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단일 및 다중 라이시안 페이딩 채널에서 M-PSK 변조기술에서의 BER 유도

(BER Derivation of M-PSK Modulation Technique for Single and Multiple Racian Fading Channel)

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요 약

무선통신에서 페이딩은 피할 수 없는 문제이다. 그러므로 전송신호에 있어서는 BER 형태의 오류개념이 도입된다. 다른 페이딩 채널들 상에서 이러한 오류들의 동작을 인식하는 것이 필요하다. Coherent MPSK의 평균 BER에 대한 수학적인 해법을 얻기 위해서 몇 가지 기법들을 제안한다. 본 논문에서, 느리고 평탄한 라이시안 페이딩 채널 상에 diversity의 영향도 또한 분석되어진다. 여기서, 변조 지표 값 M은 변화하고 이 변화의 효과들 또한 묘사되어진다. 다양한 diversity 값과 페이딩 파라미터에 따른 성능 곡선들은 믿을 수 있는 통신 시스템을 위하여 무선 채널을 설계하고 평가하는데 유용하다.

Abstract

In wireless communication system, fading is an unavoidable problem. Hence, errors in form of BER are introduced with the transmitted signal. It is necessary to recognize the behavior of these errors in different fading channels. To obtain the mathematical solution for the average bit error rate(BER) of coherent MPSK, some techniques are presented. In this paper, the impact of diversity is also analyzed over slow and flat Rician fading channel. In here, the value of modulation index, M is varied and the effects of its variation are also depicted. So, these performance curves with different diversity values and fading parameter are useful to design and evaluate the radio channel for faithful communication system.

Keywords : Bit Error Rate, M-PSK, Rician, Fading Channel, Diversity, PDF

Ⅰ. Introduction

The behavior of wireless channel is time-variant. So it is necessary to know the approximate channel status or activities for a good communication system. The quality of reception is degraded due to the effect of multipath fading and fluctuating the signal at the receiver end. The fading statistics of different radio channels are widely investigated and channel models are developed to characterize the behavior of such channels.

For GSM 800/900 MHz radio channel, some experiments show that at any fixed terminal, the temporal envelope fading is best fit by a Rician distribution with the Rician parameter k , i.e., the specular to scatter ratio which varies between 6 to 12 dB[1]. In [1], a slow, flat, Rician fading performance analysis for differential phase shift keying (DPSK) transmission is developed. In [2], the error probability for multiphase signaling is derived

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over a slow, frequency-nonselective, Rayleigh fading channel. In [3], the performance of binary coherent and non-coherent multi receiver for M equal-energy equiprobable orthogonal waveforms is demonstrated over Rician fading channel. The result doesn't extend to the M-ary case for coherent reception. In addition, Roberts in [4] derived an expression for the bit error probability (BER) of non coherent DPSK for Rician fading channel, but he didn't extend this work up to the M-ary case. For this reason, it is appreciable work to establish a general form of error rate equation for PSK modulation over Rician fading channel.

In this paper, a closed-form solution of PSK performance in a Rician fading channel is presented and it includes AWGN and Rayleigh fading as special cases. This method is applied to both a coherent multiple phase shift keying (MPSK) system and a multiple differential phase shift keying (MDPSK) system. In first case slow and flat Rician fading is assumed here. In this fading environment, the duration of a symbol waveform is sufficiently short so that the fading variations cause negligible loss of coherence within each received symbol. At the same time, the individual waveform is assumed to be sufficiently narrow-band (sufficiently long in duration) so that frequency selectivity is negligible in the fading of its spectral components. As a result, the receiver can be designed and analyzed on the basis of optimal processing of the received waveform, e.g., by a matched filter or other appropriate substitute in the same manner used in the non fading case.

In this paper, the theoretical performance analysis starts with the probability of error for a non fading channel. Then, this probability of error is averaged over the additional effects of fading. Finally, the error rate performance is expressed as a function of the mean signal-to-noise ratio (SNR) and the fading factor.

In this paper, the probability density function (PDF) approach for calculating error rates of different modulation schemes over fading channel is discussed. In the analysis of linear combining, the statistics of the instantaneous SNR after combining are determined as a function of the fading parameter k, the mean instantaneous SNR, and the order N of the diversity for the identical communication link. Thus in the PDF approach, the error probability is a function of k or N of the system and this error rate can be calculated by averaging the conditional probability of error over the PDF of γ_b, [5], i.e.

$$P_e = \int_0^\infty P_e(\gamma_b)P(\gamma_b)d\gamma_t \tag{1}$$

where $P_e(\gamma_b)$ denotes the symbol or bit error probability of an additive white Gaussian noise (AWGN) channel and $P(\gamma_b)$ corresponds to the PDF of combiner output SNR in a specified fading environment.

Ⅱ. BER Derivation of M-PSK for Single and Multiple Rician Fading Channel

It is shown that a simple formula for the probability of symbol error for coherent MPSK is obtainable from the degenerate case of the statistics of the phase angle between two vectors perturbed by Gaussian noise[7]. An application of this method yields the probability of symbol error for coherent MPSK as follows[7]:

$$P_e(\gamma_b) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{(\frac{\pi}{2}-\frac{\pi}{M}} exp[-\gamma_b \sin^2(\frac{\pi}{M})\sec^2\theta]d\theta \tag{2}$$

It is known that phase-coherent communication performs better compared to non-coherent mode of transmission. However, in fading environment signals are interfered with each other i.e., there are some constructive and destructive interferences and as a result deep fades are formed. For this reason it is very difficult for perfect synchronization in receiving end. But, still in phase shift keying modulation technique, reliable phase information should be necessary and it is done by adding the pilot tone with transmission. Thus, it is obviously required to evaluate the performance of coherent M-PSK over a fading channel.

Now the probability density function of the Rician envelope is given by [5~6].

$$P(z) = \frac{2z(K+N)}{\Omega}[\frac{N+K}{\Omega K}]^{\frac{N-1}{2}} z^{(N-1)} e^{(-k-\frac{(K+N)z^2}{\Omega})} \times I_{N-1}(2z\sqrt{\frac{k(k+N)}{\Omega}}) \qquad (3)$$

where

k = Rician parameter = $\sum_{l=1}^{N} k_l = \frac{A^2}{2\sigma^2}$

Ω = Average power of the Rician envelope

= $E\{z^2\} = A^2 + 2\sigma^2 = 2\sigma^2(1+K)$

I_{N-1} = Bessel's function of the order of N-1

N = Number of channels at the receiver

Now this density function is transformed to an expression where random variable is the instantaneous signal to noise ratio, $\gamma_b = z^2\frac{E_b}{N_0}$ instead of the signal amplitude.

$$\Gamma = E\{z^2\}\frac{E_b}{N_0} = \Omega\frac{E_b}{N_0} \qquad \therefore \Omega = \Gamma\frac{N_0}{E_b}$$

$$z = \sqrt{\frac{\gamma_b N_0}{E_b}}$$

Therefore, placing the values of Ω and z into (3), we can derive the probability density function of instantaneous signal to noise ratio over Rician fading channel which is,

$$P(\gamma_b) = \frac{(k+N)}{\Gamma}[\frac{(N+k)\gamma_b}{k\Gamma}]^{\frac{N-1}{2}} e^{\frac{-[(N+k)\gamma_b + k\Gamma]}{\gamma\Gamma}} I_{N-1}(\sqrt{\frac{4k(k+N)\gamma_b}{\Gamma}})$$

Putting the values of $P_e(\gamma_b)$ and $P(\gamma_b)$ into (1) we find that, the probability of bit error for MPSK and the equation is given below,

$$P_e = \frac{1}{\pi}\frac{(k+N)}{\Gamma}[\frac{k+N}{k\Gamma}]^{\frac{N-1}{2}} e^{-k}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}\int_0^{\infty} e^{[-(\gamma_b \sin^2(\frac{\pi}{M})\sec^2\theta + \frac{k+N}{\Gamma})]} \times \gamma_b^{\frac{N-1}{2}} I_{\frac{2(N-1)}{2}}(2\sqrt{\frac{k(k+N)\gamma_b}{\Gamma}})d\theta d\gamma_b \qquad (4)$$

Again we can write an integral relationship from [8] and it is shown below

$$\int_0^{\infty} x^v e^{-\alpha x} I_{2v}(2\beta\sqrt{x})dx = \alpha^{-(2v+1)}\beta^{2v}\exp\left(\frac{\beta^2}{\alpha}\right) \qquad (5)$$

Therefore using the relation in (5) we find

$$P_e = \frac{1}{\pi}\frac{(N+k)}{\Gamma}[\frac{N+k}{k\Gamma}]^{\frac{N-1}{2}} e^{-k}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}}[\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma}]^{-N}[\frac{k(N+k)}{\Gamma}]^{\frac{N-1}{2}} \times \exp(\frac{k(N+k)}{\Gamma(\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma})})d\theta$$

$$P_e = \frac{1}{\pi}[\frac{(N+k)}{\Gamma}]^N e^{-k(N+k)} e^{k(N+k)} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{\exp[\frac{-k\sin^2(\frac{\pi}{M})\sec^2\theta}{\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma}}]}{[\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma}]^N}d\theta$$

$$P_e = \frac{1}{\pi}[\frac{(N+k)}{\Gamma}]^N \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}-\frac{\pi}{M}} \frac{\exp[\frac{-k\sin^2(\frac{\pi}{M})\sec^2\theta}{\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma}}]}{[\sin^2(\frac{\pi}{M})\sec^2\theta + \frac{(N+k)}{\Gamma}]^N}d\theta \qquad (6)$$

Substituting N = 1 in equation (6), we find the bit error probability of MPSK over single Rician fading channel. Substitution of M = 2 in (6) yields the probability of bit error for BPSK over multiple Rician fading channel. For M = 2 and N = 1, equation (6) represents probability of bit error for BPSK over single Rician fading channel.

Ⅲ. Simulation Results

Using (6) we find the figures from 1 to 10 for MPSK modulation technique with and without diversity. Here the BER of BPSK modulation over slow, flat, i.i.d. fading channels for various values of K and N are illustrated in Fig. 1-3. If we increase the diversity then the effect of fading is gradually decreased. It is shown in Fig. 1 that the curve for AWGN channel shows better response compared to others. On the contrary Rayleigh fading channel shows worse response curve.

In Fig. 1, it shows that any fading channel is worse than non-fading AWGN channel. Rician fading channel lies between Rayleigh and AWGN channel. When Rician parameter k increases from 0 to infinity, it comes from Rayleigh to AWGN channel. In Fig. 2, it is shown that the value of BER is zero at approximately 40 dB SNR for the channels those are close to Rayleigh fading.

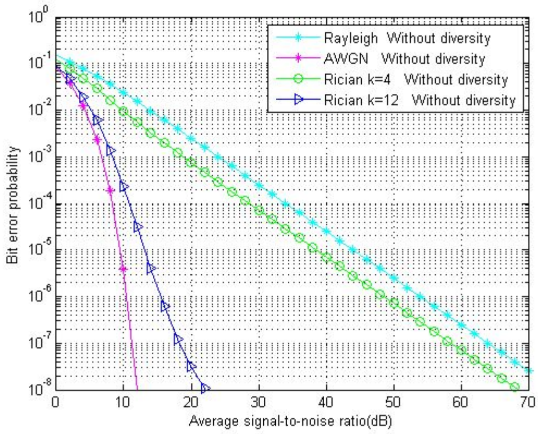

Fig. 1. AWGN, 랠리 및 라이시안 채널 상에서 N = 1 (다이버시티 존재)인 BPSK 경우의 BER

Fig. 1. BER of BPSK for N = 1 (without diversity) over AWGN, Ralyeigh and Rician channel.

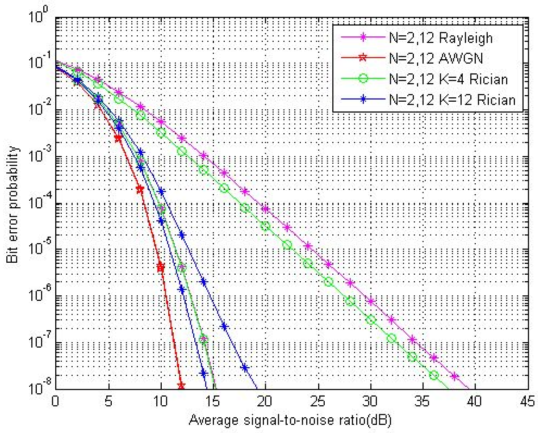

Fig. 2. AWGN, 랠리 및 라이시안 채널 상에서 N = 2, 12 (다이버시티 미존재)인 BPSK 경우의 BER

Fig. 2. BER of BPSK for N = 2, 12 (with diversity) over AWGN, Ralyeigh and Rician channel.

Increasing the diversity shows a better performance. For lower values of k, Rician channel acts like Rayleigh channel. In from Fig. 2 to Fig. 3, it is shown that the diversity N has no effect on AWGN channel. For a high diversity N = 12, the fading channel curves are vicinity to AWGN and shows almost the same performance. For N = infinite, there is no fading effect (Rayleigh and Rician fading channel) as like AWGN.

For a particular BER, Rayleigh channel requires more signal to noise ratio than any other channel. But in Rician channel, it also depends on k. As k increases it gives better performance. So, in Fig. 3, it is shown that, for a particular BER, for k = 6 (Rician) without diversity acts as like as for k = 0 (Rayleigh) with diversity N = 2. Similarly for k = 12 in Rician without diversity acts a better performance than k = 0 (Rayleigh) and k = 6 (Rician) with diversity N = 2.

Now the response for QPSK modulation are depicted with varying the different values of N. In wireless communication system, diversity and bit

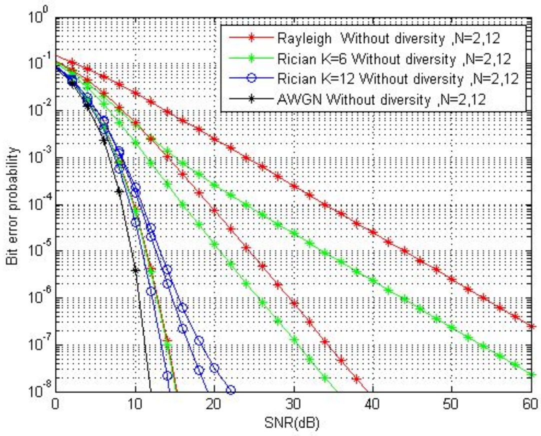

Fig. 3. AWGN, 랠리 및 라이시안 채널 상에서 N = 2, 12 (다이버시티 존재 및 미존재)인 BPSK 경우의 BER

Fig. 3. BER of BPSK for N = 2, 12 (with and without diversity) over AWGN, Ralyeigh and Rician channel.

error rate are very much related to each other. Space diversity is important to reduce the fading effect.

The BER performance of QPSK modulation over slow, flat, i.i.d. fading channels for various values of k and N are illustrated in Fig. 4, Fig. 5 and Fig. 6. In these figures, the effects of M and N are similar as previous described frames. From these figures, it can be said that in practical design consideration, the effect of fading on channel can not be removed absolutely. So by increasing the number of antennas, the fading effect is removed significantly. But comparing these three figures with previous figures, BER performance of BPSK is superior to that of QPSK. It means BPSK needs less SNR than QPSK at a fixed bit error probability.

From Fig. 7, it is shown that, in AWGN channel, it acts the same performance either for N = 1, = 2 or = 12 for a fixed modulation index M. If the index, M is increased, the performance degrades. It indicates that increasing the transmitting bits needs a high value of SNR for a particular value of bit error rate. If M is increased, it is more vulnerable to noise.

In Fig. 8-10, it can be shown that, increasing the

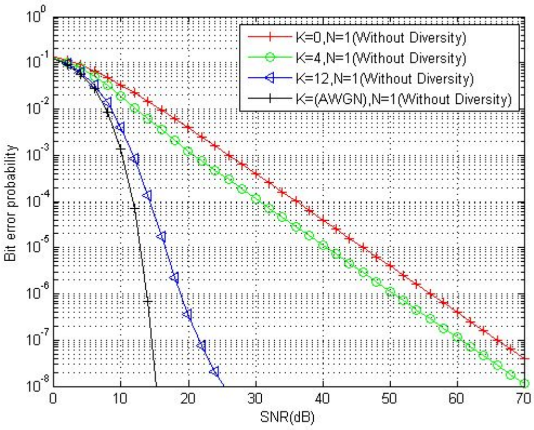

Fig. 4. AWGN, 랠리 및 라이시안 채널 상에서 N = 1 (다이버시티 미존재)인 QPSK 경우의 BER

Fig. 4. BER of QPSK for N = 1 (without diversity) over AWGN, Ralyeigh and Rician channel.

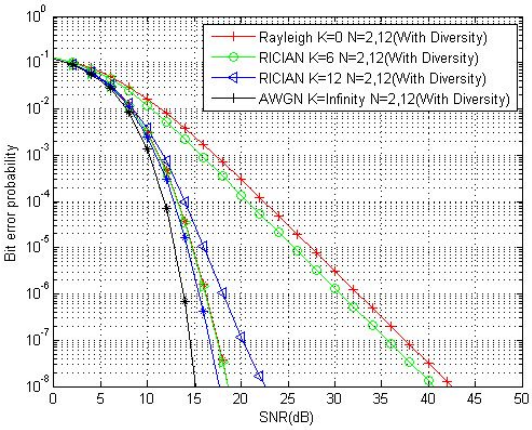

Fig. 5. AWGN, 랠리 및 라이시안 채널 상에서 N = 2, 12 (다이버시티 존재)인 QPSK 경우의 BER

Fig. 5. BER of QPSK for N = 2, 12 (with diversity) over AWGN, Ralyeigh and Rician channel.

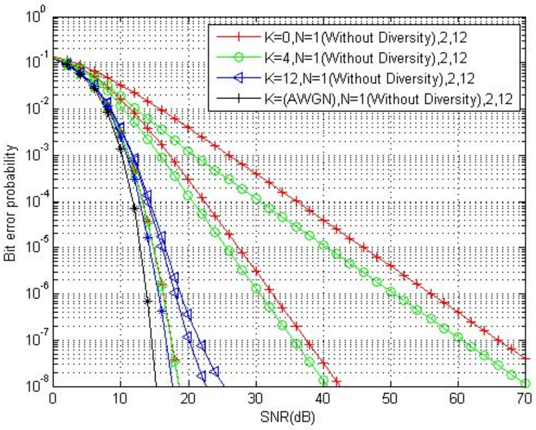

Fig. 6. AWGN, 랠리 및 라이시안 채널 상에서 N = 1, 2, 12 (다이버시티 존재 및 미존재)인 QPSK 경우의 BER

Fig. 6. BER of QPSK for N = 1, 2, 12 (with and without diversity) over AWGN, Ralyeigh and Rician channel.

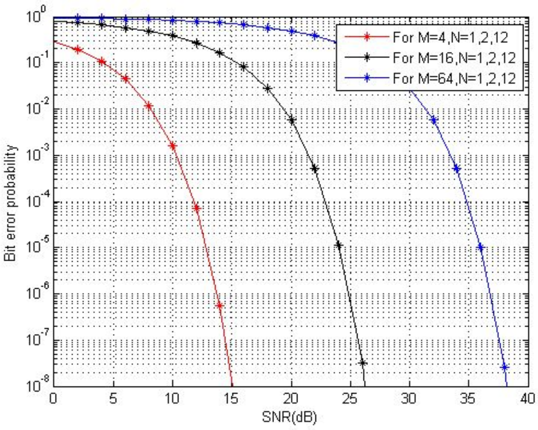

Fig. 7. N = 1, 2, 12인 AWGN에서 MPSK (M = 4, 16, 64) 경우의 BER

Fig. 7. BER for MPSK (M = 4, 16, 64) in AWGN for N = 1, 2, 12.

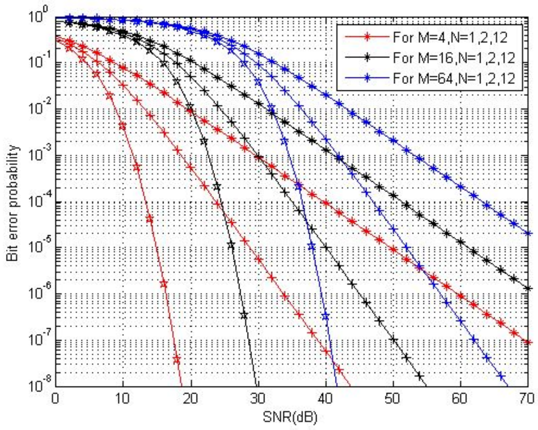

Fig. 8. N = 1, 2, 12인 랠리에서 MPSK (M = 4, 16, 64) 경우의 BER

Fig. 8. BER for MPSK (M = 4, 16, 64) in Rayleigh for N = 1, 2, 12.

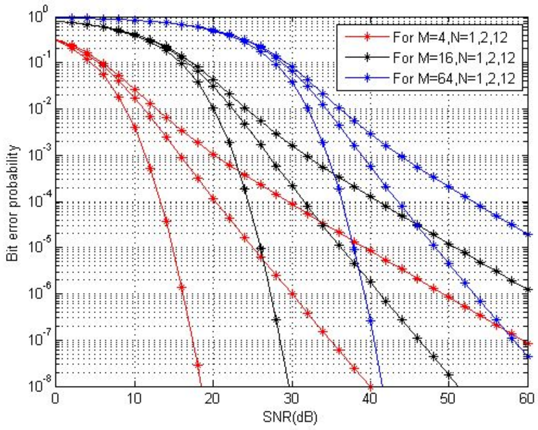

Fig. 9. N = 1, 2, 12인 라이시안에서 MPSK (M = 4, 16, 64) 경우의 BER

Fig. 9. BER for MPSK (M = 4, 16, 64) in Racian for N = 1, 2, 12.

diversity N, increases the system performance for a particular value of M in fading channel like Rayleigh or Rician. For a fixed N, the MPSK (M = 4, 16, 64) behaves always the same. Moreover, comparing these three curves, it is found that, Rayleigh fading is the worst channel. For a fixed M, it is found that, higher diversity (in here, N = 12) performance is very close to AWGN (N = Infinity diversity).

Again, in Rician channel for a fixed M the system performance is also dependent on k (Rician parameter) for a lower diversity or without diversity. As k increases, the fading characteristic decreases for a fixed value of diversity N and modulation index M. Or, as diversity N increases for a fixed M and k the fading behavior also decreases.

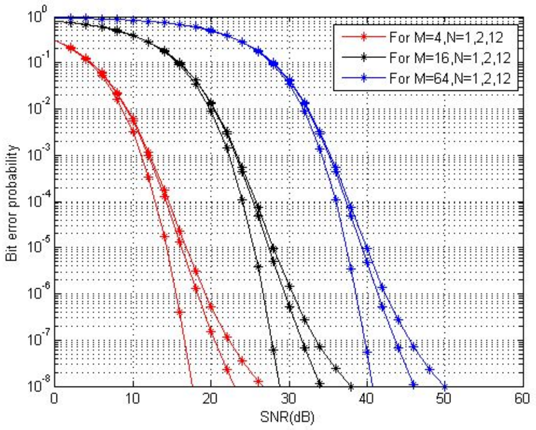

Fig. 10. N = 1, 2, 12인 라이시안(K = 12)에서 MPSK (M = 4, 16, 64) 경우의 BER

Fig. 10. BER for MPSK (M = 4, 16, 64) in Racian(K = 12) for N = 1, 2, 12.

Ⅳ. Conclusions

In this paper, we obtained simple closed form expression of bit error rate (BER) to determine the performance of M-PSK modulation technique transmitted overslow, flat, identically independently distributed i.i.d fading channels by using space diversity. There are two fading channels considered as Rayleigh and Rician. In here, we tried to make a relationship between modulation index M and diversity order N, and fading parameter(k for Rician). Obviously, different curves of BER versus SNR are plotted by varying the above parameters. According to the above discussion the following points should be noted.

- As k increases, the fading effect reduces and reducing fading effect decreases the bit error probability in MPSK modulation scheme over slow, flat fading channel.
- Rayleigh fading channel shows worst performance comparatively with other fading channels on MPSK performance.
- Diversity N has no effect on AWGN channel.

- Fading effect reduces when data rate or modulation index of PSK decreases.
- BPSK shows better BER performance than QPSK over fading channels.
- 4-PSK has better performance than 16 or 64 PSK over fading channels.

The work of this paper can be extended by analyzing the response under other fading characteristic like Nakagami-m and non identical channel.

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