한국지반공학회논문집 제28권 4호 2012년 4월 pp. 35 ～ 42

Seepage Face and Reliability Indexes of Anisotropic Homogenous Dam at Steady State Condition

비등방 균질 댐의 정상상태에서의 침투면과 신뢰성지수

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요 지

균질한 재료로 설계된 댐에서 수직 수평 투수계수가 서로 다를 경우에 이 비등방성 비율이 침투수의 침투면과 신뢰성지수에 어떤 영향을 주는지를 분석하였다. 정상상태에 있는 포화-불포화 침투수 현상을 유한요소법을 사용하여 분석하였다. 수평방향 투수계수를 고정한 상태에서 수직방향 투수계수만을 줄여주는 방법으로 여러가지 다른 비등방성 비율을 해석하였다. 댐제체의 전단강도는 수정된 모어-쿨롱 파괴기준을 사용하여 결정하였다. 해석 결과에 따르면 비등방성 비율이 증가함에 따라 침투면의 거리와 신뢰성지수는 증가하는 것으로 나타났으나 댐 상하류의 전체 수두차에 따라 차이를 보였다. 이러한 침투면과 신뢰성지수의 차이는 비등방성 비율이 변동함에 따라 등가수두선도 변화하는 현상에 의한 것으로 판단된다.

Abstract

This paper evaluates the effect of anisotropic conductivity on the seepage face and reliability index of an homogeneous dam with and without toe drain. The analysis are conducted under steady state saturated-unsaturated seepage condition using finite element method. Various anisotropic conductivity ratios were interpreted under such conditions as the vertical conductivity is reduced while the horizon conductivity is fixed. The shear strength of soil is defined by the modified Mohr-Coulomb failure criterion. The analysis results demonstrate that the length of seepage face and reliability index at the downstream and upstream of the dam increase with an increasing anisotropic ratio. These results of the seepage face and reliability index, however, depend on the total head difference between the upstream slope and downstream toe. The difference in seepage face and reliability index is attributed to the different equipotential head with different anisotropic ratios of the dam material.

Keywords : Seepage face, Anisotropic conductivity, Saturated-unsaturated seepage, Homogenous dam, Finite element method

1. Introduction

Seepage faces occur commonly in unconfined groundwater flows. Dams and dikes, whatever their types, experience some seepage, even if they can be made relatively impermeable by using either excavated materials with or without some treatment or other selected, possibly artificial materials (Robert et al., 2001). Accurate estimation

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* 본 논문에 대한 토의를 원하는 회원은 2012년 10월 31일까지 그 내용을 학회로 보내주시기 바랍니다. 저자의 검토 내용과 함께 논문집에 게재하여 드립니다.

of seepage face height is important in investigating the stability of porous structures (Freeze and Cherry, 1979). A complication in modeling unconfined ground water flow is the need to simulate seepage flow when the water table intersects the land surface (Knupp, 1996). Even when ground water seepage is not apparent in the field, the existence of a seepage face is necessary to provide a physical transition between a phreatic surface and equipotential boundary when the water table approaches a water body (Bear 1972). Another important issue regarding steady state seepage is the stability of upstream slope. The transient numerical analyses such as drawdown, infiltration etc are always based on the initial steady state condition. The accuracy of consecutive results of transient analysis thus depends on the accurate establishment of initial steady state condition. The hydraulic conductivity of dam material is the only sole property that affects this steady state condition. The effect of this important parameter for isotropic case has been well established for seepage faces and slope stability. However for anisotropic case the literature is very rare specifically for steady state condition. This paper therefore evaluates the effect of anisotropic hydraulic conductivity condition on seepage face and stability of dam.

This paper presents two simple example problems of homogenous dam with and without toe drain for which the authors performed finite element (FE) calculations. The numerical calculations are based on the principle of saturated-unsaturated flow problem that takes into account both positive and negative pore-water pressures and thus speicifies the location of water-table more precisely. The problems are studied for different case of anisotropic condition using different hydraulic conductivity of dam material.

2. Example Problem

2.1 Geometry and material properties of dam

The geometry and finite element discretization of homogeneous dam with and without toe drain along with definition of parameters are given in Fig. 1 and Fig. 2 respectively. It is worth noting that the toe drain material as shown in Fig. 2b has been left out of the mesh. The reason is that this material is very coarse compared to the dam material and thus has much higher hydraulic conductivity. Thus it is assumed that there will be little or no head loss in this material, so there is no reason to include it in the mesh. Leaving it in the mesh would only add to numerical convergence difficulty, as this type of material has a high non-linear conductivity function.

The seepage line is the uppermost level of water within the dam, where the water emerges along a seepage face (shown as “a” in Fig. 1a and Fig. 2a). At both seepage line and seepage face the water is at atmospheric pressure. The steady state seepage analysis in this paper has been carried out using the finite element software Seep/W (Geo-Slope 2007) at different head △h. The △h is defined as the difference in hydraulic head between reservoir and downstream toe of the dam.

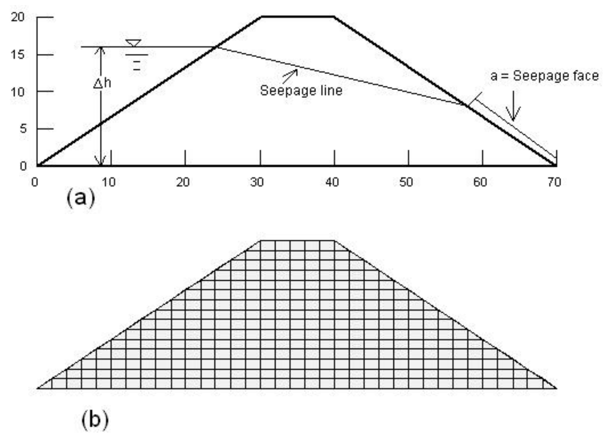

Fig. 1. Homogeneous dam without toe drain: (a) Geometry: (b) FE discretization

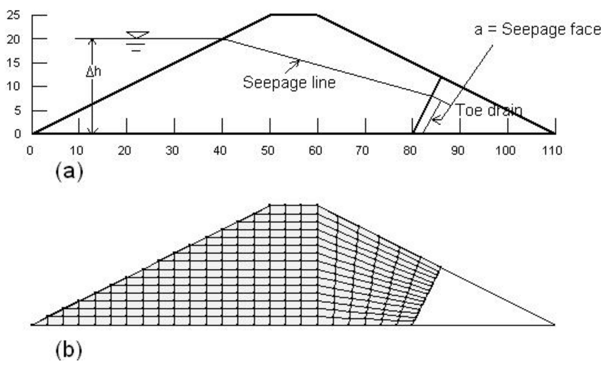

Fig. 2. Homogeneous dam with toe drain: (a) Geometry: (b) FE discretization

Table 1. Material properties used in present paper

Properties	Mean	*COV	Min	Max	*PDF	*ρ
Effective cohesion c'(*kpa*)	10	0.2	7	13	Normal	
Effective friction angle φ'(°)	30	0.1	25.5	34.5	Normal	−0.25
Unit weight γ (kn/m^3)	18	−	−	−	−	−

*COV=Coefficient of variation; *PDF=Probability distribution function; *ρ=Correlation coefficient between c' and φ'

Table 2. Saturated hydraulic conductivities in x direction and SWCC parameters

Soil	Sat. Conductivity in x−direction (m/s)	SWCC Parameter			
		a (kPa)	n	m	*θ_s
Soil 1	1×10^{-4}	10	2	1	0.45
Soil 2	1×10^{-5}	50	2	1	0.45
Soil 3	1×10^{-6}	100	2	1	0.45

*θ_s=Saturated Volumetric Water Content

The material properties along with probability distribution characteristics are given in Table 1.

2.2 Steady State Saturated-Unsaturated Analysis

The geological formation of residual soil is mostly in distinctive layers that may have different conductivity in different direction. The range of anisotropic ratio for a wide variety of cohesive and cohesionless soil is available in the paper of Robert and Denis (1989). Most of the dam embankments are constructed in stages and in layers and thus, can result in different hydraulic conductivities in different directions. The assumption of isotropic hydraulic conductivity may thus lead to wrong interpretation of results for anisotropic soil when subjected to seepage condition. This situation is dangerous and can lead to uncertainty in slope or embankment in terms of stability. The failure of Gouhou dam in China is one of the least examples. The preliminary analysis of the causes of the break of this dam shows that, the horizontal seepage coefficient is 45 times larger than that of the vertical (Chen and Zhang, 2006).

The anisotropic ratio in this paper is defined as (k_x/ k_y), where, k_x is the hydraulic conductivity in x (horizontal) direction and k_y is the hydraulic conductivity in y (vertical) direction. The finite element seepage program (Seep/W 2007) evaluates the hydraulic conductivity in vertical direction for fixed horizontal conductivities and known anisotropic ratio. In the present paper the anisotropic ratios are set to 1, 5, 10, and 20 for three different soils for which the fixed saturated horizontal conductivity are given in Table 2.

The soil water characteristic curves (SWCCs) for three kinds of soil are shown in Fig. 3a. These curves that define the relationship between volumetric water content θ_w and pore-water pressure u_w have been established using Fredlund and Xing (1994) method for known parameters given in Table 2. It is important to note that the SWCC is not important for steady state problem. However in the present case these curves are used to evaluate the hydraulic conductivity versus pore- water pressure relationship (hydraulic conductivity function). The hydraulic conductivity functions for each soil are constructed from its respective SWCC according to Fredlund and Xing (1994) method with known saturated hydraulic conductivity (Table 2). The hydraulic conductivity function for three soils is given in Fig. 3b. It is important to note that these are the horizontal hydraulic conductivity functions. In case of the same SWCC the hydraulic conductivity function in anisotropic direction has the same curve shape with only the difference of saturated hydraulic conductivities (Freez and Cherry, 1979). Thus the vertical conductivity function has the same curve shape as that of horizontal conductivity function with saturated vertical hydraulic conductivity.

The different head conditions adopted for homoge-

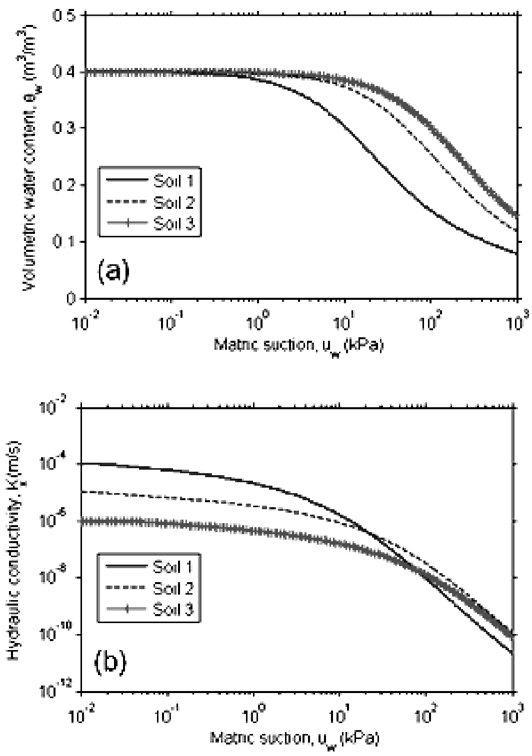

Fig. 3. (a) Soil water characteristic curves (SWCCs): (b) Hydraulic conductivity functions

Table 3. Head difference used in present paper

Homogeneous dam	Δh in meter
Without toe drain	17.5, 15, 12.5, 10, 7.5, 5, 2.5
With toe drain	20, 17.5, 15, 12.5, 10, 7.5, 5

neous dam with and without toe drain are given in Table 3.

2.3 Reliability Analysis

The pore-water pressure profile generated in the finite-element seepage analysis is imported into the limit-equilibrium software Slope/W (Geo-Slope 2007). In limit-equilibrium software the shear strength in the saturated and unsaturated zone is based on the empirical relationship proposed by Frendlund et al. (1978). This relationship, a modified form of the Mohr-Coulomb criteria, is defined as:

$$\tau = c' + (\sigma_n - u_a)\tan\varphi' + (u_a - u_w)\tan\varphi^b \qquad (1)$$

where $(\sigma_n - u_a)$ is the net normal stress; u_a the pore air pressure; u_w the pore water pressure; φ^b the angle expressing the rate of increase in shear strength relative to matric suction.

The φ^b depends on matric suction in the unsaturated zone. The limit equilibrium software has the ability to evaluate the value of φ^b if the SWCC of material is known. The value of φ^b in this paper is thus evaluated for each soil according to its SWCC.

The terms on the right side of Eq. 1 add cohesive, frictional and matric suction strength respectively. It is worth noting that the pore air pressure u_a is usually set to zero value. In case of full saturation when the soil is below flow-line (water table) Eq. 1 reduces to the well known Mohr-Coulomb failure criteria. In this full saturated zone the pore-water pressure affects the frictional strength and thus the shear strength of soil. In case of partial saturation when the soil is above flow-line the cohesion and frictional strength are not influenced by pore-water pressure, however additional shear strength is introduced in the form of matric suction strength. Thus it is obvious that in case of soil below and above flow-line the pore-water pressure affects the shear strength of soil as a change in frictional strength and matric suction strength respectively.

The mean safety factor of the upstream slope is evaluated using the Morgenstern-Price method with the same grid and radius for the critical slip surface. The reliability index of slope is evaluated using a total number of 20,000 Monte-Carlo simulations. This number is established on repeated runs and found sufficient to converge the reliability index results. The reliability index in this paper is obtained according to the following relationship.

$$\beta = \frac{(\mu_F - 1)}{\sigma_F} \qquad (2)$$

where β is the reliability index; μ_F the mean safety factor; and σ_F the standard deviation of the safety factor.

3. Analysis Results

The seepage face on the downstream side in term of head difference is shown in Fig. 4. These results have been evaluated at different anisotropic ratio of three soils

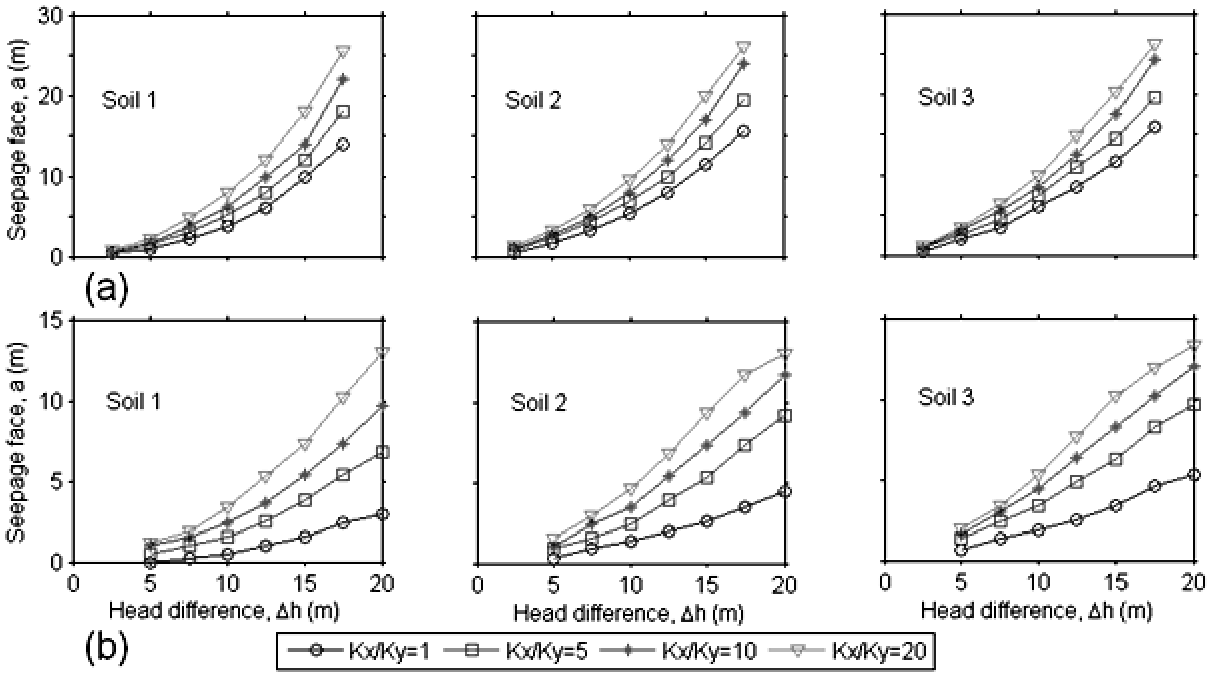

Fig. 4. Seepage face in term of head difference (a) Dam without toe drain: (b) Dam with toe drain

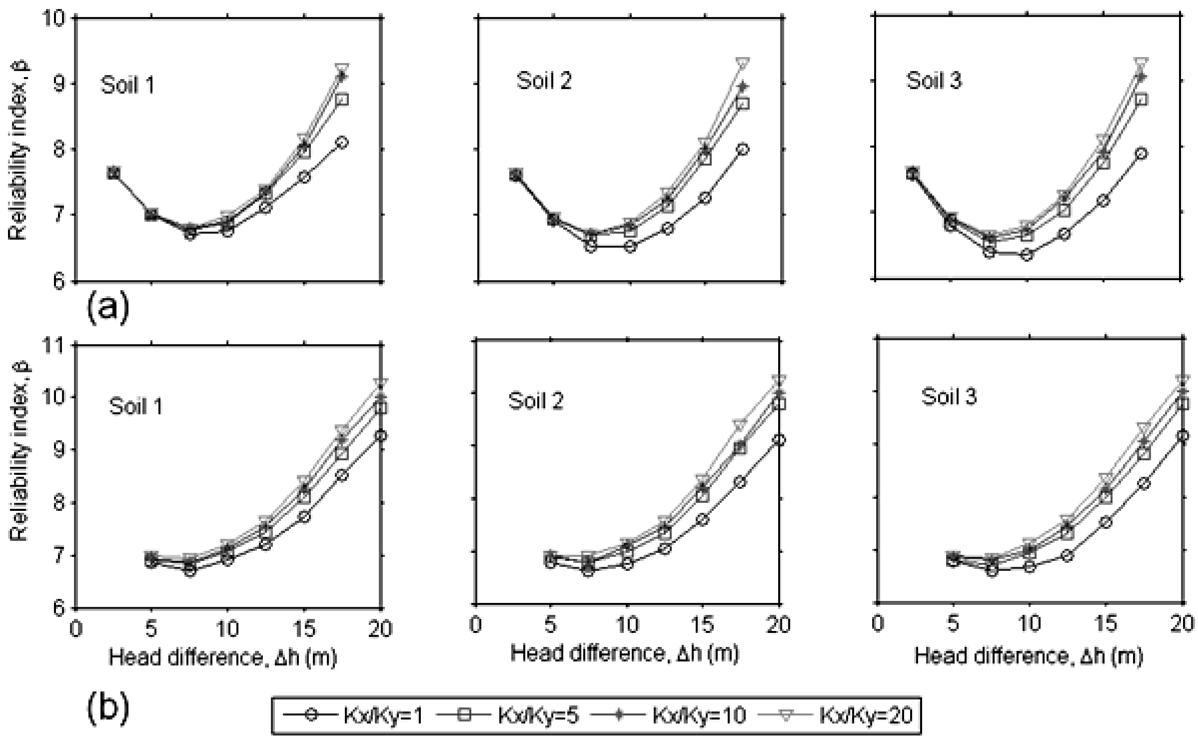

Fig. 5. Reliability index in term of head difference for upstream slope of dam (a) Dam without toe drain: (b) Dam with toe drain

for dam with and without toe drain. The results show that the length of seepage face on the downstream slope depends on the anisotropic ratio of dam material. The discrepancy in results is higher for high head difference value and vice versa. The reason for different seepage face in terms of anisotropic ratio is further explained in the discussion part of this paper.

The reliability index in terms of head difference for dam with and without toe drain is shows in Fig. 5. Fig. 5 shows that, the reliability index for upstream slope of dam has different value at different anisotropic ratio. This discrepancy in reliability indexes however depends on the head difference. At certain high head difference the discrepancies in reliability indexes are higher for different anisotropic ratio of dam material. However this difference decreases with decreasing head difference and at a certain lower value practically the same reliability indexes are obtained with different anisotropic ratio. The reason for the effect of different anisotropic ratio on the reliability index of upstream slope of dam has fully explained in the discussion part of this paper.

4. Discussion

The pressure head or pore-water pressure at a specific location in dam at steady state condition is established based on the following relationship.

$$\frac{u_w}{\gamma_w} = h - z \qquad (3)$$

where u_w is the pore-water pressure; h is the equipotentials head; z is the elevation head and; γ_w is unit weight of water.

In finite element seepage analysis, the anisotropy ratio physically means that the material is perfectly stratified. It assumes further that, all layers are the same in thickness and extends throughout the problem domain. The anisotropic ratios of 1, 5, 10, and 20 mean that, the hydraulic conductivity in horizontal direction is greater by these numbers than that of vertical direction. Thus with increasing anisotropic conductivity ratio the flow tends to be more lateral than vertical. It has been well established in geotechnical and hydrological engineering that the shape of flow nets is different in isotropic and anisotropic case. The steady state seepage analysis in the present study shows that different equipotential lines are obtained with different anisotropic ratio. The equipotential lines shift more to the upstream and downstream slope respectively below and above the flow-line (phreatic surface) as the anisotropic ratio increases. This situation thus results in different equipotential head at a specific distance from upstream slope that is further clarified in Fig. 6 and Fig.

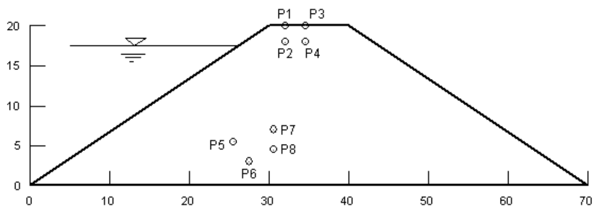

Fig. 6. Points for evaluation of equipotential head at different anisotropic ratio

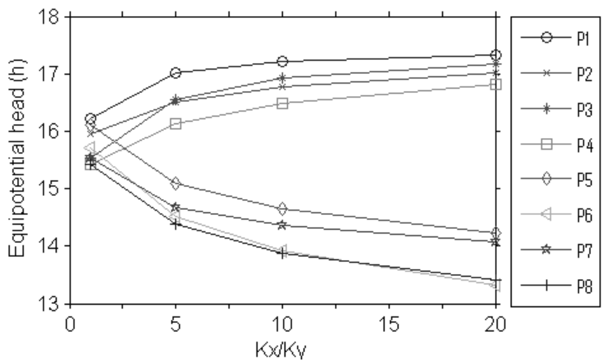

Fig. 7. Values of equipotential head at different points of figure 6 in term of anisotropic ratio

7. Fig. 6 shows different points above and below the flow-line in dam (without drain). At these points equipotential head has been evaluated at a head difference of 17.5 m for different anisotropic ratio of soil 1. The evaluated equipotential heads for these specific points are shown in Fig. 7. This figure shows that for different anisotropic ratio different values for equipotential head are obtained below and above the flow-line. In case of points above and below the flow-line the equipotential head increases and decrease with increasing anisotropic ratio and vice versa. According to the definition of seepage face it is a belt on the downstream of the slope along which the water emerges at atmospheric pressure (zero head pressure) and flows down the slope. The increasing of seepage face with anisotropic ratio is thus understandable in terms of Eq. 3. This equation shows that as the equipotential head increases there requires more elevation head to get the zero pressure head. The elevation head for this geometry of slope can be related to the seepage face according to the simple trigonometry rules as below

$$seepage\ \ face = a = 1.8 \times z \qquad (4)$$

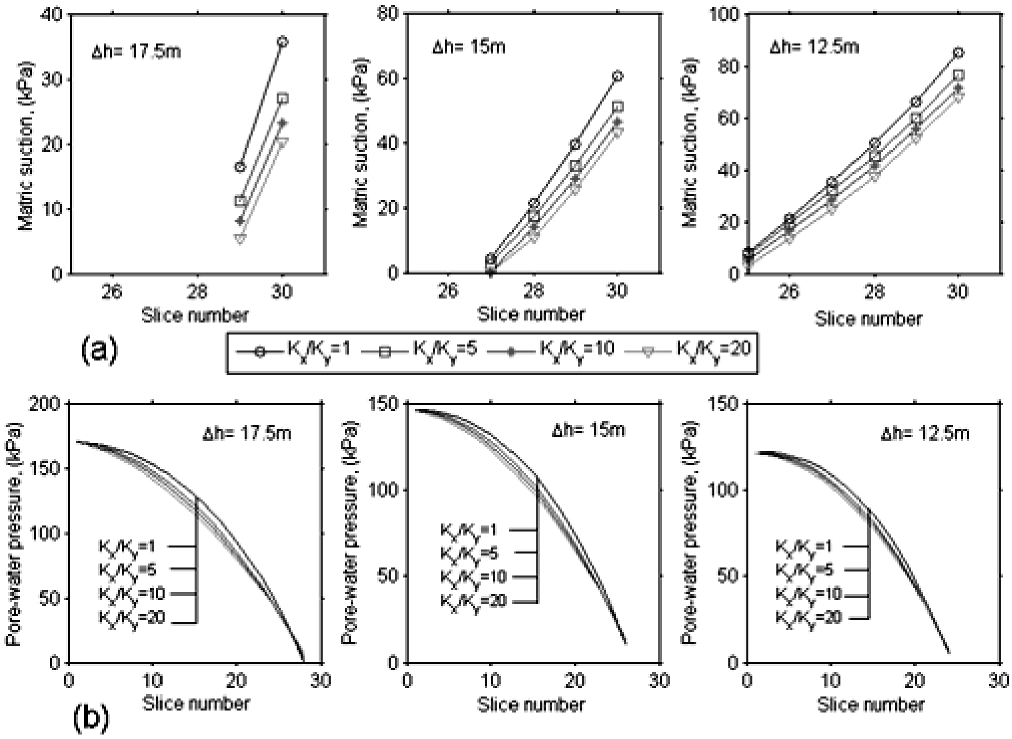

Fig. 8. Pore-water pressure at the slice surface at different anisotropic ratio for soil 1 (a) △h = 17.5m: (b) △h = 15m: (c) △h = 12.5m

The above Eq. 4 thus shows that the length of seepage face is proportional to the elevation head z.

It is worth noting that elevation head here means the most upper point on the downstream slope where the pressure head is zero.

Figure 8 shows the evaluated pore-water pressure and matric suction at the slices of slip surface for different anisotropic ratio. These results have been evaluated for dam without toe drain using head difference of 17.5 m, 15 m and 12.5 m. Figure 8 shows that, the matric suction above and pore-water below the flow-line decreases as the anisotropic ratio increases. It can be noticed that the depth of matric suction increases while that of pore-water pressure decreases with the decrease in elevation head. This is quite understandable because for fixed geometry of dam the portion of saturated zone decreases as the elevation head decreases and that of partial saturation (unsaturated zone) increases. Further, the difference in pore-water pressure for different anisotropic ratio also decreases as the elevation head decreases. This difference of matric suction and pore-water pressure results in terms of different anisotropic ratio is quite understandable in terms of previous discussion of Fig. 6 and Fig. 7 of this section.

At large head difference (Fig. 7a) the pore-water pressure in the saturated zone decreases with increasing anisotropic ratio. According to Eq. 1 the contribution of frictional strength in the saturated zone is thus more for high anisotropic ratio than low ratio. Further the matric suction in the unsaturated zone is lower for high anisotropic ratio than low ratio. The contribution of apparent cohesion due to matric suction in the unsaturated zone is thus low for high anisotropic ratio than low ratio. However the portion of saturated zone is quite larger than that of unsaturated zone and thus the discrepancy in stability results is mostly controlled by frictional strength in the saturated zone.

As the head difference decreases (Fig. 8b, Fig. 8c) the discrepancy in pore-water pressure in the saturated zone in terms of different anisotropic ratio also decreases. At the same time the portion of unsaturated zone increases. However it is noticeable that the pore-water pressure and matric suction respectively below and above flow-line are still lower for high anisotropic ratio than low ratio.

According to Eq. 1 the contribution of frictional strength and apparent cohesion due to matric suction are respectively higher and lower for high anisotropic ratio than low ratio. This situation thus results in little or no discrepancy in reliability indexes in term of different anisotropic ratio for dam material.

5. Conclusions

The effect of anisotropic conductivity ratio on the length of seepage face and reliability index of dam has been investigated using saturated-unsaturated seepage condition. The following main conclusions are drawn:

The length of seepage face at the downstream of dam increases with increasing anisotropic ratio and vice versa. The different results however depend on the pressure head difference between upstream slope and downstream toe of the dam. This discrepancy is clearer at high head difference and becomes negligible at low head difference.

The difference in seepage face in terms of anisotropic ratio is attributed to the increased equipotential head at a specific location above the seepage-line.

At certain high head difference the reliability index increases with increasing anisotropic ratio and vice versa. However beyond certain head difference practically the same reliability indexes are obtained with different anisotropic ratio of dam material.

The upstream slope stability or reliability index is controlled by the frictional strength and apparent cohesion due to matric suction strength as a result of change in water content. At high head difference the frictional strength mainly controls the stability of slope and is higher for high anisotropic ratio than low ratio. In case of low head difference the contribution of frictional strength and apparent cohesion due to matric suction are respectively higher and lower for high anisotropic ratio than low ratio. This situation thus results in little or no discrepancy in reliability indexes in terms of different anisotropic ratio for dam material.

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0014592).

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(접수일자 2011. 10. 4, 심사완료일 2012. 4. 19)