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Pressure Drop Reduction and Heat Transfer Increase with Rheological Fluid Flows in a Circular Conduit
원형 도관 내에서의 유변 유체에 대한 압력손실 감소 및 열전달 증가

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Key Words : 유변 유체(Rheological Fluid), 압력손실(Pressure Drop), 열전달 증가(Heat Transfer Increase)

Abstract : 본 연구는 컴팩트한 열교환기의 설계를 위하여 열교환기 내의 원형 단면 도관의 유변 유체의 압력강하 및 대류 열전달률을 수치해석적으로 수행하였다. 유변 유체의 구성방정식은 기존의 비뉴톤 유체 멱법칙을 보완한 수정 멱법칙 모델을 채택하였다. 도관 내의 압력강하를 의미하는 마찰계수와 수정 레이놀즈 수의 곱은 기존 문헌치와 비교할 때 뉴톤 유체 영역과 유변 멱법칙 영역에서 각각 0.01% 및 0.004% 내에서 일치함을 보였고 유변 수정멱법칙 유체 모델의 형태를 띠는 유변 유체를 열교환기 내의 원형 단면 도관 내에서 사용하면 뉴톤 유체보다 최대 58%의 압력강하를 감소시켰고 최대 9%의 대류 열전달 증가을 발생시킬 수 있었다.

Nomenclature

A_c: Cross-sectional area of conduit [m^2]
C_p: Specific heat [$J/kg \cdot K$]
d: Diameter of the conduit [m]
f: Darcy friction factor($-2(dp/dx)d/\rho\overline{u}^2$) [-]
$f \cdot Re_m$: Dimensionless pressure drop [-]
K: Power Law Consistency [Ns^n/m^2]
Nu : Nusselt number [-]
Nu_H : Nusselt number of CHF [-]
n: Power law flow index [-]
Re_d: Newtonian Reynolds number($\rho\overline{u}d/\eta_0$) [-]
Re_g: Power law Reynolds number ($\rho\overline{u}^{2-n}d^n/K$) [-]
Re_m: Modified power law Reynolds number ($\rho\overline{u}d/\eta^*$) [-]
T : Temperature [K]
T^+: Dimensionless temperature [-]
T^{++}: Dimensionless temperature [-]
T_B : Bulk temperature [K]
T_w : Wall temperature [K]
u : Velocity in flow direction [m/s]
$\overline{u}$: Bulk velocity in flow direction [m/s]
u^+: Dimensionless velocity in flow direction($u/\overline{u}$) [-]
u^{++}: Dimensionless velocity in flow direction ($2u^+/f \cdot Re_m$) [-]
r : Coordinate in radial direction [m]
r^+: Dimensionless coordinate in radial direction [-]
x : Coordinate in flow direction [m]
x^+ : Dimensionless coordinate in flow direction [-]

Greek letters

α : Thermal diffusivity [m^2/s]
β : Shear rate parameter($(\eta_0/K)(\overline{u}/d)^{1-n}$) [-]
$\dot{\gamma}$: Shear rate [$1/s$]
η_a: Apparent viscosity ($\tau/\dot{\gamma}$) [Ns/m^2]
η_0: Zero shear rate viscosity [Ns/m^2]

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η^*: Reference viscosity ($\eta_0/(1+\beta)$) [Ns/m^2]
η^+: Dimensionless viscosity (η_a/η^*) [-]
ρ : Fluid density [kg/m^3]
τ : Shear stress [Ns/m^2]

1. Introduction

Many attempts have been made in determining the pressure drop and heat transfer characteristics of commercial conduits in wide applications in engineering, especially in the design of compact heat exchangers. Consequently, extensive analytical and experimental investigations have been conducted on such heat and flow systems. The investigation of the laminar flow and heat transfer behavior in a circular conduit has been important as a result of the ongoing research of an advanced liquid cooling module for heat exchangers by a existing circular commercial conduits. For fully developed laminar flow of Newtonian and rheological power law fluids in a circular conduit, the solutions have been suggested for both the classical thermal boundary conditions of constant wall temperature (CWT) and constant wall heat flux (CHF) and the pressure drop.

For Newtonian fluids, pressure drop and heat transfer coefficients were calculated by Shah and London[1]. For rheological power law fluids, Bird[2], Grigull[3], and Kozicki et al.[4] obtained those analytically and experimentally.

A grasping of rheological fluid flow behavior will contribute directly to the solution of a variety of commercial conduits with arbitrary cross-sectional shapes. It is very significant to have a knowledge of the characteristics of the pressure drop and the forced convection heat transfer in fully developed laminar rheological fluid flow in a circular conduit to exercise an appropriate control over the performance of the heat exchanger and to economize the process. Furthermore, the results provide an proper basis for estimating the effects of the reduction of fluid frictional drag and heat transfer enhancement. Recently a large number of heat exchangers are designed and manufactured for the chemical and biological process industries to heat or cool rheological fluids such as shear-thinning or shear-thickening fluids.

Rheological fluids usually have been assumed as power law fluids in the analysis. Many rheological fluids, however, have viscous properties which are different in the various shear rate ranges. Although a power law model has been used extensively for calculating velocity profile and temperature field in engineering, it has significant disadvantages that it only applies to the power law region in the flow curve and the apparent viscosity at the centroid of the conduit becomes infinite.

Such a rheological behavior in the transition zone causes the critical problem. It should be determined in which shear rate range the system is operating and if either of the Newtonian or power law solutions can be applied. This is not always simple because there is not a suitable shear rate parameter available and also the solutions were obtained independently. If the shear rate range falls within the transition zone then a "transition equation" must be applied for the type of rheological fluid considered here. What is required to overcome this problem is a solution for a fluid which has rheological characteristics in the wide shear rate zone.

A plenty of constitutive equations can describe the apparent viscosity-shear rate relation for fluids such as rheological fluid. A convenient and useful equation of rheological fluid is the "modified power law model" which was first proposed by Dunleavy and Middleman[5].

$$\eta_a = \eta_0 [1 + \frac{\eta_0}{K}(\dot{\gamma})^{1-n}]^{-1} \quad (1)$$

Examination of equation (1) shows that the apparent viscosity becomes equal to zero shear rate viscosity at very low shear rates and the fluid is operating in the Newtonian region. At the higher shear rates the fluid becomes a power law fluid. At intermediate shear rates, there is a transition zone. An additional advantage of the modified power law model over other constitutive equations such as Sutterby[6], Cross[7], Carreau[8], etc. is that the familiar Newtonian and power law Reynolds numbers are retained in the analysis.

The purpose of the present research is to upgrade the related knowledge by presenting solutions for fluids having the rheological characteristics illustrated and to develop the relationships between the dimensionless pressure drop and dimensionless convective heat transfer for a rheological fluid with modified power law fluid model. Such a solution should have the characteristics that at low shear rates (low Reynolds number) the Newtonian solution is an asymptote while at large shear rates the power law solution is an asymptote. In addition, the solution should predict the appropriate friction factor-Reynolds number relation and heat transfer coefficients in the transition zone. Finally a specific parameter explaining the shear rate is needed to predict the shear rate range in terms of the operating characteristics of the system.

2. Theoretical background

The study of fully developed laminar flow in conduits consists of one of the fundamental and classical problems in fluid mechanics and heat transfer. Solutions to such problems are obtained by solving the appropriate forms of the momentum and energy equations along with the associated boundary conditions.

It is convenient to start with the conservation equations to solve a problem related to fluid flowing through a circular conduit. For steady flow of an incompressible fluid with negligible viscous dissipation, the governing equations depend on the apparent viscosity that related to the shear stress and shear rate.

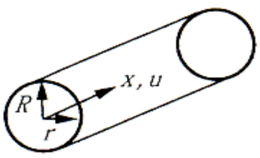

Fig. 1 Schematic diagram of a circular conduit

For a rheological modified power law fluid flow through a circular conduit as shown in Fig. 1, the fully developed velocity field is described by the following momentum equation.

$$\frac{1}{r}\frac{d}{dr}\left(r\,\eta_a\frac{du}{dr}\right) = -\frac{dp}{dx} \tag{2}$$

with boundary conditions

$$u(R) = 0, \quad \frac{du(0)}{dr} = 0$$

The analytical models of the apparent viscosity for modified power law fluids are as following.

$$\eta_a = \frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{du}{dr}\right)^{1-n}}$$

Following dimensionless quantities may be defined

$$r^+ = \frac{r}{R}, \quad f = \frac{2d\frac{dp}{dx}}{\rho\bar{u}^2}, \quad u^+ = \frac{u}{\bar{u}}$$

where the Darcy friction factor ($f=-\frac{8\tau_w}{\rho\bar{u}^2}$) is defined by a dimensionless pressure drop and *R(=d/2)* is a radius of the circular conduit.

$$\eta_a^+ = \frac{\eta_a}{\eta^*}, \quad \eta^* = \frac{\eta_0}{1+\frac{\eta_0}{K}\left(\frac{\bar{u}}{d}\right)^{1-n}}$$

$$Re_d = \frac{\rho\bar{u}d}{\eta_0}, \quad Re_g = \frac{\rho\bar{u}^{2-n}d^n}{K}$$

$$Re_m = \frac{\rho\bar{u}d}{\eta^*}, \quad \eta^* = \frac{\eta_0}{1+\beta}$$

$$\beta = \frac{Re_g}{Re_d} = \frac{\eta_0}{K}\left(\frac{\bar{u}}{d}\right)^{1-n}, \quad u^{++} = \frac{u^+}{\left(\frac{f\cdot Re_m}{8}\right)}$$

$$Re_m = Re_d + Re_g = \frac{\rho\bar{u}d}{\eta_0} + \frac{\rho\bar{u}^{2-n}d^n}{K} = \frac{\rho\bar{u}d}{\eta_0}(1+\beta)$$

$$\eta_a^+ = \frac{1+\beta}{1+\beta\left(\frac{f\cdot Re_m}{2}\right)\left(\frac{du^{++}}{dr^+}\right)^{1-n}} \tag{3}$$

From equations (1) and (3),

as $\beta \rightarrow 0$, $\eta_a \rightarrow \eta_0$ and $Re_m \rightarrow Re_d$

as $\beta \rightarrow$ very large, $\eta_a \rightarrow K(\dot{\gamma})^{n-1}$and $Re_m \rightarrow Re_g$

For a rheological modified power law fluid through a circular conduit, the continuity equation can be expressed by the following equation

$$\bar{u} = \frac{1}{A_c}\int_{A_c} u \; dA_c \tag{4}$$

$$= \frac{1}{\pi R^2}\int_0^R u \cdot 2\pi r dr$$

The dimensionless forms of equations (2) and (4) are

$$f \cdot Re_m = \frac{4}{\int_0^1 r^+ \cdot u^{++} dr^+} \tag{5}$$

$$\frac{1}{r^+}\frac{d}{dr^+}(r^+ \cdot \eta_a^+ \frac{du^{++}}{dr^+}) = -1 \tag{6}$$

with boundary conditions

$$u^{++}(1) = 0, \quad \frac{du^{++}(0)}{dr^+} = 0$$

Thus equation (6) could give the complete solution for the fluids and the final results can be presented as the dimensionless pressure drop (product of $f \cdot Re_m$) versus the shear rate parameter β.

The energy equation for the thermally developed flow in a circular conduit neglecting viscous dissipation and rate of energy generation [9] with thermal boundary condition of constant heat flux (CHF, H) can be written as

$$\frac{k}{r}\frac{d}{dr}(r\frac{dT}{dr}) = \rho c_p u \frac{dT_B}{dx} \tag{7}$$

with boundary conditions

$$T(R) = T_w, \quad \frac{dT(0)}{dr} = 0$$

Following dimensionless quantities may now be defined

$$T^+ = \frac{T - T_w}{T_B - T_w}, \qquad T^{++} = \frac{T^+}{Nu_H}$$

The dimensionless form of equation (7) becomes

$$\frac{1}{r^+}\frac{d}{dr^+}(r^+ \frac{dT^{++}}{dr}) = u^+ \tag{8}$$

with boundary conditions

$$T^{++}(1) = 0, \quad \frac{\partial T^{++}(0)}{\partial y^+} = 0$$

Considering the definition of bulk temperature, T_B:

$$T_B = \frac{\int_{A_C} u T dA_c}{A_c u} \tag{9}$$

For the circular conduit geometry, Eg.(9) may be rewritten in dimensionless form

$$1 = 2\int_0^1 u^+ T^+ r^+ dr^+ \tag{10}$$

Introducing the definition of T^{++} and solving for the Nusselt number gives

$$Nu_H = \frac{1}{2\int_0^1 u^+ T^{++} r^+ dr^+} \tag{11}$$

Equations (11) was solved numerically to obtain the relationship of Nusselt number vs. the shear rate parameter β for constant heat flux with the dimensionless velocity distribution, u^+ calculated from the solution of the previous momentum equation.

3. Results and discussion

A plenty of numerical solutions for shear-thinning fluid flows by rheological modified power law fluid model have been acquired for present research. For fully developed non-Newtonian laminar flows, the present results include the variation of the dimensionless pressure drop reduction(product of friction factor and modified Reynolds number) and dimensionless heat transfer enhancement(increase of Nusselt numbers) with shear rate parameter (β) in a circular conduit. The results of these analyses will be revealed in this chapter and presented in Fig. 2 to Fig. 3.

Numerical solutions to Eq. (5) for a circular conduit are shown in Fig. 2. The figures explain that in a quantitative sense, β defines the three regions, Region I – Newtonian, Region II – Transition, and Region III – Power Law. The parameter β is the shear rate parameter which specifies in which region of the flow curve the system is operating. Large values of β will indicate that the system is operating in region III, low values of β indicate Region I and intermediate values of β indicate the transition region (Region II).

The shear rate parameter defines the transition region (approximately $10^{-2.5} \le \beta \le 10^{2.5}$) and is useful to estimate whether the fluid is a fully developed Newtonian fluid ($\beta \le 10^{-2.5}$) or a fully developed Power Law fluid ($\beta \ge 10^{2.5}$). Thus the shear rate parameter β can be used to determine which of the three regions a particular system is operating in.

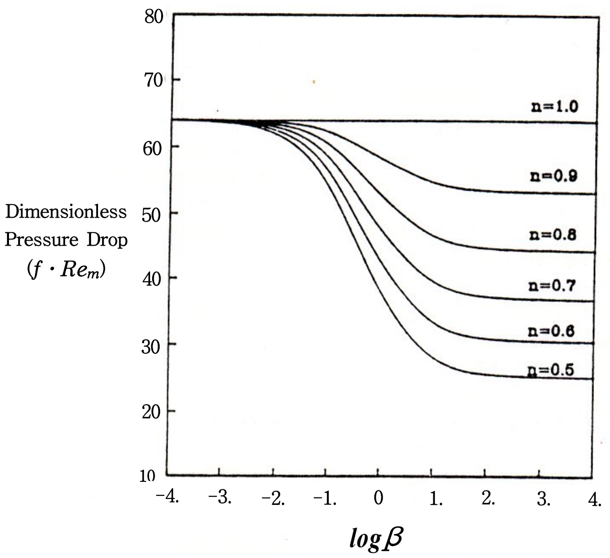

Fig. 2 Variation of dimensionless pressure drop with shear rate parameter (β) for shear-thinning fluids in a circular conduit

Fig. 2 shows the relationship between $f \cdot Re_m$ and the shear rate parameter and also expresses several important features of rheological modified power law fluid flow. First, for complete similarity modeling, the modified Reynolds number and the parameter β must both be considered. Also, a considerable difference exists if it is assumed that the system is operating in region III when it actually is operating in Region I. Simple calculations show that errors in pressure drop predictions can be as large as several hundred percent if such an uncertainty exists in correct operating region.

As the shear rate parameter increases, the Reynolds number increases. As the power law flow index(n) increases, the tendency increases to retain Newtonian characteristics at low Reynolds numbers. As the flow index decreases, the tendency increases to retain the characteristics of power law fluid at high Reynolds numbers.

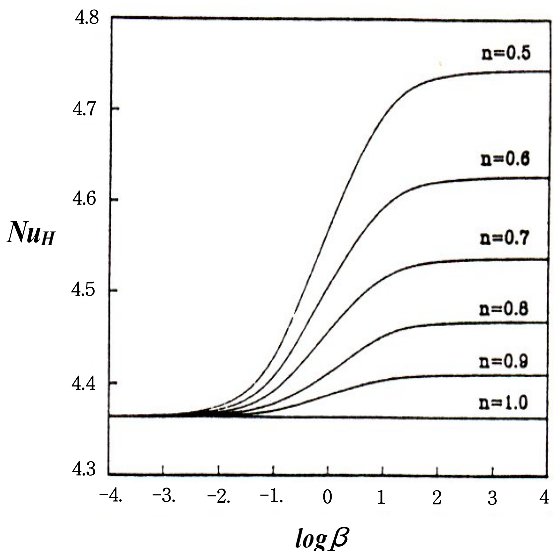

Fig. 3 Variation of Nusselt number (CHF) with shear rate parameter (β) for shear-thinning fluids in a circular conduit

The numerical calculations of the dimensionless pressure drop (product of the friction factor and Reynolds numbers) and forced convective heat transfer enhancement for the shear-thinning fluids in a circular conduit (increase of Nusselt numbers) for the Newtonian and the power law region were compared with other previously published asymptotic results[Shah and London[1], Bird[2], Grigull[3] , and Kozicki et.al[4]]. For Newtonian fluid flow in a circular conduit, the differences of the friction factors times the Reynolds numbers between the results of Shah and London[1)] and the present results are less than 0.01 % . These results are presented in Table 1. For power law fluids which various flow indices (n=0.4, 0.5, ⋯, 1.0), the differences of the friction factors times the generalized Reynolds numbers between the results of Kozicki et al. 4) and the present results are less than 0.004 %. These results are shown in Table 2.

At the power region-Ⅲ, where high Reynolds number exists, the pressure drop with shear-thinning fluids in a circular conduit decreased up to about 58% compared to that at Newtonian Region. The pressure drop also decreased with shear rate and Reynolds number.

For Newtonian fluids in a circular conduit, the uncertainties of the Nusselt number (CHF and CWT) between the results of Shah and London[1)] and the present results are less than 0.03%. These results are also presented in Table 1. For power law fluids in a circular conduit, the differences of the Nusselt numbers (CHF and CWT) between the results of the results of Bird[2)], Grigull[3)] and the present results are less than 1.1% and 0.3 % for the thermal boundary conditions of CHF and CWT, respectively. These results are shown in Table 3.

Table 1 Present and previous value of $f \cdot Re_{D_h}$, Nu_H, and Nu_T of Newtonian Fluid

$f \cdot Re_{D_h}$ (previous)[1)]	$f \cdot Re_{D_h}$ (present)	Nu_H (previous)[1)]	Nu_H (present)	Nu_T (previous)[1)]	Nu_T (present)
64.000	64.000	4.364	4.363	3.657	3.656

Table 2 Present and previous value of $f \cdot Re_g$ of Power Law Fluids

n	(previous)[6)]	(present)
1.0	64.000	64.000
0.9	53.282	53.282
0.8	44.322	44.323
0.7	36.829	36.830
0.6	30.557	30.557
0.5	25.298	25.299

Fig. 3 shows the laminar and fully developed Nusselt numbers versus the shear rate parameter for a circular conduit for the thermal boundary conditions of constant heat flux (CHF). Depending on the power-law flow index(n), this results showed the maximum 9% increase of convective heat transfer compared with Newtonian heat transfer. According the numerical solutions in Fig. 3, the influence of the shear rate parameter β is much less for the fully developed Nusselt numbers than for the product of f and Re_m . Thus it would appear that the influence of β on the hydrodynamic internal flow is much more critical than the convective heat transfer.

Table 3 Present and previous value of Nu of Power Law Fluids

n	Nu_H (previous)[3)]	Nu_T (previous)[3)]	Nu_H (previous)[4)]	Nu_T (previous)[4)]	Nu_H (present)	Nu_T (present)
1.0	4.363	3.657	4.360	-	4.364	3.657
0.9	4.410	3.691	4.400	-	4.411	3.693
0.8	4.467	3.732	4.449	-	4.468	3.738
0.8	4.538	3.783	4.510	-	4.539	3.792
0.6	4.628	3.850	4.589	-	4.629	3.861
0.5	4.745	3.949	4.696	-	4.746	3.949

4. Conclusions

Numerical solutions for rheological fluid flows by rheological modified power law model have been determined with the conditions of fully developed non-Newtonian laminar flows. These calculations include the variation of the dimensionless pressure drop and dimensionless heat transfer enhancement with qualitative Reynolds number in a circular conduit.

By utilizing a constitutive equation of the rheological modified power law model, solutions considered this shear rate dependence on pressure drop and convective heat transfer and through a dimensionless shear rate parameter enabled an appropriate choice of the pressure drop and heat transfer solutions.

From literatures and present calculations between Newtonian and rheological fluid flow it is evident that for the thermal boundary conditions (CHF) a rheological fluid with power flow index less than one shows a convective heat transfer enhancement than a Newtonian fluid. Owing to the reduction in pressure loss and the increase in heat transfer rates, rheological fluids seem to be better working fluids in commercial conduits and heat exchangers compared to Newtonian fluids. Thereby, the use of appropriate rheological fluids by modified power law fluid model may lead to heat transfer enhancement without the handling difficulties.

The applicability of this friction factor and Reynolds number relation will be useful for the determination of cross-sectional shapes for pressure drop in commercial conduits and the heat transfer enhancement for rheological fluids in a circular conduit can be applied for the design of a liquid cooling module in heat exchanger.

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