Capacity Bound for Discrete Memoryless User-Relaying Channel

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Abstract

In this paper, we consider the discrete memoryless user relaying channel (DMURC) in which a user-relay switches its operational mode symbol-by-symbol. In particular, we obtain upper and lower bounds on the channel capacity for the general DMURC and then show that these the upper and lower bounds coincide for degraded DMURC. It is also shown that the capacity of the degraded DMURC can be achieved using two separate codebooks corresponding to the two UR states. While the UR is assumed to switch states symbol-by-symbol, the results in this paper is the same as when the UR switches states packet-by-packet.

Keywords: User-relay, relay channel, channel capacity, channel state information, CSI, Cognitive radio,

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1. Introduction

With the advent of cooperative communication technologies, the concept of cooperative relaying has been receiving growing attention in recent years. Cooperative relays are commonly assumed to be subordinate to source and destination nodes playing only the relaying role for maximum source-destination capacity. However, relays in cooperative networks are ordinary users that transmit and receive their own messages taking up the relaying role only from time to time. We call such a dual-functioning node a user relay (UR) and obtain upper and lower bounds on the channel capacity for the general DMURC and then show that these the upper and lower bounds coincide for degraded DMURC.

During the last decade, there have been various efforts to understand the impact of introducing cooperative relays into wireless networks [1][2][3][4][5]. In the seminal work of Cover and El Gamal [1], capacity theorems are derived for various dedicated single-relay channels. Only recently, these theorems have been extended to cooperative mupltipe-relay channels in [2][3][4][5]. As shown in [2], the relay selection scheme has a significant impact on the performance of a relay network. In [3], it is shown that space-time coded relays can effectively mitigate the detrimental effect of multipath fading under fully distributed relay nodes. Since the algorithms in [3] require the channel state information (CSI) among the nodes in the entire network, a semi-distributed cooperative relaying algorithm is proposed in [4] which greatly reduces the computational complexity while keeping the system capacity comparable. In these studies, even though the results can be directly applied to user relay networks, it is presumed that the relays must always be available for relaying under the control of the source and/or central network.

Recently, Cai et-al, in [5], introduced the term user relay and emphasized the impact of user role on the cooperative relay. They assumed that URs choose by themselves whether to transmitt their own messages or to relay the source, putting higher priority on their own demands. Such a situation can be found in various wireless networks. A typical example can be found in a cognitive radio network where user node may or may not relay other nodes to get a certain form of benefits, for example, monetary returns. For another example, let us consider a user node who occasionally needs to transmit urgent messages such as military data or emergency call. Such a user can play a relaying role only when it does not have its own urgent demand. Focusing on the user role of the UR, in [5], the queuing performance of the UR network is analyzed and a new relay node selection scheme is proposed. However, [5] did not consider source-destination capacity. Moreover, there is still no published research efforts to determine appropriate source encoding strategies for coopertive URs which put higher priority on its own demand. Hence, we study, in this paper, capacity theorems for discrete memoryless user relaying channel (DMURC) in which URs have higher priority on their user role. We assume that the UR switches symbol-by-symbol between two operational modes, namely, user and relay modes. Such a UR is referred to as a fast switching (FS) UR in this paper. Results on FSUR can be directly applied to the packet-by-packet switching user relay channel by simply regarding a packet as a symbol. Consequently, the results in this paper are more general than those for slow switching (SS) URs that switches the operational modes packet-by-packet.

The source-destination capacity depends fundamentally on whether the UR operates as a relay or a user. Theoretically, the UR operational mode can be incorporated into the channel model by adopting a corresponding channel state variable. Previous research on the capacity of state dependent channel can be found in [6]][7][8][9][10]. In [6], a pioneering work on channels with states, Shannon showed that the capacity of a state dependent channel can be derived by adding a fictitious device in front of the channel when the CSI is causally (meaning that CSI is given just before the transmission of the symbol) available at the encoder. In [7], the capacity of state dependent relay channel (SDRC) was first introduced. There was found the capacity for degraded discrete memoryless relay channel when casual CSI is available both at transmitter and relay. This result was extended to variety of state dependent relay models in [8][9][10]. Most common issues on SDRCs are how the relay deliver the message (decode and forward, compress and forward, or amplify and forward) and to whom the state information be given (to all nodes or to a selection of nodes). Zaidi et-al, in [8], obtained a capacity lower bound for SDRC with non-causal CSI (meaning that whole packet CSI is given before the beginning of encoding) available both at source and relay, and they assumed that decode and forward strategy is employed by the relay. In [9], three different situations are considered; CSI is avaliable only at the source, only at the relay, and both at source and relay using compressed and forward relaying scheme. More recently, capacity lower and upper bounds of SDRC are investigated with non-causal CSI avaliable only at the relay in [10].

As shown in [7]][8][9][10], most previous works on SDRCs have focused on subordinate relay node. We, in this paper, consider a UR node with higher priority on its user role. We assume that CSI, namely, the information regarding the operational mode of the UR, is causally given both at source and destination nodes, and derive capacity theorems of the channel. To be more specific, we model a general DMURC using CSI and obtain a capacity upper bound and an achievable rate region for the channel. Then, we show that, when the channel is degraded, the upper and lower bounds coincide becoming the channel capacity. Even though we only consider the FS UR channel, the results can be extended to the SS UR channel directly.

The rest of this paper is organized as follows. In Section 2, we describe the channel model considered through this paper. In Section 3, we derive the capacity upper bound for the general DMURC. Then, in Section 4, we get the achievable rate region of the general DMURC and show that the region becomes the channel capacity for the degraded DMURC. Finally, we draw conclusions in Section 5.

2. System Model

To make the problem tractable, we restrict our attention to a system consisting of a single source, a single destination, and a single UR as shown in **Fig. 1**. We assume that the source has unlimited amount of data to send. In each symbol time, the UR determines whether to relay the source message or to transmit its own message, which are defined as state 0 and 1, respectively. At the state 0, the source, UR and destination nodes form a traditional relay channel shown in [1]. Otherwise, at the state 1, the source and destination nodes form a traditional point-to-point channel and we assume that signal of the UR is independent of both source and destination nodes. One of the most important issues in determining the capacity of a state-dependent channel is the availability of the channel state information. If all the channel states (corresponding to a given packet) are known in advance and can be used for encoding, then the system is said to be non-causal. However, in practice, the channel states are known only until the time of each symbol transmission. In general, CSI is said to be causally available, if the information is given only upto the present time. In this paper, we study the case that the CSI is given causally both at the source and destination nodes.

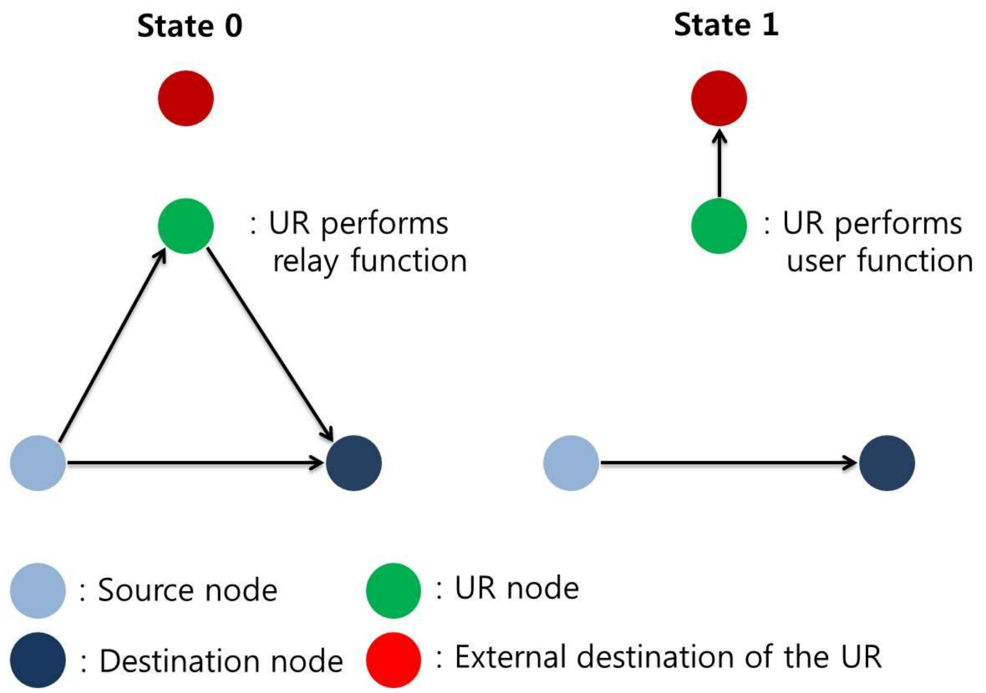

Fig. 1. User-Relaying Model

Let $\mathcal{S} = \{0,1\}$ and let $\mathcal{X}, \mathcal{X}_r, \mathcal{Y}$, and $\mathcal{Y}_r$ be any given finite non-empty sets. Let $p(s)$ be a probability mass function (pmf) on $\mathcal{S}$ and $p(y, y_r|x, x_r, s)$ be a collection of pmf's on $\mathcal{Y} \times \mathcal{Y}_r$, one for each $(x, x_r, s) \in \mathcal{X} \times \mathcal{X}_r \times \mathcal{S}$, satisfying

$$p(y, y_r, x|x_r, s = 1) = p(y, y_r, x|s = 1), \quad (1)$$

which means that (Y, Y_r, X) and X_r are conditionally independent for given $(S = 1)$. A triplet $(\mathcal{X} \times \mathcal{X}_r \times \mathcal{S}, p(s) \cdot p(y, y_r|x, x_r, s), \mathcal{Y} \times \mathcal{Y}_r)$ is called a Discrete Memoryless User-Relaying Channel (DMURC). The variables introduced above have the following meanings:

- x: the input of the channel chosen by the source
- x_r: the input of the channel chosen by the UR
- s: the state of the UR operational mode. In state 0 ($s = 0$), the UR operates like a traditional relay, and, in state 1 ($s = 1$), the UR transmits its own message. We note that (1) implies that UR does not affect the received signals at the destination and the UR if $s = 1$.
- y: the channel output
- y_r: the received signal at the UR

The n^{th} extension of the above DMURC is the channel $(\mathcal{X}^n \times \mathcal{X}_r{}^n \times \mathcal{S}^n,\ p(s) \cdot p(y, y_r|x, x_r, s),\ \mathcal{Y}^n \times \mathcal{Y}_r{}^n)$ satisfying

$$p(y^n, y_r^n|x^n, x_r^n, s^n) = \prod_{i=1}^{n} p(y_i, y_{ri}|x_i, x_{ri}, s_i). \quad (2)$$

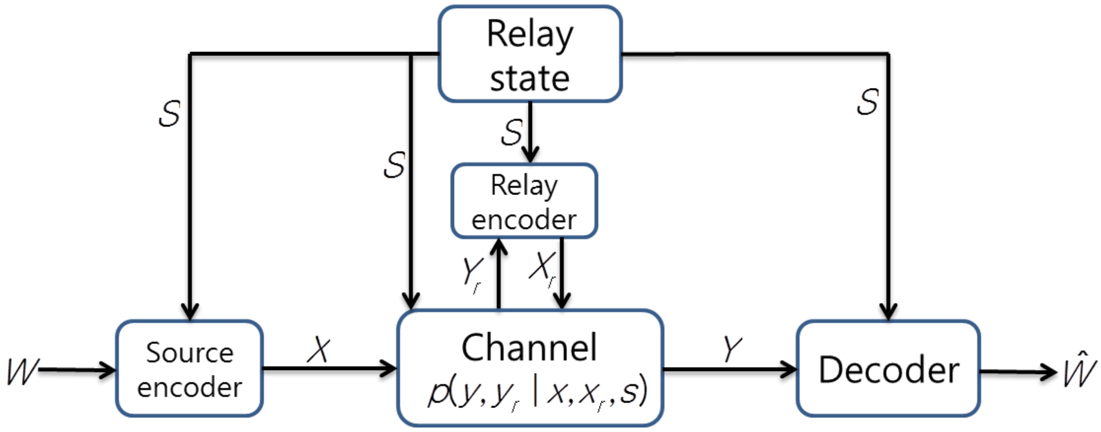

Fig. 2. Coding scheme for DMURC

An $(n, 2^{nR})$ code for DMURC, shown in **Fig. 2**, consists of the followings

- An index set $\mathcal{W}$

$$\mathcal{W} = \{1,2,\cdots,2^{nR}\}, \quad (3)$$

- A set of encoding functions f_i's (at the source)

$$x_i = f_i(w, s^i), \quad 1 \le i \le n,\ w \in \mathcal{W},\ s^i \in \mathcal{S}^n, \quad (4)$$

- A set of encoding functions f_{ri}'s (at the UR)

$$x_{ri} = f_{ri}(y^{i-1}, s^i), \quad 2 \le i \le n,\ s^i \in \mathcal{S}^i, \quad (5)$$

- A decoding function

$$g\colon \mathcal{Y}^n \times \mathcal{S}^n \to \mathcal{W}, \quad (6)$$

- A distribution $p(w)$ on $\mathcal{W}$.

Due to the memoryless property of the channel, the joint probability distribution on $\mathcal{W} \times \mathcal{X}^n \times \mathcal{X}_r^n \times \mathcal{S}^n \times \mathcal{Y}^n \times \mathcal{Y}_r^n$ satisties

$$p(w, x^n, x_r^n, y^n, y_r^n, s^n) = p(w) \prod_{i=1}^{n} p(s_i) \cdot p(x_i | w, s^i) \cdot p(x_{ri} | y_r^{i-1}, s^i) \times \mathrm{p}(y_i, y_{ri} | x_i, x_{ri}, s_i). \quad (7)$$

We note that (7) implies the followings :

- $p(w, s^n) = p(w) \prod_{i=1}^{n} p(s_i)$

- (y_i, y_{ri}) and $\left(x^{i-1}, x_r^{i-1}, y^{i-1}, y_r^{i-1}, s^{i-1}, s_{i+1}^n, w\right)$ are conditionally independent given (x_i, x_{ri}, s_i)
- x_i and $(x^{i-1}, x_r^i, y^{i-1}, y_r^{i-1}, s_{i+1}^n)$ are conditionally independent given (w, s^i)
- x_{ri} and $(x^{i-1}, x_r^{i-1}, y^{i-1}, s_{i+1}^n, w)$ are conditionally independent given $(y_r^{i-1},, s^i)$.

The average probability of error, $P_e^{(n)}$, is given by

$$P_e^{(n)} = \sum_{w \in \mathcal{W}} p(w) \cdot p(g(Y^n, S^n) \neq w \mid w \text{ is sent}). \tag{8}$$

A rate R is said to be achievable if there exists an $(n, 2^{nR})$ code such that $P_e^{(n)} \to 0$ for sufficiently large n. The capacity C of DMURC is the supremum of the achievable rates. A DMURC is said to be degraded if $p(y, y_r|x, x_r, s)$ is written as

$$p(y, y_r|x, x_r, s = 0) = p(y_r|x, x_r, s = 0) \cdot p(y|y_r, x_r, s = 0) \tag{9}$$

and

$$p(y, y_r|x, x_r, s = 1) = p(y_r|y, s = 1) \cdot p(y|x, s = 1). \tag{10}$$

Consequently, given $s = 0$ and $s = 1$, '$X, (X_r, Y_r)$, and Y' as well as 'X, Y, and Y_r', forms a Markov chain, namely, $X \to (X_r, Y_r) \to Y$ and $X \to Y \to Y_r$ for a degraded DMURC.

3. Capacity Bound

In this section, we obtain a capacity upper bound C for the general DMURC modifying the approaches employed in [1] and [7][8][9][10] with additional conditioning on the UR state S.

Lemma 1 For the n^{th} extension of a DMURC,

(1) $I(W; Y^n) \leq \sum_i I(Y_i; X_i, X_{ri}|S_i)$ (11)

(2) $I(W; Y^n) \leq \sum_i I(X_i; Y_i, Y_{ri}|X_{ri}, S_i)$ (12)

Proof: Before attempting to prove (11) and (12), we note the following

$$I(W; Y^n) = H(W) - H(W|Y^n) \overset{(a)}{\leq} H(W|S^n) - H(W|Y^n, S^n) = I(W; Y^n|S^n), \tag{13}$$

in which the inequality (a) follows from the independence between W and S^n and $H(W|Y^n) \geq H(W|Y^n, S^n)$. Now, to prove (11), we first observe that

$$I(W; Y^n|S^n) = H(Y^n|S^n) - H(Y^n|W, S^n)$$

$$= \sum_i \{H(Y_i|Y^{i-1}, S^n) - H(Y_i|Y^{i-1}, W, S^n)\}. \tag{14}$$

Next, we note that the right hand side of (14) is bounded as

$$\begin{aligned}\sum_i \{H(Y_i|Y^{i-1}, S^n) - H(Y_i|Y^{i-1}, W, S^n)\} \\ &\le \sum_i \{H(Y_i|Y^{i-1}, S^n) - H(Y_i|Y^{i-1}, W, S^n, X_i, X_{ri})\} \\ &\le \sum_i \{\{H(Y_i|S_i) - H(Y_i|S_i, X_i, X_{ri})\}\},\end{aligned} \tag{15}$$

where the first inequality follows from the fact that conditioning only reduces the entropy and the second one from the memoryless property of the channel. Finally, the right hand side of (15) becomes

$$\sum_i \{\{H(Y_i|S_i) - H(Y_i|S_i, X_i, X_{ri})\}\} = \sum_i I(Y_i; X_i, X_{ri}|S_i), \tag{16}$$

which implies (11).

To prove (12), we begin with the following inequality

$$I(W; Y^n|S^n) \le I(W; Y^n, Y_r^n|S^n). \tag{17}$$

Here, the right hand side of (17) leads to

$$\begin{aligned}I(W; Y^n, Y_r^n|S^n) &= \sum_i I(W; Y_i, Y_{ri}|Y^{i-1}, Y_r^{i-1}, S^n) \\ &= \sum_i \{H(W|Y^{i-1}, Y_r^{i-1}, S^n) - H(W|Y^i, Y_r^i, S^n)\} \\ &= \sum_i \{H(W|Y^{i-1}, Y_r^{i-1}, S^n, X_{ri}) - H(W|Y^i, Y_r^i, S^n, X_{ri})\},\end{aligned} \tag{18}$$

where the third equality follows from the fact that X_{ri} is a function of (Y^{i-1}, S^i). We can further manipulate the right hand side of (18) to obtain

$$\begin{aligned}\sum_i \{H(W|Y^{i-1}, Y_r^{i-1}, S^n, X_{ri}) - H(W|Y^i, Y_r^i, S^n, X_{ri})\} \\ &= \sum_i I(W; Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}) \\ &= \sum_i \{H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}) - H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}, W)\} \\ &= \sum_i \{H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}) - H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}, X_i, W)\},\end{aligned} \tag{19}$$

in which the last equality follows from the fact that X_i is a function of (W, S^n). Finally, by the conditoning property of the entropy and memoryless property of the channel, we obtain

$$\sum_i \{H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}) - H(Y_i, Y_{ri}|S^n, Y^{i-1}, Y_r^{i-1}, X_{ri}, W, X_i)\}$$
$$\leq \sum_i \{H(Y_i, Y_{ri}|S_i, X_{ri}) - H(Y_i, Y_{ri}|S_i, X_i, X_{ri})\}$$
$$= \sum_i I(X_i; Y_i, Y_{ri}|X_{ri}, S_i). \tag{20}$$

which implies (12). ∎

Theorem 1 *The capacity of DMURC is bounded by*

$$C \leq \sup_{p(x,x_r|s)} \min\{I(X; Y, Y_r|X_r, S), I(X, X_r; Y|S)\} \tag{21}$$

Proof: Consider the n^{th} extension of the given DMURC described in Section 2. Let R be an achievable rate. Then, from the Fano's inequality,

$$H(W|Y^n) \leq p_e^{(n)} nR + 1 \equiv n\varepsilon_n \tag{22}$$

which implies

$$\lim_{n\to\infty} \frac{1}{n} H(W|Y^n) = 0, \tag{23}$$

since

$$\lim_{n\to\infty} \frac{1}{n} H(W|Y^n) \leq \lim_{n\to\infty} \left\{p_e^{(n)} R + \frac{1}{n}\right\} = 0. \tag{24}$$

Now, since

$$nR \leq H(W) = I(W; Y^n) + H(W|Y^n), \tag{25}$$

it follows

$$R \leq \lim_{n\to\infty} \frac{1}{n} I(W; Y^n). \tag{26}$$

Now, from Lemma 1, it follows that

$$R \leq \lim_{n\to\infty} \frac{1}{n} \sum_i I(X_i, X_{ri}; Y_i|S_i) \tag{27}$$

and

$$R \leq \lim_{n\to\infty} \frac{1}{n} \sum_i I(X_i; Y_i, Y_{ri} | X_{ri}, S_i). \tag{28}$$

Let Z be a random variable which is distributed uniformly on $\{1,2,\cdots,n\}$ and independent of X, X_r, Y, Y_r, and S. Put $X \equiv X_Z, X_r \equiv X_{rZ}, Y \equiv Y_Z, Y_r \equiv Y_{rZ}, S \equiv S_Z$. Then,

$$\frac{1}{n} \sum_i I(X_i, X_{ri}; Y_i | S_i) = I(X, X_r; Y | S, Z) = I(X, X_r; Y | S) \tag{29}$$

and similarly,

$$\frac{1}{n} \sum_i I(X_i; Y_i, Y_{ri} | X_{ri}, S_i) = I(X; Y, Y_r | X_r, S, Z) = I(X; Y, Y_r | X_r, S). \tag{30}$$

Nowt, it follows that R is bounded by $I(X, X_r; Y|S)$ and $I(X; Y, Y_r | X_r, S)$, completing the proof of Theorem 1. ∎

4. Achievable Rate Region

In this section, we get a capacity lower bound C_{lo} of the general DMURC and show that this lower bound coincides with the upper bound when the channel is degraded.

Theorem 2 *The capacity lower bound C_L of the DMURC is given by*

$$C_L = \sup_{p(x,x_r|s)} [\, p(s=1) \cdot I(X; Y | s=1) + p(s=0) \cdot \min\{I(X; Y_r | X_r, s=0), I(X, X_r; Y | s=0)\}] \tag{31}$$

Proof

Overview : To show the achievability of the capacity lower bound C_L, we employ a block Markov encoding scheme in which B blocks, each consisting of n channel use, are used to transmit $B-1$ messages since the last block is used to transmit some predefined symbol. One of the key ideas is in dividing transmitted symbols into two groups, one for state 0 and the other for state 1. Let $N_1 = S_1 + \cdots + S_n$, the number of times the channel is in state 1 during a given transmission block. Similarly, $N_0 = n - N_1$ denotes the number of times the channel is in state 0. We note that $\frac{1}{n} N_0 \to p(s=0)$ and $\frac{1}{n} N_1 \to p(s=1)$, in probability, as $n \to \infty$. Consequently, the channel will be in state 0 (and hence in state 1) approximately fixed number of times for different transmission blocks if n is very large. Noting this fact, we generate two separate codebooks, $\mathcal{C}_0$ for $n_0 \equiv n \cdot p(s=0)$ channel use of state 0 and $\mathcal{C}_1$ for $n_1 \equiv n \cdot p(s=1)$ channel use of state 1. Here, we note that the codebook $\mathcal{C}_0$ must be defined to consist of pairs of n_0 dimensional codewords, one for the source and the other for the UR, while $\mathcal{C}_1$ consists of n_1 dimensional codewords.

To be more specific, let us denote by $\mathcal{W}_0$ and $\mathcal{W}_1$ the message index sets corresponding to $\mathcal{C}_0$ and $\mathcal{C}_1$, respectively. Then, the set $\mathcal{W}_0 \times \mathcal{W}_1$ can be regarded as the message index set $\mathcal{W}$ for overall transmission. In what follows, we denote by $\big(w_0(b), w_1(b)\big)$ the message transmitted during the b^{th} transmission block. Given the message $\big(w_0(b), w_1(b)\big)$, a codeword $x_1^{n_1}\big(w_1(b)\big)$ corresponding to $w_1(b)$, is selected in $\mathcal{C}_1$ while a codeword pair $\Big(x_0^{n_0}\big(w_0(b)\big), x_r^{n_0}\big(w_1(b)\big)\Big)$ corresponding to $(w_0(b), w_1(b))$ is selected in $\mathcal{C}_0$. Here, we note that the codewords $x_0^{n_0}\big(w_0(b)\big)$ and $x_1^{n_1}\big(w_1(b)\big)$ are transmitted from the source in state 0 and 1, respectively, and the codeword $x_r^{n_0}\big(w_1(b)\big)$ is sent from the UR in state 0. For example, to send the $w_0(b)$, the source transmits the codeword symbols $x_{01}\big(w_0(b)\big), \cdots, x_{0n_0}\big(w_0(b)\big)$ one by one during the symbol durations in which the channel is in state 0. Let $N_k(b)$ $(k = 0,1)$ be the number of symbol durations (within the transmission block b) during in which the channel is in state k. Since $N_0(b)$ may not be exactly the same as the n_0, there are two possible cases: 1) $N_0(b) > n_0$ and 2) $N_0(b) \le n_0$. If $N_0(b) \le n_0$, the source transmits first $N_0(b)$ symbols of $x_0^{n_0}\big(w_0(b)\big)$. If $N_0(b) > n_0$, after the whole sequence $x_0^{n_0}\big(w_0(b)\big)$ is transmitted, the source repeats, $N_0(b) - n_0$ times, to transmit the last symbol $x_{0n_0}(w_0(b))$. In a similar manner, the $x_1^{n_1}(w_1(b))$ and $x_r^{n_0}(w_1(b))$ are transmitted from the source (in state 1) and the UR (in state 0), respectively.

In decoding, the destination divides the received sequence y^n into length $(1-\varepsilon)n_k$ subsequences $\tilde{y}_k$ consisting of the first $(1-\varepsilon)n_k$ symbols corresponding to state k $(k = 0,1)$ and then decodes w_k separately using $\tilde{y}_k$. We assume that the codebooks are defined (or the code rates are chosen) so that the message w_k is completely specified by the first $(1-\varepsilon)n_k$ codeword symbols. Here, we note that the probability that $N_k(\mathrm{b}) < (1-\varepsilon)n_k$ holds approaches 0 as $n \to \infty$ for any $\varepsilon > 0$. Consequently, the decoder almost always enough number of received symbols for decoding as long as the codebook is so designed that $(1-\varepsilon)n_k$ channel use is enough to transmit the message w_k. We note that the channel capacity decreases by factor of $(1-\varepsilon)\cdot p(s=0)$ if the channel is used $(1-\varepsilon)n_k$ times during a transmission block which consists of n symbol durations.

Codebook generation : Let $p(x, x_r|s=0)$ and $p(x|s=1)$ be given probability distributions and let R_0, R_0', and R_1 be rates satisfying $R = R_0 + R_1$ and $R_0 > R_0'$. Noting that, R_0 and R_1 stand for the rates at which $w_o \in \mathcal{W}_0$ and $w_1 \in \mathcal{W}_1$ are sent, respectively and R_0' for the rate at which the UR routes the source message in state 0. As we described above, we generate two separate code books codebooks $\mathcal{C}_0$ and $\mathcal{C}_1$ for use in state 0 and 1, respectively. Due to the simplicity, we begin with the generation of $\mathcal{C}_1 = \{\, x_1^{n_1}(w_1) : w_1 \in \{1,2,\cdots,2^{nR_1}\}\,\}$. According to the probability distribution $p(x|s=1)$, $n_1 \cdot 2^{nR_1}$ symbols are generated randomly and independently one by one. The first n_1 symbols are collected to form the first codeword $x_1^{n_1}(1)$, the next n_1 symbols to form the second codeword $x_1^{n_1}(2)$, and so on.

The generation of the codebook $\mathcal{C}_0 = \{(x_r^{n_0}(t), x_0^{n_0}(w_o|t)) \colon t \in \{1,2,\cdots,2^{nR_0'}\},\ w_0 \in \{1,2,\cdots,2^{nR_0}\}\}$ is a little bit more complex. This is primarily because both the source and the UR transmit signals in state 0 and hence a pair of codewords must be generated instead of a single codeword for each message. To be more specific, the UR determines what to transmit in $\{1,2,\cdots,2^{nR_0'}\}$ from the signal received in the previous transmission block and sends an appropriate codeword. The UR codewords $x_r^{n_0}(1), \cdots, x_r^{n_0}\big(2^{nR_0'}\big)$ are generated in the same way as $x_1^{n_1}$'s by randomly generating $n_0 \cdot 2^{nR_0}$ symbols independently according to the probability distribution $p(x_r|s=0)$ which is obtained by marginalizing $p(x, x_r|s=0)$. The source must also know what the UR transmits and sends an correspondingly generated codeword. For this reason, 2^{nR_0} source codewords are generated for each UR message

$t \in \{1,2,\cdots,2^{nR_0'}\}$, or equivalently, for each UR codeword $x_r^{n_0}(t)$. For a given t, n_0 symbols are generated randomly and independently according to $p(x|s=0,x_{r1}^{n_0}(t)),\cdots,\ p(x|s=0,x_{rn}^{n_0}(t))$ and are collected to form the first source codeword $x_0^{n_0}(1|t)$. This process of random n_0-symbol generation is repeated independently until the last source codeword $x_0^{n_0}(2^{nR_0}|t)$ is generated.

Encoding : As we mentioned previously, we employ a block Markov encoding method. We assume that the source transmits information bearing method during the first $B-1$ transmission blocks while it transmits, in the last (=B^{th}) transmission block, a known message which we set to be 1, while the UR relays information bearing signal during the last $B-1$ transmission blocks. Let $w(b)$ be the message to be sent in block $b \in \{1,\cdots,B\}$. As we just described, we assume that $w(B)=1$. The message $w(b)$ is first mapped into a unique pair $(w_0(b),w_1(b)) \in \mathcal{W}_0 \times \mathcal{W}_1$ of indices, assuming that conversion algorithm is known to all nodes. We assume that the index set $\mathcal{W}_0$ is pre-partitioned into $2^{nR_0'}$ equal-sized sets $\mathcal{W}_{0,t} \subset \mathcal{W}_0,\ t \in \{1,\cdots,2^{nR_0'}\}$. After $(b-1)^{th}$ transmission block, the UR determines the set $\mathcal{W}_{0,t(b)}$ to which the message $w_0(b-1)$ belongs. Here we assume that $w_0(0)$ is predefined to be 1. Then, the UR chooses $x_r^{n_0}(t(b))$ as the codeword to be transmitted during the transmission block b. Naturally, the source also determines $t(b)$ and choose its state 0 codeword $x_0^{n_0}(w_0(b)|t(b))$. We note that the source state 1 codeword $x_1^{n_1}(w_1(b))$ is chosen block by block independently of the previous or the further message.

As mentioned before, since $N_k(b)$ may not be exactly the same as the n_k there are two possible cases; 1) $N_0(b) \le n_0$ and $N_1(b) > n_1$ or 2) $N_0(b) > n_0$ and $N_1(b) \le n_1$. We note that, at the beginning of the block b, the source and the UR do not know the case of block b since states are given causally, or equivalently, s_i is given just before transmitting i^{th}symbol. Counting the number of states, the source transmits symbols from $x_k^{n_k}(w_k(b))$ one by one during the symbol durations in which the channel is in state $k, k=0,1$. The UR transmits symbols from $x_r^{n_0}(w_0(b))$ one by one only if the channel is in state 0.

If the case is determined as 0 (counting number of 1 states are greater than n_1), the source re-transmits symbols from $x_1^{n_1}(w_1(b))$ in which the channel state is 1. If channel is in state 0, otherwise, the source and the UR keep transmitting remaining symbols of $x_0^{n_0}(w_0(b))$ and $x_r^{n_0}(w_0(b))$ one by one, respectively. Likewise, if the system is in case 1 (counting number of 0 states are greater than n_0), the source and the UR re-transmit symbols from $x_0^{n_1}(w_0(b))$ and $x_r^{n_0}(w_0(b))$, respectively, in which the channel state is 0, and the source transmitting remaining symbols of $x_1^{n_1}(w_1(b))$ one by one in state 1.

Decoding and Achievable Rates: In the followings, we derive the decoding algorithm of the UR and the destination after b^{th} block transmissions. By hypothesis, the destination and the UR know the channel state sequence $s^n(b)$. Given $s^n(b)$, the destination and the UR divide the received sequence according to the channel state. To be more specific, the destination divides $y^n(b)$ into $\tilde{y}_0(b)$ and $\tilde{y}_1(b)$ where $\tilde{y}_k(b)$ denotes the subsequence consisting of the first $(1-\varepsilon)n_k$ symbols corresponding to state k in transmission block b. Likewise the UR divides the received sequence $y_r^n(b)$ into $\tilde{y}_{r,0}(b)$ and $\tilde{y}_{r,1}(b)$.

We first derive the decoding algorithm of the UR. At the end of the block b, upon receiving $\tilde{y}_0(b)$, the UR estimates the $w_0(b)$. The UR declares $\widehat{w}_0(b)=w$ if there exists unique w such that $(x_0^{(1-\varepsilon)n_0}(w|t(b)),\ x_r^{(1-\varepsilon)n_0}(t(b)),\ \tilde{y}_0(b))$ are jointly ε-typical. Otherwise, the UR declares an error. Here, we note that $t(b)$ is assumed to be determined by the UR after

$(b-1)^{th}$ transmission block. Similarly to the proof in [1], it can be shown that $\widehat{w}_0(b) = w_0(b)$ with an arbitrary small error if

$$R_0 < (1-\varepsilon)\cdot p(s=0)\cdot I(X;Y_r|X_r, s=0). \quad (32)$$

At the destination, after b^{th} block transmission, the $w_0(b-1)$ and the $w_1(b)$ are decoded. To decode the $w_0(b-1)$, upon receiving $\tilde{y}_0(b)$, the destination declares $\hat{t}(b) = t$ if there exists a unique t such that $(\tilde{y}_0(b),\ x_r^{(1-\varepsilon)n_0}(t))$ are jointly ε-typical. Otherwise, the destination declares an error. It can be shown that $\hat{t}(b) = t(b)$ with an arbitrary small error if

$$R_0' < (1-\varepsilon)\cdot p(s=0)\cdot I(Y;X_r|s=0). \quad (33)$$

Estimating $\hat{t}(b)$, the destination declares $\widehat{w}_0(b-1) = w$ if there exists unique w such that $w \in \mathcal{W}_{0,t(b)}$, simultaneously $(x_0^{(1-\varepsilon)n_0}(w|t(b-1)),\ \tilde{y}_0(b-1),\ x_r^{(1-\varepsilon)n_0}(t(b-1)))$ are jointly $\varepsilon-$typical. Otherwise, the destination declares an error. Here, we note that $t(b-1)$ is assumed to be determined by the destination after $(b-1)^{th}$ transmission block. Similarly to the proof in [1], it can be shown that $\widehat{w}_0(b-1) = w_0(b-1)$ with an arbitrary small error if

$$R_0 < (1-\varepsilon)\cdot p(s=0)\cdot I(X;Y|X_r, s=0) + R_0'. \quad (34)$$

From the (33) and (34) we can obtain

$$R_0 < (1-\varepsilon)\cdot p(s=0)\cdot I(X, X_r; Y|s=0). \quad (35)$$

To decode $w_1(b)$, upon receiving $\tilde{y}_1(b)$, the destination declares that $\widehat{w}_1(b) = w$ if there exists an unique w such that $(x_1^{(1-\varepsilon)n_1}(w),\ \tilde{y}_1(b))$ are jointly ε-typical. Otherwise, the destination declares an error. By the Shannon's channel coding theorem, $\widehat{w}_1(b) = w_1(b)$ with an arbitrary small error if

$$R_1 < (1-\varepsilon)\cdot p(s=1)\cdot I(X;Y|s=1). \quad (36)$$

With a sufficiently large n, ε goes to zero. From the (32) and (35), we can conclude that the effective rate $\tilde{R} = \frac{R_0(B-1)+R_1B}{B}$ is achievable with an arbitrary small error if

$$\tilde{R} < p(s=1)\cdot I(X;Y|s=1) + p(s=0)\times \min\{I(X;Y_r|X_r, s=0), I(X, X_r; Y|s=0)\}. \quad (37)$$

By a sufficiently large B, the $\tilde{R}$ goes to the R, completing the proof of Theorem 2. ∎

Corollary *The capacity lower bound C_L becomes the channel capacity when the channel is degraded.*

Proof

Given state 0 and 1, $X \to (X_r, Y_r) \to Y$ and $X \to Y \to Y_r$, respectively, for the degraded channel. Hence, it immediately follows that

$$I(X;Y_r|X_r,s=0)=I(X;Y,Y_r|X_r,s=0), \tag{38}$$

and

$$I(X;Y|s=1)=I(X;Y,Y_r|s=1). \tag{39}$$

Also, from (1), namely, from the fact that (X,Y,Y_r) and X_r are conditionally independent given state 1, we obtain

$$I(X;Y,Y_r|s=1)= I(X;Y,Y_r|X_r,s=1) \tag{40}$$

and

$$I(X;Y|s=1) \quad = I(X,X_r;Y|s=1). \tag{41}$$

Plugging (38) - (41) into right side of the (31), we can show that a lower bound of the degraded DMURC $C_{L,d}$ is given by

$$C_{L,d}= \sup_{p(x,x_r|s)} \min\{I(X;Y,Y_r|S),I(X,X_r;Y|Xr,S)\}, \tag{42}$$

which coincides the upper bound in Theorem 1. ■

5. Conclusion

In this paper, we considered the capacity of DMURC in which the UR puts higher priority on its user role and switches its operational modes symbol-by-symbol. We regarded the operational modes, namel, relay and user modes, of the UR as the channel states, and assumed that the state information is causally available at the receiver and the transmitter in addition to the UR. We first obtained the capacity upper bound of the general DMURC. The result has the form of the cut set upper bound shown in [1] with additional conditioning on the channel state, namley, the UR operational mode. Then, we derived the capacity lower bound of the general DMURC. Finally, we showed that the lower bound is the same as the upper bound when the channel is degraded. While we assumed the FSUR channel, the results can be applied directly to the SSUR channel by regarding a symbol as a packet.

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