbf{y}^{2}) (\alpha^{2} \rho + \alpha \mathbf{x}_{s}^{2} + \alpha \mathbf{y}^{2}) - (1 - \alpha) (\mathbf{x}_{s}^{2} + \mathbf{y}^{2} + 1) \leq 0.$$
 (28)

After the rearrangement of variables, we have

$$\alpha \left(\mathbf{x}^{2} + \mathbf{y}^{2} \right)^{2} + k_{1} \left(\mathbf{x} - k_{2} \right)^{2} + k_{3} \mathbf{y}^{2} \le k_{4}.$$
⁽²⁹⁾

Since we can decompose $k_4 = k_{4,1} + k_{4,2}$, then this inequality becomes the same as the sum of two closed contours, one of which is $\alpha (\mathbf{x}^2 + \mathbf{y}^2)^2 \le k_{4,1}$ and the other is $k_1 (\mathbf{x} - k_2)^2 + k_3 \mathbf{y}^2 \le k_{4,2}$. Therefore it is expected that the region satisfying the inequality should also be certain closed region. Interestingly, as will be observed in the numerical results, this region looks like a circle.

5. Numerical Results

In this section we verify our derivation with numerical methods. First we use 20,000 randomly generated Rayleigh fading channels to reliably get a numerical average throughput for a

specific α and ρ value. Note that the resolution of α is 0.001. In order to obtain the curve of $f(\alpha, \rho) = 0$, we numerically find the solution. For each SNR ρ which increases from -10 dB to 40 dB with a 2 dB step, we evaluate $f(\alpha, \rho) = E[I_{NAF} - I_{OAF}]$ for a specific α which increases from 0 to 1 with a 0.001 step. We then obtain the two zero-crossing points where

 $f(\alpha, \rho)$ changes the sign from negative to positive. Finally, we set the solution of

 $f(\alpha, \rho) = 0$ as the center of the two zero-crossing points for a given SNR ρ .



Fig. 3. Relation curve for power allocation ratio α and total network SNR ρ .

Fig. 3 shows $f(\alpha, \rho) = 0$ and $h(\alpha, \rho) = 0$ curves. By definition $f(\alpha, \rho) = 0$ corresponds to $E[I_{OAF}] = E[I_{NAF}]$. Therefore solid line divides the region in which protocol of AF should be selected for better performance. The dashed line obtained by numerical upper bound of E[N/D] is really lower bound of the exact boundary curve. This results verify the derivation

of Section 3. From (23) we can check the limit values of α when ρ approaches zero or infinity, $\lim_{\rho \to 0} \alpha = 2/3$ and $\lim_{\rho \to \infty} \alpha = 0$. This says if we allocate the network power ratio of more than 2/3 into the source or we have very large network power then NAF is always better than OAF, which is the same as the asymptotical results. For NAF protocols, the source participates during both phases, $\alpha = 2/3$ corresponds to the transmission with equal power of $E_T/3$ at the source and the relay in each phase.

Careful examination tells us that two curves are located with almost the same SNR gap for most of the ρ values in dB scale. This gap is about 2.43 dB. This means that $f(\alpha, \rho) = 0$ curve is the shifted version of $h(\alpha, \rho) = 0$ curve. Since 2.43 dB corresponds to 1.75 in linear scale, we can get the approximate solution as

$$f(\alpha, \rho) \approx \frac{4}{7}\rho\alpha^2 + 3\alpha - 2 = h\left(\alpha, \frac{\rho}{1.75}\right)$$

This curve is plotted by the dash-dot (red) curve and we can see the two curves are almost the same. Since the network SNR is independent of the exponential random variables, we will use this approximate solution for the relay location problem with the assumption that the same

SNR gap will be held for the path loss channel model. Therefore this changes ρ to $\rho/1.75$ in (28).



Fig. 4. 3-D Illustration for power allocation ratio α and total network SNR ρ .

In **Fig. 4**, we also illustrate the same results in three dimensions. Here $f(\alpha, \rho) = 0$ curve matches with blue curve in bottom face. Although the maximum gain of average capacity for NAF is achieved at $\alpha = 1$, this corresponds to direct transmission. However, as opposed to average capacity, error rate performance can be improved by cooperative relay communication than direct transmission. Therefore, where some fixed α are used and the system operates in the low SNR regime, it may be better to use OAF which requires low complexity receiver than NAF. OAF does not care about synchronization problem which may occur in the second phases of NAF.

Fig. 5 and Fig. 6 show the relay position satisfying the sufficient condition in which OAF has better performance. Each figure shows the results for small $\alpha = 0.3$ and large $\alpha = 0.7$. In both figures, the inner region of circle-like contour corresponds to relay positions at which the OAF protocol should be used. As expected, it is observed that the OAF region shrinks as the ρ increases, and becomes smaller for large α . This contour may be disappeared eventually as the network SNR increases, so the NAF protocol should be used irrespective of where a relay is located. If ρ goes to zero, the contour does not become larger but converges to a

specific one for a fixed α . This can be seen from (28) by setting ρ to zero, and we have

$$\alpha (\mathbf{x}_{d}^{2} + \mathbf{y}^{2}) (\mathbf{x}_{s}^{2} + \mathbf{y}^{2}) - (1 - \alpha) (\mathbf{x}_{s}^{2} + \mathbf{y}^{2} + 1) \leq 0.$$

The positions marked by black circle at $(0, \pm \sqrt{3}/2)$ correspond to the case that all the distances are the same $(d_x = d_y = d_z = 1)$, which makes the path loss model (Section 4) and the non-path loss model (Section 3) coincide.



Fig. 5. Region in which OAF has better performance than NAF when $\alpha = 0.3$.



Fig. 6. Region in which OAF has better performance than NAF when $\alpha = 0.7$.

From Fig. 6, the marked positions are located outside the contours of all ρ values when α is larger than 2/3, i.e., (0.7). When α is low such as 0.3, the marked position are located inside the contour of $\rho = 10$ dB as shown in Fig. 5. This matches with the results of the Fig. 3. However as ρ increases, the marked positions move outside. For a specific α value, the exact ρ value of the boundary between the two regions (OAF and NAF) can be obtained from (23) with modification of the approximate gap of network SNR.

6. Conclusions

In this paper, we compare the two different AF relaying protocols in terms of average throughput and relay position. For the average throughput, it was shown that OAF may perform better in the range of a certain network SNR ρ and power allocation ratio α . This fact cannot be directly obtained from the existing asymptotic results. We then obtain the lower bound of exact relation curve for (α, ρ) using the Jensen's inequality and a numerical method. This lower bound may work as a sufficient condition for which OAF has better performance. By observing the empirical SNR gap between lower bound boundary and the exact boundary curves, we derive an approximate relationship between the power allocation ratio and the network SNR when OAF and NAF are equally preferred. We apply the results to a path loss channel model where the relay position which prefers OAF is determined. The derived boundary between the OAF and the NAF regions can be a useful tool to adapt the relaying protocol according to the SNR and the relay location, which improves the average throughput of cooperative relay systems.

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