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The Construction of Children's Partitioning Strategy on the Equal Sharing Situation

Kim, Ahyoung*

This paper investigated the conceptual schemes in which four children constructed a strategy representing the situation as a figure and partitioning it related to the work which they quantify the result of partitioning to various types of fractions when an equal sharing situation was given to them in contextual or an abstract symbolic form of division. Also, the paper researched how the relationship of factors and multiples between the numerator and denominator, or between the divisor and dividend affected the construction.

The children's partitioning strategies were developed such as: repeated halving stage → consuming all quantity stage → whole number objects leftover stage → singleton object analysis/multiple objects analysis → direct mapping stage. When children connected the singleton object analysis with multiple object analysis, they finally became able to conceptualize division as fractions and fractions as division.

I. Introduction

1. The necessity and the purpose of the research

The understanding of fraction notation rational number concept is the first huge obstacle for many students in school mathematics. Unfortunately, many of them memorize the operation of fractions without enough conceptual construction, so mathematics begins to be separated from the numeric sense of the students and becomes a monster in which they cannot understand.

One of the reasons students cannot construct fraction notation rational number properly is the majority of the time for fraction concept teaching at school is devoted to part-whole sub-concept of rational number (Steerfland, 1991). In other words, students lose the opportunity to recognize fractions as numbers by being emphasized on the part-whole sub-concept of rational number. For example, when one of three objects which are of the same size and shape is referred to as one third, students cannot think of the fraction as a number (Ohlsson, 1988). On the other hand, quotient fraction provides students an opportunity to recognize fractions as numbers (Toluk, 1999; Kim, 2009). That means students can recognize fractions as numbers by getting the quotient through partitioning on equal sharing situations and expressing it as a fraction. Also, partitioning is the model of verifying their answer even after they learn how to get a quotient fraction using division operation. Therefore, students' partitioning strategies and their ability of quantifying the

* Soosong Junior High School (akim4@ewha.ac.kr)

partitioning results to fraction quotients seriously affect their conceptualization of fraction notation of rational number.

However, there are few longitudinal researches about students' partitioning strategies and there is no research on how students develop abstract quotient fraction concept by partitioning strategy on the equal sharing situation. Therefore, this paper investigates how students' partitioning strategies are constructed longitudinally, related to quantify the result of partitioning to various forms of fractions.

2. Research questions

A. How do children partition quantities and express them in various fraction forms?

B. How do children' partitioning strategies change chronologically?

3. Definition of terms

A. Unit

The notion of unit is usually distinguished to both singleton and composite units by the fact of whether or not an object or a collection of objects is regarded as a whole. Even though diagrams and real-world situations are used to describe whole number arithmetic problem situations in the school mathematics curriculum, the quantities expressed in problem situations typically are only singleton units rather than various counting unit types (Steffe, 1988). However, there are problem situations in which it is more efficient to use a counting unit other than a unit of 1 and the counting unit as norming (Lamon, 1994). Behr and his colleagues (1994) argued that norming in whole number situations facilitated the learning and understanding of rational number concepts. In their paper, they used two nonstandard representational systems to describe units and quantities. One system used drawing representation to give the psychological aspect and the other system used symbolic representation to give corresponding mathematical aspects. For example, both singleton units and composite units as shown in <Table I-1> can represent the number 4.

Using this system, a unit fraction $\frac{1}{4}$ can be

<Table I-1> The Units Analysis of Natural Number 4

	Drawing Representation	Symbolic Representation
Singleton Units Analysis	[O][O][O][O]	4[1-unit]s
Composite Units Analysis	[OO][OO][OO][OO]	4[2-unit]s

<Table I-2> The Units Analysis of Fraction $\frac{1}{4}$

	Drawing Representation	Symbolic Representation
Composite unit made of single objects analysis	[O O O O]	$\frac{1}{4}$[4-unit]
Composite unit made of single units analysis	[[O][O][O][O]]	$\frac{1}{4}$[4[1-unit]s-unit]
Composite unit made of composite units analysis	[[OO][OO][OO][OO]]	$\frac{1}{4}$[4[2-unit]s-unit]

represented as shown in <Table I-2>.

It is assumed that these flexible mental models are a powerful tool to understand and compute fractions (Steffe, 1988).

B. Partitioning strategies at pre-stage of partitioning for equal sharing

1) unequal sharing strategy: a child exhausts the whole shared quantity but distributes unequal quantitiy to shared people

2) sorting out strategy: a child distrubuted equal amount of shared objects to shared people but does not exhaust the whole shared quantity

II. Review of the Literature

Children develop various partitioning strategies for equal sharing, depending on their previous social practice, the shapes of shared objects, the number of shared objects, and the number of people sharing. According to Piaget, Inhelder and Szeminska (1960, p333), understanding the relations involved in a complex of discontinuous logical or numerical elements is a harder process than the thinking of subdivision of continuous spatial quantities because when a class or a collection is divided into sub-classes, it cannot be easy to remember the initial whole and its reconstruction must imply a precise operation of logical addition or of re-assembly. In addition, they revealed that successful partitioning needs an anticipatory schema in which the desired part is recognized beforehand as something bound up with a divisible whole equal to the sum of its parts because at the lower stages, children did not use the whole area of a given whole to share equal parts among shares. Pothier and Sawada (1983) classified a first and second grade students' partitioning developmental process into a sequence of five levels of the partitioning process with the property of numbers(odd/even, prime/composite, factor/multiple) and the transformation of figure (translation, symmetry, rotation, similarity, congruence). They explained the first level was allocating the same number of pieces regardless of size, the second level was algorithmic halving, the third level was evenness for real "fair share" and making an even number of pieces by geometrical transformation, the fourth level was making both even and odd number partitions by counting an algorithm, and the fifth level was composition to get complex multiplicative numbers of partitions: for example, to get fifteenths, a child might trisect fifths. In their experiment however, they could not discover the real example of the fifth level; they just assumed that more mature children could do the fifth method. Corresponding to these levels, children could only count pieces (e.g. one for me, and one for you...) in the first level, represent a fraction in which a denominator was the power of 2 in the second level, represent a unit fraction in which the denominator was an even number in the third level, and then represent all unit fractions in the fourth level. Also, Empson and her colleagues (2005) documented the case of equal sharing problems in which the number of shared objects was smaller than the number of sharers and in which they shared a common factor stimulated the problem solver's multiplicative thinking. On the other hand, Lamon (1996) reported children having a tendency to develop more economically efficient ways of partitioning–preserving wholes, cutting fewer pieces. Charles and Nason (2000) argued that children need three conditions to build partitive quotients as a general concept because the children can get various forms of fraction quotients by using their partitioning strategies: i. e. construction of conceptual mapping through partitioning strategies for generating equal and quantifiable shares to abstract the partitive quotient notion of a fraction, direct mapping from the number of subjects to the unit fraction name of each share, direct mapping from the number of objects being shared to the number of the unit fraction in each share (i.e. as a dividend). They reported that if one of them could not be satisfied, then children could not recognize direct mapping between the elements in a quotient fraction and the elements in given division.

From the previous research, the change of partitioning strategy does not parallel with recognizing direct mapping among sharing objects and shares in equal sharing situation, the numerator and denominator in a quotient fraction, and the dividend and divisor in a division expression. It shows the need of research to find the link.

III. Method

1. Participants

Four students from a inner city fifth grade mathematics class the Southwestern United States participated in the study. One boy and one girl were chosen from the high mathematics performance group and the other boy and girl were chosen from the low mathematics performance group, each based on their mathematics teacher nomination (see <Table III-1>). The high performing students were Caucasian and the low performing students were Hispanic. However, they were all members of middle-class families.

2. Research Method and Research Design

The individual teaching experiment method which consisted of a clinical interview phase, a teaching phase, and an analysis phase was used (Steffe & Thompson, 2000). For this study, the researcher interviewed each student respectively, and each interview took about an hour. Identical questions were used for two clinical interviews (pre-test and post-test). The purpose of the pre-test interview was to assess each student's initial mathematical knowledge under investigation and to target the changes in their mathematical thinking. After that, six teaching episodes were progressed using three different types of questionnaires which induced specific type of fraction answers: proper fractions, improper fractions, and mixed numbers each. An equal sharing situation was given to them in contextual or abstract symbolic form of division, which was similar to the questionnaire in the clinical interview. Teaching episodes were used to construct models of the children's mathematical thinking and guide them to develop more reflective ways of thinking about partitive quotient fraction that built on the results from the initial clinical interview.

The set of proper fraction questions was given to the children first. At this time, the effect of contemplating equivalent fractions was the major focus of the study. After that, to reduce the biased result caused by introducing fraction forms in different orders, and to maximize the effect of the order of introducing fraction forms, the researcher divided the children into different orders of presentation.

The questions were arranged to give the children a chance to develop various partitioning strategies in different social situations especially at first teaching episode (see Appendix 1). Also, the questions which were used in each teaching episode were designed so that the children would develop various partitioning strategies by being given specially selected quantities. The post-test was used to observe the change of students' partitioning strategies and the quantifying methods the partitioning to a quotient fraction.

During the interview, the researcher used probing questions such as "Show me what you did", "Why did you do that?" and "Show me how that works" in an attempt to draw out verbal, gestural, and written evidence of their thinking. Each clinical interview and teaching interview was videotaped and transcribed by the researcher. A pseudonym was used to represent each participant.

The analysis phases conducted both ongoing and retrospective analysis (Huberman & Miles, 1994). <Table III-1> is the overall interview schedule.

IV. Results

At the pre-test clinical interview, all of the students' partitioning strategies met two necessary conditions for equal sharing: First, each sharer got an equal amount of the shared objects and second, the entire shared quantity had to be exhausted. However, one of them (Hunter) went backward to pre-stage of partitioning whenever his partitioning strategy failed to solve a equal sharing problem later. He just cut each object into two unequal pieces (see Figure IV-1 for the example) and then tried to make three similar amounts of groups by adding several different sized pieces.

All of the students had similar characteristics during the pre- test. First, they had a tendency to distribute shared objects equally, not partitioning them as much as possible. Specifically, if the number of shared objects were bigger than the

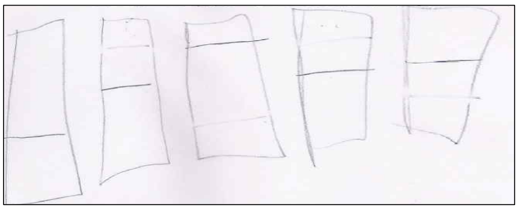


Figure IV-1. Hunter's pre-stage partitioning strategy for equal sharing five different flavored fruit roll-ups among three friends in 5th interview.

<Table III-1> Overall Schedule for Clinical Interviews and Teaching Episodes
(Question Set 1(Q.S1): proper fraction questions, Question Set 2(Q.S. 2): mixed number questions, Question Set 3(Q.S. 3): improper fraction questions)

Subject	Initial Performance Level	Week 1 (interview 1, 2)	Week 2 (interview 3, 4)	Week 3 (interview 5, 6)	Week 4 (interview 7, 8)	Week 5 (interview 9, 10)
S1 (girl: April)	High	Pre-test	Q.S.1	Q.S.2	Q.S.3	Post-test
S2 (girl: Corey)	Low	Pre-test	Q.S.1	Q.S.2	Q.S.3	Post-test
S3 (boy: Hunter)	High	Pre-test	Q.S.1	Q.S.3	Q.S.2	Post-test
S4 (boy: Sam)	Low	Pre-test	Q.S.1	Q.S.3	Q.S.2	Post-test

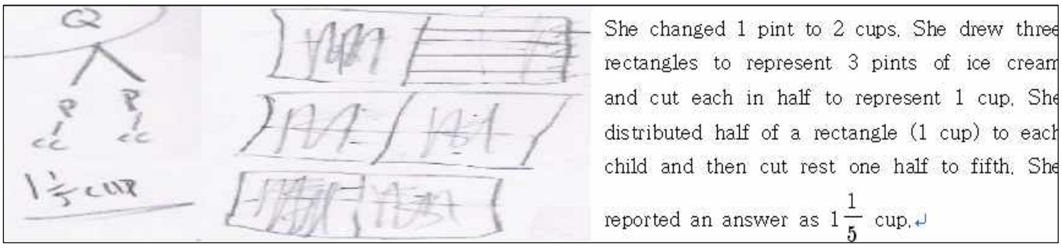


Figure IV-2. Corey's partitioning strategy for sharing 3 pints of ice cream among 5 children in pre-test interview.

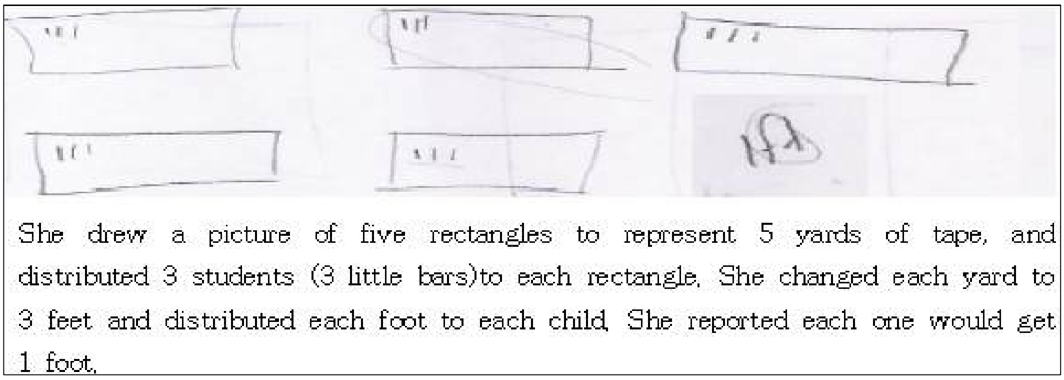


Figure IV-3. Corey's partitioning strategy for equal sharing 5 yards of tape among 15 students in pre-test interview.

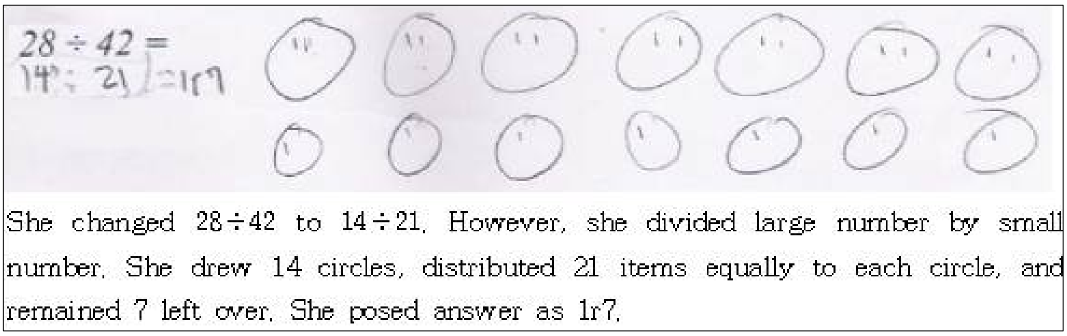


Figure IV-4. Corey's partitioning strategy for $28 \div 42$ in pre-test interview.

number of sharers, then all of the students distributed whole objects to the sharers equally, and then distributed the leftover shared objects to sharers. Second, if the unit of the shared objects had a sub-unit, then they would distribute the shared objects at the sub-unit level and as a result the activity of distribution was changed to sorting out rather than partitioning (see Figure IV-2 for the example).

Third, if the number of shared objects was a multiple of the number of sharers, then students distributed the number of sharers to the number of shared objects. Therefore, they searched for how many sharers were in one shared object (see Figure IV-3 for the example).

Fourth, if the number of shared objects and the number of sharers had a common factor, students searched for a simple ratio between them before partitioning (see Figure IV-4 for the example). However, most of those strategies faded or disappeared as students moved to a higher level of partitioning strategy.

When they could not avoid partitioning for equal sharing situations, the student's first partitioning strategy tended to be repeated halving until there was at least one piece for each sharer (see Figure IV-5 for the example).

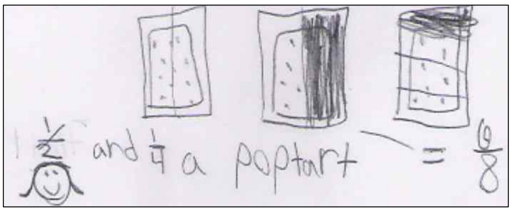


Figure IV-5. Sam's partitioning strategy for equal sharing three Pop-tarts among four girls in third interview.

At this level, students understood marked lines as cutting lines, so they could not adjust the lines after marking. Therefore, with leftover pieces, they repeated the same procedure until they consumed all the parts. When they quantified the result, some of them expressed the result of partitioning as the list of the fractions in which the denominators were powers of 2.

When students recognized that they could not consume all shared objects using this partitioning strategy in some cases, they moved to a second stage of partitioning. This new strategy consisted of marking each shared object equally using easy-to-make partitions, such as halving, until there were enough pieces to distribute a piece to each sharer. If they did not have enough pieces to distribute to the sharers, then they re-marked each object equally, increasing the number of marking one by one, such as trisecting or quartering until they had enough pieces to distribute to the sharers. With the leftover pieces, they repeated the procedure. In this stage, they did not think of marked lines as cutting lines, so they could remark each object by trisecting, quartering etc. Their activity focused on consuming all shared objects (see Figure IV-6). At this stage, most of them still failed to quantify the result, so they represented the result of partitioning using a diagram and a fraction combined.

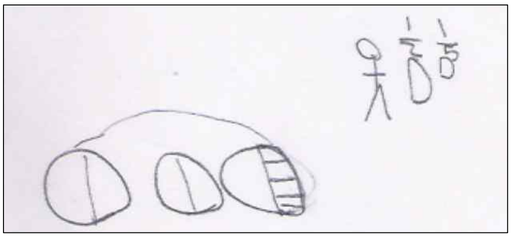


Figure IV-6. Sam's partitioning strategy for equal sharing 3 pints of ice cream and the quantification of the result.

When they started to focus on quantifying the result of partitioning, they moved to the next partitioning stage. The previous partitioning strategy often generated pieces that were hard to quantify; when the leftover pieces after distribution were not whole objects, students had to deal with quantifying part of a part. However, combining two levels of partitioning was not an easy step for most students. They wanted only whole objects leftover. At this time, students' schemes for unitizing objects were differentiated

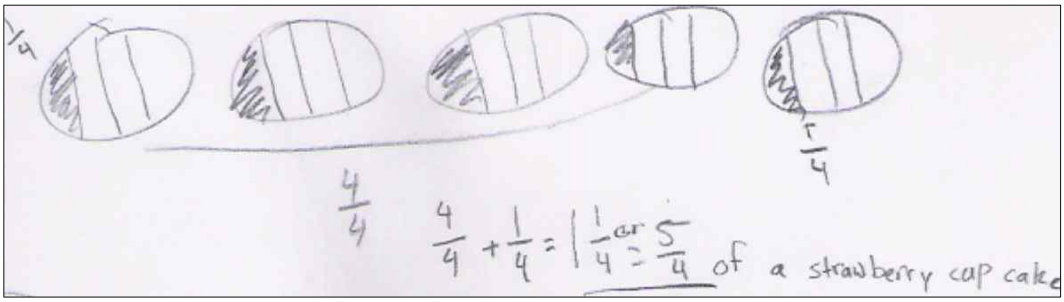
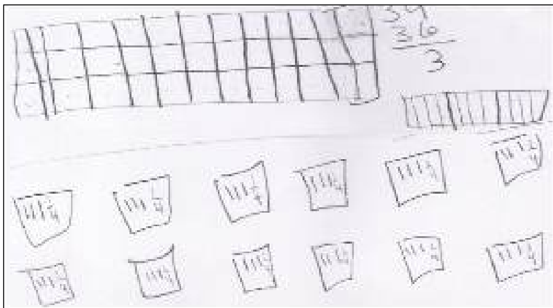


Figure IV-7. Corey's partitioning strategy for equal sharing five strawberry cupcakes among four boys by a singleton object scheme in seventh interview.



She first represented 39 inches of fruit roll-ups with 39 consecutive rectangles. Because they were hard to count, she changed the picture like above. She drew a rectangle and cut it to 3 by 13 small pieces. She distributed three inches (three little bars) to each child (a rectangle in the lower side of the picture). Then she represented the leftover three rectangles with three consecutive rectangles in the right-middle of the picture again. She cut them into 12 pieces, distributed one piece to each child, and recorded it as $\frac{1}{4}$.

Figure IV-8. April's partitioning strategy for equal sharing 39 inches fruit roll-ups among 12 children in fifth interview.

as a singleton units scheme and the other was a multiple unit scheme.

The students who had a singleton units scheme (Corey) marked each object equally with the number of sharers with anticipation that they would distribute one piece from each object to each sharer (see Figure IV-7 for the example). According to how the student anticipated the whole, she marked each shared object one by one or marked all of the shared objects simultaneously. Later, she mapped directly from the number of shared objects to the numerator of a fraction quotient. Therefore, singleton units analysis was easy to partition, to quantify the result of partitioning, to perceive the direct mapping from the numbers in equal sharing situation to numerator and denominator. However, when the purpose of the activity was equal sharing itself, students did not use this analysis much because of the inefficiency of partitioning.

The students who had a multiple units scheme chose multiple objects that could be marked into the number of parts equal to the number of sharers (see Figure IV-8 for the example). With any leftover objects, the student continued choosing multiple objects that could be marked into the number of parts equal to the number of sharers. From the very beginning, the student's focus was first, securing enough pieces to give each sharer one piece by marking all sharing objects with an equal number of marked lines, by increasing marked lines one by one. Second, he/she would keep the leftover whole objects. Therefore, even though their partitioning created enough pieces for each sharer to have one piece, if the leftover was not a whole number of objects, then they would tend to increase the number of partitions. When they got a whole number of objects leftover, they distributed the pieces from the shared objects, except for the leftover, and then he repeated the same procedure with the leftover whole objects.

Through this activity, the students developed multiplication number facts and this made marking more efficient. They started to predict how many objects they had to choose to leave a whole number of objects leftover and found the number to cut for each object at once using multiplication fact relationships between the number sharers and the number of parts of shared objects reserving the whole object leftover. The development of multiplication number facts enforced their multiple units scheme once again. Especially when the number of sharers was a multiple of the number of shared objects, the students who knew multiplication number facts created a number of parts equal to the number of sharers using all shared objects at once by recalling the factors.

However, at the beginning of the development of this strategy, they sometimes missed this multiplication number fact, so they chose a different factor of the number of sharers that was not a factor of the number of shared objects. Since they were much familiar with using multiplication number facts, when the number of shared objects that had a common factor with the number of sharers, they created a number of parts that was a multiple of both the shared objects and the number of sharers. Finally, when they used a common multiple of the number of sharers and the number of shared objects as the total number of pieces they wanted to generate from the whole objects, they conceptualized all partitioning in a single rule. However, the form of fraction answers did not show direct mapping from the numbers in equal sharing situation to the numbers in their fraction answer. In conclusion, students who used multiple units analysis developed multiplicative thinking, economical partitioning strategies, and a good procedural skill with multiplication number facts.

Students who had already learned multiplication number facts hardly chose singleton object analysis. However, when each shared object had a different quality or each shared object was given with a time interval, students chose singleton object analysis. Throughout problem solving, they understood the relationship between improper fractions and their corresponding mixed fractions, as well as the relationship between the direct-mapped proper fraction and its simplest proper fraction.

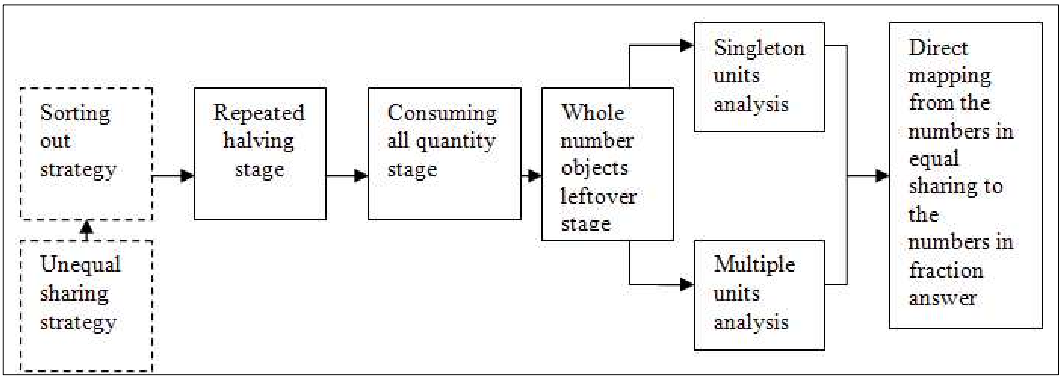


Figure IV-9. The development trajectory of students' partitioning strategies.

Furthermore, they conceptualized direct mapping.

When students distributed whole objects from shared objects to sharers equally in situations where the number of shared objects was bigger than the number of sharers, they distributed objects one by one, or equal small numbers like by fives to sharers without anticipation of the quotient even though they got the whole number portion of the fraction quotient using multiplication fact families when they represented equal sharing situations as division number sentences. When the researcher asked for more efficient methods for distribution of whole objects in the eighth interview, they changed their method to use multiplication facts. In addition, the whole number portion of a quotient from partitioning helped the students quantify the remaining fractional pieces correctly. At the early partitioning stage, the students had a tendency to quantify the result of partitioning as a part-whole relationship, but this whole number portion of a quotient prevented the incorrect quantifying by comparing a fractional portion to a whole number portion of the quotient.

Among the four interviewed students, April was the only one to use the first strategy of non-partitioning strategies at the beginning of the study. She started from a repeated halving stage and had difficulty quantifying the result. She avoided the obstacle by using multiple units analysis. Over the course of the study, she also gained singleton units analysis and she conceptualized the direct mapping between the numbers in equal sharing situations and the numbers in the fraction quotient.

Corey had all four strategies of non-partitioning strategies at the beginning of the study, so she hardly succeeded to partition on the pre-test. However, when she started to do partitioning correctly, she quickly moved from the first to the fourth stage step by step. She primarily chose singleton object analysis at the fourth stage, so she quickly perceived the direct mapping between the numbers in equal sharing situations and the numbers in fraction quotient. She reached the direct mapping stage by the end of the study.

Sam's mathematical development stage was lower than his peers. First, he was still interested in the qualities of the objects rather than the quantity of the objects. It made it hard for him to understand the shared objects as mathematical objects. Second, he often represented the shared objects as short lines similar to tallies when the number of objects was big, but when he did partition them, he was confused as to what he had to count, so instead of counting partitioned lines, he counted partitioned spaces between tallies. Third, he was not interested in quantifying the result of partitioning, so he often represented the result with diagrams. Those things prevented him from getting fractional quotients. He had second and third non-partitioning strategies (sorting out and halving) at the beginning of the study. When he did partition, he did repeated halving regardless of the fact that the number of shared objects was bigger than the number of sharers. He moved to the consuming all quantity stage, but he could not go further because he had no way to check if his quantification of the result was correct or not; he did not have other mathematical knowledge that could generate the fraction quotient. Since he could not recognize his wrong quantification of the result of his partitioning, he did not feel the necessity to develop a new partitioning strategy. As a result, he could not develop his partitioning strategy further during the study.

Hunter had the first three non-partitioning strategies at the beginning of the study. In particular, he used the second non-partitioning strategy (sorting out strategy) a lot. If the number of shared objects was smaller than the number of sharers, then he reconceptualized one of the shared objects as one hundred pieces in his mind. He partitioned it by the number of sharers, and multiplied the result by the number of shared objects. Therefore, if the number of sharers was a factor of 100, then he easily found a percent quotient and converted it to a fraction using the denominator 100. However, he could not get a fraction answer in which the numerator and the denominator were whole numbers. In this case, the precutting of one hundred pieces' image disappeared in his mind. He chose all shared objects and he went back to his unequal sharing strategy. Through the Problem Set 1 that was designed to generate proper fraction answers, he started to partition without thinking of cutting into one hundred pieces and got proper fraction quotients directly without converting them from percent quotients. When the number of shared objects was bigger than the number of sharers, he used a quotitive division interpretation for the fractional portion of a quotient, so he did not need to do partitioning. Because more than half of the problems in the problem sets had the number of shared objects bigger than the number of sharers, he had to partition these questions to develop his partitioning strategy, but he refused to do partitioning after getting quotients using his quotitive division interpretation. Therefore, he could not develop his partitioning strategy for mixed fractions and improper fractions and he lost the chance to practice his new partitioning strategy for proper fractions. As a result, he reverted back to his original stage of partitioning at post-test.

V. Discussion and Conclusion

Compared to most previous studies of partitioning involving quantitative research with different age groups, this study followed the change of several individual students' partitioning strategies. The results showed not only the development trajectory of children's partitioning strategy, but it also revealed how and why they changed their partitioning strategies until they discovered direct mapping from the numbers in equal sharing situations to the numerator and denominator in fraction answer, so it tied the development children's partitioning strategies to the development of partitive quotient fraction.

Their partitioning stages were repeated halving stage → consuming all quantity stage → whole number objects leftover stage → singleton units or multiple units analysis → direct mapping from the numbers in equal sharing to the numbers in fraction answer. Whenever their purpose of partitioning was changed, their partitioning strategy was changed, too. The purpose of partitioning started with consuming all quantities for equal sharing → quantifying the result of partitioning without dealing with composition of partitioning → making general rules for partitioning. In addition, the development of students' partitioning strategies showed how fractions and multiplicative thinking affected each other's development from a certain point in time. When children refocused on whole number objects leftover from partitioning to quantifying the cutting pieces easily, they started to think multiplicatively, and the development of this multiplicative thinking made children partition efficiently and quantify the pieces to fraction form easily.

Again, well-partitioned diagrams by multiplicative thinking became mental images for multiplication number facts, so it re-enforced their multiplicative thinking. After they were fluent using multiplication number facts, they could analyze one problem both in single units analysis and in multiple units analysis. As a result, they understood partitioning strategies in single rule; direct mapping from the numbers in equal sharing to the numbers in the fraction answer. When equal sharing word problems were used to develop the quotient interpretation of fractions, situation in which the number of shared objects is bigger than the number of sharers was easier for students to understand fraction division than the problem in which the number of shared objects was smaller than the number of sharers. In addition, with specially designed equal sharing problems to shared objects attaching different qualities or time intervals, students generated improper fraction quotients with less aversion and more quickly perceived the direct mapping from the dividend and the divisor of the division number sentences to the numerator and the denominator of the fraction quotient.

Therefore, division number sentences or equal sharing word problems which generate fractions bigger than 1 may fruitfully given to student prior to asking students to perceive proper fractions as division. Furthermore, in this step, the teacher should check if students understand whole numbers as fractions or not. It was not enough representing 1 as a correct fraction. The result of the study showed that all interviewees represented 1 as a fraction in which the numerator and the denominator were the same, but only half of them could represent other whole numbers as fractions correctly. In particular most students represented any whole number n as $\frac{n}{n}$.

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균등분배 상황에서 아이들의 분할전략의 구성

김 아 영 (수송중학교)

이 논문은 균등분배상황이 문장제나 수식 형태의 나눗셈으로 주어졌을 때, 아이들이 그 문제 상황을 도형으로 표현하고 그 도형을 분할하는 전략이 그 분할 결과를 다양한 형태의 분수로 수량화시키는 작업과 연관해 구성하는 개념적 스키머에 대하여 조사하였다. 그리고 이 때 분자와 분모 간, 제수와 피제수 간의 인수와 배수 관계들이 그 과정에 어떤 영향을 미치는지를 연구하였다.

아이들의 분할 전략은 다음 순서로 발달했다: 반복적인 이등분 수준→ 전체 양 모두 사용하기 수준→ 자연수 물건 남기기 수준→ 단수 물건 해석/복수 물건 해석 수준→ 직접 사상(mapping) 수준. 또한, 아이들이 단수 물건 해석을 복수 물건 해석과 연관시킬 수 있을 때, 그들은 마침내 나눗셈을 분수로, 분수를 나눗셈으로 개념화할 수 있었다.

*key words : partition(분할), fraction(분수), division(나눗셈), quotient concept(몫개념), unit(단위), constructivism(구성주의)

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<Appendix 1> The Questions Used in the First Teaching Episode

* Solve the questions thinking aloud.

Q1. Six boys want to share a brownie cake equally. How much does each get?

Q2. Six boys are going to share a brownie cake equally. However, before they cut it, they get one more brownie cake. If they get equal amount of the brownie cakes, how much of a brownie cake does each get?

Q3-1. Even though these brownie cakes are the same size, one brownie cake has walnuts, and the other has Marshmallows. If six boys want to share the cakes so that each gets equal amount of the different flavors, how do they have to cut the brownie cakes? How much of a brownie cake does each get now?

Q3-2. Six boys want to share the two brownie cakes equally, but they want to get one big piece rather than several smaller pieces. How do they have to cut the brownie cakes? How much of a brownie cake does each get now?

Q4. Four girls want to share three Pop-Tarts equally. Does each get more or less than one Pop-Tarts? Then, how much of a Pop-Tart does each get?

Q5. With six granola bars, eight boys want to share them equally. Does each get more or less than one granola bar? Then, how much of a granola bar does each one get?

Q6. Four friends got two dozen eggs to decorate for Easter egg hunting. If each of them paints an equal number of them, how much of a dozen does each paint?

Q7. You can wrap twelve equal size boxes with three yards of wrapping paper. How much of a yard do you need to wrap each box?