# 비젼 기반 자율주행을 위한 다중비율 예측기 설계와 모델예측 기반 능동조향 제어

# MPC-based Active Steering Control using Multi-rate Kalman Filter for Autonomous Vehicle Systems with Vision

김 보 아<sup>\*</sup>·이 영 옥<sup>\*\*</sup>·이 승 희<sup>\*\*\*</sup>·정 정 주<sup>†</sup> (Bo-Ah Kim·Young Ok Lee·Seung-Hi Lee·Chung Choo Chung)

Abstract – In this paper, we present model predictive control (MPC) applied to lane keeping system (LKS) based on a vision module. Due to a slow sampling rate of the vision system, the conventional LKS using single rate control may result in uncomfortable steering control rate in a high vehicle speed. By applying MPC using multi-rate Kalman filter to active steering control, the proposed MPC-based active steering control system prevents undesirable saturated steering control command. The effectiveness of the MPC is validated by simulations for the LKS equipped with a camera module having a slow sampling rate on the curved lane with the minimum radius of 250[m] at a vehicle speed of 30[m/s].

Key words : Lane keeping system(LKS), Model predictive control(MPC), Multi-rate system, Steering control, Lane detection

### Nomenclature

 $\{XYZ\}$  : global coordinate frame

 $\{xyz\}$ : local coordinate frame

x: longitudinal position of the origin of  $\{xyz\}$  coordinate to the font fixed point along the longitudinal axis

to the folit fixed point along the folightudinal axis

y : lateral position of the origin of  $\{xyz\}$  coordinate to the rotation center 'O' along the lateral axis

 $V_x$ : longitudinal velocity of the vehicle at c.g.

 $\psi$ : yaw angle

 $\dot{\psi}$  : yaw rate

 $\beta$ : side slip angle

 $e_{yL} = y_L - y_L^d$ : lateral position error w.r.t. reference at look-ahead distance

 $e_{\psi} = \psi^d - \psi$ : yaw angle error w.r.t. road

 $C_a$ : cornering stiffness of tire

 $I_z$ : yaw moment of inertia of vehicle

m: total mass of the vehicle

l: distance of the tire respective from c.g. of the vehicle  $\delta$ : steer angle

L: look-ahead distance from c.g. to look ahead point

- $N_P$ : predictive horizon
- $N_C$ : control horizon

*	준	회	원	:	한양대	공대	전기공학과	석사과정
---	---	---	---	---	-----	----	-------	------

\*\* 정 회 원 : 한양대 공대 전기공학과 박사과정

\*\*\* 정 회 원 : 한양대 공대 전기생체공학부 교수

 \* 교신저자, 정회원 : 한양대 공대 전기생체공학부 교수 E-mail : cchung@hanyang.ac.kr 접수일자 : 2012년 4월 2일
 최종완료 : 2012년 4월 28일 Q, R: MPC weighting for states and control command u: input

## Subscripts

f: front

r: rear

L : value at look-ahead distance

#### Superscripts

d: desired value

# 1. INTRODUCTION

The active safety systems have increased in the automotive industry [1–3]. Anti–lock braking system and electronic stability program enhance the vehicle stability by controlling the brake systems effectively. Lane keeping system (LKS) controls the front steer angle in order to improve lateral vehicle stability. Through implementation of the LKS, the steer angle can be assisted by getting the information of road environment and vehicle's states. Moreover, LKS can be implemented for the autonomous vehicle systems by tracking the reference trajectory. The robust switching LKS controller was proposed in [1], where the vehicle has magnetic sensors and it tracks the lane reference with magnetic markers. Recent trends in LKS research are using the vision system to obtain the road information. The lane keeping assistant system

(LKAS) with vision module is also studied with the desired reference path generation algorithm and optimal controller [2]. It was reported that the model based predicted controller for LKAS with a vision module is robust against time delay [3].

For industrial application of vision systems, fast road information measurement is not available due to cost restriction on vision modules. Thus, the slow sampling period and time delay of a camera module compared to the car electronic control unit (ECU) and inertia measurement units (IMU) should be considered in the design of controller. The conventional method for resolving such slow sampling rate and time delay is generating the steer angle at the same period as the measurement period of the vision module. This method, however, leads to an inaccurate solution of the control command, and as a result the vehicle may have undesirable lateral behavior such as oscillatory vaw rate. The previous results[1-3], however, do not consider these problems. For resolving these problems, we propose multi-rate Kalman filter that can estimate the informations of the vision data at the period of the control command. Multi-rate Kalman filter can make optimal signal having the minimum-variance reconstruction based on the probability theory [4],[5].

In this paper, we propose a model predictive control (MPC) for multi-rate LKS. MPC computes a sequence of control inputs to optimize the future behavior under various constraints [6],[7]. The MPC has been applied to active front steering control [8-10]. These papers also do neither consider slow sampling rate nor the camera module's time delay. And performances were evaluated at only low vehicle speeds. In this paper, the MPC is applied to the LKS under the constraints of steer angle and rate. We demonstrate the effectiveness of the proposed multi-rate estimation and control system through simulations based on three scenarios: (1) a slowly controlled system at a slow rate as the vision module. (2) a fast controlled system at each ECU computation instance utilizing a high-speed vision module. (3) the proposed method : multi-rate system with the multi-rate Kalman filter utilizing a slow rate vision module.

Simulation results show that the system using the multi-rate estimator improves lane keeping performance even at high speeds. we confirm that the system using multi-rate estimator with a low cost vision module was competitive performance with the system having the fast vision module.

This paper is structured as follows : Section 2 describes the lateral dynamics of a vehicle. Section 3 describes the multi-rate Kalman filter, the MPC method and control structure of the system. Section 4 shows simulation results under various scenarios.





Fig. 1 Lateral vehicle dynamics

The simple bicycle model [11] is used to model the lateral dynamics of a vehicle. Fig. 1 shows the lateral vehicle dynamics on the global coordinate. We treat the lateral control system using the vision processing system, thus the lateral model in terms of lateral offset at look-ahead distance is useful [12]. The time derivative of lateral offset at the look-ahead distance is described by

$$\dot{e}_{yL} = V_x \left( \theta_{yf} + e_{\psi L} \right) - L \dot{\psi}^d. \tag{1}$$

Since  $\theta_{y} = \beta + L\dot{\psi} / V_x$  and  $\dot{e}_y = V_x (e_y - \beta)$ , we see that

$$\begin{split} \dot{e}_{yL} &= V_x \beta + L \dot{\psi} + V_x e_{\psi L} - L \dot{\psi}^d \\ &= \dot{e}_y - V_x e_{\psi} + L \dot{\psi} + V_x e_{\psi L} - L \dot{\psi}^d \,. \end{split}$$

Here, by defining

$$e_{\psi L} = e_{\psi} + \eta_L$$

where  $\eta_L$  denotes the difference between yaw angles at the vehicle's center of gravity (c.g.) and at the look-ahead distance, equation (1) can be rewritten as

$$\dot{e}_{yL} = \dot{e}_y + L\dot{\psi} + V_x\eta_L - L\dot{\psi}^d$$

Then, the state-space model in terms of the state vector  $\mathbf{x} = \begin{bmatrix} e_{yL} & \dot{e}_y & e_{\psi} & \dot{\psi} \end{bmatrix}^T$ , the control input  $\mathbf{u} = \delta$  and output  $\mathbf{y} = \begin{bmatrix} e_{yL} & e_{\psi} & \dot{\psi} \end{bmatrix}^T$  is obtained as :

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + B_2 \mathbf{u} + B_q \mathbf{q} \\ \mathbf{y} &= C\mathbf{x}, \end{aligned} \tag{2}$$

where

$$\begin{split} &a_{22} = -\frac{2C_{af} + 2C_{ar}}{mV_x}, \quad a_{23} = -a_{22}V_x, \quad a_{24} = -1 - \frac{2C_{af}l_f - 2C_{ar}l_r}{mV_x^2}, \\ &a_{24}' = (a_{24} - 1)V_x, \quad a_{42} = -\frac{2C_{af}l_f - 2C_{ar}l_r}{I_z}, \quad a_{42}' = \frac{a_{42}}{V_x}, \quad a_{43} = -a_{42}, \\ &a_{44} = -\frac{2C_{af}l_f^2 + 2C_{ar}l_r^2}{I_zV_x}, \quad b_{21} = \frac{2C_{af}}{mV_x}, \quad b_{21}' = b_{21}V_x, \quad b_{41} = \frac{2C_{af}l_f}{I_z}, \\ &\mathbf{q} = \begin{bmatrix} \dot{\psi}^d \\ \eta_L \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & -L \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & -1 \\ 0 & a_{42}' & a_{43} & a_{44} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ b_{21}' \\ 0 \\ b_{41} \end{bmatrix}, \\ &B_q = \begin{bmatrix} L & V_x \\ V_x & 0 \\ 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{split}$$

The discretization of (2) using the zero-order hold (ZOH) equivalence at sampling rate  $1/T_c$  leads to the discrete-time matrices  $(\Phi, \Gamma_2, \Gamma_q, C)$  from  $(A, B_2, B_q, C)$  such that

 $\mathbf{x}(k+1) = \Phi \mathbf{x}(k) + \Gamma_2 \mathbf{u}(k) + \Gamma_q \mathbf{q}(k)$  $\mathbf{y}(k) = C \mathbf{x}(k),$ 

where

$$\Phi = e^{At_c}, \ \Gamma_2 = \int_0^{T_c} e^{A(T_c - v)} B_2 dv, \ \Gamma_q = \int_0^{T_c} e^{A(T_c - v)} B_q dv.$$

# 3. MODEL PREDICTIVE CONTROLLER FOR LANE KEEPING SYSTEM

In this section, we design the MPC for lane keeping system based on the prediction equation. The prediction equation is developed based on the estimated signals from the multi-rate Kalman filter.

#### 3.1. Design of multi-rate Kalman filter

The output *y* can be measured or available from the vision processing system and IMU. The update periods of states, however, are different according to sensors configurations : The vision processing system provides the information of road lane such as lateral offset, heading angle, curvature and curvature derivative with respect to the vehicle's c.g. at a slow sampling rate of  $1/T_{cam}$ . The IMU provides the values of steering wheel angle, ratio of steering wheel angle, and yaw rate at the same sampling rate as that of car ECU,  $1/T_c$ . For the simplicity of presentation, we assume that  $T_{cam}$  is an integer-multiple of  $T_c$  and all measurements are synchronized, that is,  $T_{cam} = R_m T_c$ ,  $R_m \ge 1$ ,  $R_m \in \Box$ . We can, thus, represent a time instant

$$t = (k + i / R_m) T_{cam}, k = 0, 1, \dots, \text{ and } i = 0, 1, \dots, R_m - 1,$$

where k and i indicate the vision processing update and the control update instance, respectively. To estimate the all states in (2) at the rate of  $1/T_c$ , a multi-rate Kalman filter is designed for the partitioned dynamics  $\mathbf{x} = [\mathbf{x}_v \ \mathbf{x}_m]^T$ comprising the state  $\mathbf{x}_v = [e_{yL} \ \dot{e}_y \ e_w]^T$  having slow measurement sampling rates and the state  $\mathbf{x}_m = [\psi]$  having fast measurement sampling rates [12].

The slow dynamics related with the data from vision processor can be arranged as :

$$\begin{aligned} \dot{\mathbf{x}}_{v} &= A_{v}\mathbf{x}_{v} + B_{v}\mathbf{u} + B_{\psi}\mathbf{x}_{m} + B_{q}\mathbf{q} \\ \mathbf{y}_{v} &= C_{v}\mathbf{x}_{v}, \end{aligned} \tag{3}$$

where

$$\begin{split} A_{\nu} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}, \ B_{\nu} &= \begin{bmatrix} 0 \\ b_{21}^{'} \\ 0 \end{bmatrix}, \ B_{\nu} &= \begin{bmatrix} -L \\ a_{24}^{'} \\ -1 \end{bmatrix}, \\ B_{q} &= \begin{bmatrix} L & V_{x} \\ V_{x} & 0 \\ 1 & 0 \end{bmatrix}, \ C_{\nu} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \ \mathbf{q} = \begin{bmatrix} \dot{\psi}^{d} \\ \eta_{L} \end{bmatrix}. \end{split}$$

The discretization of (3) using the ZOH equivalence at sampling rate  $1/T_c$  leads to the discrete-time matrices  $(\Phi_v, \Gamma_v, \Gamma_v, \Gamma_q)$  from  $(A_v, B_v, B_q, B_q)$ . Then the estimated slow state vector  $\hat{\mathbf{x}}_v$  is obtained by designing the following Kalman filter :

$$\begin{aligned} \overline{\mathbf{x}}_{\nu}(k,i+1) &= \mathbf{\Phi}_{\nu} \widehat{\mathbf{x}}_{\nu}(k,i) + \Gamma_{\nu} \mathbf{u}(k,i) + \Gamma_{\psi} \widehat{\mathbf{x}}_{m}(k,i) + \Gamma_{q} \mathbf{q}(k,i) \\ \widehat{\mathbf{x}}_{\nu}(k,i) &= \overline{\mathbf{x}}_{\nu}(k,i) + L_{\nu} \big( \mathbf{y}_{\nu}(k,0) - C_{\nu} \overline{\mathbf{x}}_{\nu}(k,0) \big), \end{aligned}$$
(4)

where  $L_v$  is state estimator gain to be chosen off-line such that the mean square state estimation error is the smallest possible. Through this current estimator, the predicted state vector  $\bar{\mathbf{x}}_v$  is corrected as  $\hat{\mathbf{x}}_v$  based on the measurement data  $\mathbf{y}_v(k,0)$  [13].

The fast dynamics related with the data from IMU can be described as:

$$\begin{aligned} \dot{\mathbf{x}}_m &= A_m \mathbf{x}_m + B_m \mathbf{u} + B_{xv} \mathbf{x}_v \\ \mathbf{y}_m &= C_m \mathbf{x}_m, \end{aligned} \tag{5}$$

where

$$A_m = [a_{44}], B_m = [b_{41}], B_{xv} = \begin{bmatrix} 0 & a_{42} & a_{43} \end{bmatrix}, C_m = \begin{bmatrix} 1 \end{bmatrix}$$

The discretization of (5) using the ZOH method at

sampling rate  $1/T_c$  leads to the discrete-time matrices  $(\Phi_m, \Gamma_m, \Gamma_{xv})$  from  $(A_m, B_m, B_{xv})$ . Then the estimated state vector  $\hat{\mathbf{x}}_m$  is obtained by designing the following Kalman filter :

$$\begin{aligned} \overline{\mathbf{x}}_{m}\left(k,i+1\right) &= \Phi_{m}\widehat{\mathbf{x}}_{m}\left(k,i\right) + \Gamma_{m}\mathbf{u}\left(k,i\right) + \Gamma_{xy}\widehat{\mathbf{x}}_{v}\left(k,i\right) \\ \widehat{\mathbf{x}}_{m}\left(k,i\right) &= \overline{\mathbf{x}}_{m}\left(k,i\right) + L_{m}\left(\mathbf{y}_{m}\left(k,i\right) - C_{m}\overline{\mathbf{x}}_{m}\left(k,i\right)\right), \end{aligned}$$
(6)

where  $L_m$  is state estimator gain. Due to the designed multi-rate Kalman filter, all of states in (6) can be estimated at the same rate as ECU rate. Even though we only treat the case when  $R_m$  is a fixed integer, the Kalman filter can be generalized straightforwardly to case when  $R_m$  is a randomly varied integer [14].

#### 3.2. Design of model predictive controller



Fig. 2 Controller state at the k-th sampling instant

For the LKS, the control objective is following the desired lateral offset trajectory with the fulfillment under various constraints reflecting the vehicle physical limits and industrial control specifications. The MPC is an effective method to treat the tracking system by incorporating the constraints into the controller formulation. The MPC computes a set of optimal inputs that will drive the plant to the desired trajectory without violating constraints [7] as shown in Fig. 2. The input constraints for the LKS such as steer angle and steer angle ratio are from the attributes of steering electric power steering. The constrained optimization problem can be solved by online quadratic programming (QP) at each sampling time using the current states and previous input value [15].

Let us define lifting matrices  $\tilde{\mathbf{u}}(k)$ ,  $\Delta \tilde{\mathbf{u}}(k)$ ,  $\tilde{\mathbf{x}}(k)$ ,  $\tilde{\mathbf{y}}(k)$  for the input  $\mathbf{u}(k)$ , incremental input  $\Delta \mathbf{u}(k) = \mathbf{u}(k) - \mathbf{u}(k-1)$ , state  $\mathbf{x}(k)$ , and output prediction  $\mathbf{y}(k)$  such as

$$\begin{split} \tilde{\mathbf{u}}(k) &= \begin{bmatrix} \mathbf{u}(k) \\ \vdots \\ \mathbf{u}(k+N_c-1) \end{bmatrix}, \Delta \tilde{\mathbf{u}}(k) = \begin{bmatrix} \Delta \mathbf{u}(k) \\ \vdots \\ \Delta \mathbf{u}(k+N_c-1) \end{bmatrix}, \\ \tilde{\mathbf{x}}(k) &= \begin{bmatrix} \mathbf{x}(k) \\ \vdots \\ \mathbf{x}(k+N_p-1) \end{bmatrix}, \\ \tilde{\mathbf{y}}(k) &= \begin{bmatrix} \mathbf{y}(k) \\ \vdots \\ \mathbf{y}(k+N_p-1) \end{bmatrix}. \end{split}$$

Then, the lifted input and output matrices can be rewritten as

$$\Delta \tilde{\mathbf{u}}(k) = \tilde{\mathbf{u}}(k) - \tilde{\mathbf{u}}(k-1)$$
$$\tilde{\mathbf{y}}(k) = \tilde{C}\tilde{\mathbf{x}}(k),$$

where

$$\tilde{C} = diag(\underbrace{C, \cdots, C}_{N_p}).$$

The variables  $\mathbf{r}(k+i)$  denotes reference trajectory at time k+i based on the information available at time k. Then, the solution of MPC problem, i.e.  $\Delta \tilde{\mathbf{u}}(k)$ , at time k is obtained by solving the following dynamic objective function J:

$$J = \min_{i \in \mathbf{u}} \sum_{i=1}^{N_{p}} \mathcal{Q} \{ \mathbf{y}(k+i) - \mathbf{r}(k+i) \}^{2} + \sum_{i=0}^{N_{c}-1} R \Delta \mathbf{u}(k+i)^{2},$$
(7)

subject to

$$\tilde{\mathbf{u}}_{\min} \leq \tilde{\mathbf{u}}(k) \leq \tilde{\mathbf{u}}_{\max}$$
$$\Delta \tilde{\mathbf{u}}_{\min} \leq \Delta \tilde{\mathbf{u}}(k) \leq \Delta \tilde{\mathbf{u}}_{\max}$$
$$\tilde{\mathbf{y}}_{\min} \leq \tilde{\mathbf{y}}(k) \leq \tilde{\mathbf{y}}_{\min}.$$

The objective function involves two contributions. The first term in (7) represents the penalty on trajectory tracking error and the second term in (7) penalizes the steering effort. Usually the output prediction horizon  $N_P$  is larger than the control horizon  $N_C$ .

From lateral dynamics model (2), the output prediction is described by

$$\tilde{\mathbf{y}}(k) = \tilde{C}\tilde{\mathbf{x}}(k)$$
$$= \Psi \mathbf{x}(k) + \Theta \Delta \tilde{\mathbf{u}}(k) + \Upsilon \mathbf{u}(k-1), \qquad (8)$$

$$\Psi = \tilde{C} \begin{bmatrix} I \\ \Phi \\ \vdots \\ \Phi^{N_{c}-1} \\ \vdots \\ \Phi^{N_{p}-1} \end{bmatrix}, \ \Theta = \tilde{C} \begin{bmatrix} 0 & 0 & 0 \\ \Gamma_{2} & 0 & 0 \\ \vdots & \ddots & 0 \\ \sum_{i=0}^{N_{c}-1} \Phi^{i} \Gamma_{2} & \cdots & \Gamma_{2} \\ \vdots & & \vdots \\ \sum_{i=0}^{N_{p}-N_{c}-1} \Phi^{i} \Gamma_{2} & \cdots & \sum_{i=0}^{N_{p}-N_{c}-1} \Phi^{i} \Gamma_{2} \end{bmatrix}, \ \Upsilon = \tilde{C} \begin{bmatrix} 0 \\ \Gamma_{2} \\ \vdots \\ \sum_{i=0}^{N_{c}-1} \Phi^{i} \Gamma_{2} \\ \vdots \\ \sum_{i=0}^{N_{p}-2} \Phi^{i} \Gamma_{2} \\ \vdots \\ \sum_{i=0}^{N_{p}-N_{c}-1} \Phi^{i} \Gamma_{2} \end{bmatrix}.$$

Then the object function J in (7) can be rewritten as

$$J = (\tilde{\mathbf{y}}(k) - \tilde{\mathbf{r}}(k))^{T} Q(\tilde{\mathbf{y}}(k) - \tilde{\mathbf{r}}(k)) + \Delta \tilde{\mathbf{u}}^{T}(k) R \Delta \tilde{\mathbf{u}}(k)$$
  
$$= (\Theta \Delta \tilde{\mathbf{u}}(k) - \tilde{\mathbf{c}}(k))^{T} Q(\Theta \mathbb{I} \tilde{\mathbf{u}}(k) - \tilde{\mathbf{c}}(k)) + \Delta \tilde{\mathbf{u}}^{T}(k) R \Delta \tilde{\mathbf{u}}(k)$$
  
$$= \frac{1}{2} \Delta \tilde{\mathbf{u}}^{T}(k) H \Delta \tilde{\mathbf{u}}(k) + \Delta \tilde{\mathbf{u}}^{T}(k) f + f_{0}, \qquad (9)$$

where

$$\tilde{\mathbf{c}}(k) = \tilde{\mathbf{r}}(k) - \Psi \mathbf{x}(k) - \Upsilon \mathbf{u}(k-1)$$

$$H = 2(\Theta^T Q \Theta + R), f = -2\Theta^T Q \tilde{\mathbf{c}}(k), f_0 = \tilde{\mathbf{c}}^T(k) Q \tilde{\mathbf{c}}(k).$$

The constraints of the lifted output, steer angle and rate can be put in the form of

$$\begin{split} \mathbf{y}_{\min} &\leq \mathbf{y}(k) \leq \mathbf{y}_{\max} \\ \mathbf{u}_{\min} &\leq \mathbf{u}(k) \leq \mathbf{u}_{\max} \\ \Delta \mathbf{u}_{\min} &\leq \Delta \mathbf{u}(k) \leq \Delta \mathbf{u}_{\max} \,. \end{split}$$

The constraint of the lifted output is described by  $\tilde{\mathbf{y}}(k) \leq \mathbf{y}_{max}, -\tilde{\mathbf{y}}(k) \leq -\mathbf{y}_{min}$ , then the constraint is rewritten as

$$\begin{bmatrix} I_{n_{y} \times N_{P}} \\ -I_{n_{y} \times N_{P}} \end{bmatrix} \tilde{\mathbf{y}} \leq \begin{bmatrix} \tilde{\mathbf{y}}_{\max} \\ -\tilde{\mathbf{y}}_{\min} \end{bmatrix}.$$

Likewise, the constraints for the horizon from the control specifications and the physical limits are given by

$$\begin{split} &G_{y}\tilde{\mathbf{y}}(k)\leq g_{1}\\ &G_{u}\tilde{\mathbf{u}}(k)\leq g_{2}\\ &G_{\Delta \mathbf{u}}\Delta\tilde{\mathbf{u}}(k)\leq g_{3}, \end{split}$$

where

$$\begin{split} G_{y} = \begin{bmatrix} I_{n_{y} \times N_{P}} \\ -I_{n_{y} \times N_{P}} \end{bmatrix}, \ G_{u} = \begin{bmatrix} I_{n_{u} \times N_{C}} \\ -I_{n_{u} \times N_{C}} \end{bmatrix}, \ G_{\Delta u} = \begin{bmatrix} I_{n_{u} \times N_{C}} \\ -I_{n_{u} \times N_{C}} \end{bmatrix} \\ g_{1} = \begin{bmatrix} \tilde{\mathbf{y}}_{\max} \\ -\tilde{\mathbf{y}}_{\min} \end{bmatrix}, \ g_{2} = \begin{bmatrix} \tilde{\mathbf{u}}_{\max} \\ -\tilde{\mathbf{u}}_{\min} \end{bmatrix}, \ g_{1} = \begin{bmatrix} \Delta \tilde{\mathbf{u}}_{\max} \\ -\Delta \tilde{\mathbf{u}}_{\min} \end{bmatrix}. \end{split}$$

We can find  $\Xi$  that satisfies this equality  $\Xi C \mathbf{x}(k) = \Phi \mathbf{x}(k)$ . Then, from the output prediction (8), the constraints for the output are obtained

$$G_{y}\tilde{\mathbf{y}}(k) = G_{y}\left(\Xi\mathbf{y}(k) + \Theta\Box\tilde{\mathbf{u}}(k) + \Upsilon\mathbf{u}(k-1)\right) \le g_{1}$$
  

$$\Leftrightarrow G_{y}\Theta\Delta\tilde{\mathbf{u}}(k) \le g_{1} - G_{y}\Xi\mathbf{y}(k) - G_{y}\Upsilon\mathbf{u}(k-1), \qquad (10)$$

where  $\Xi \mathbf{y}(k) = \Phi \mathbf{x}(k)$ . The constraints for the steer angle and rate are obtained

$$\underbrace{G_{u}\left(I_{N_{c}}-\begin{bmatrix}0&0\\I_{(N_{c}-1)}&0\end{bmatrix}\right)^{-1}}_{F_{d}}\Delta\tilde{\mathbf{u}}(k)+G_{u}\begin{bmatrix}\mathbf{u}(k-1)\\\vdots\\\mathbf{u}(k-1)\end{bmatrix}$$
$$=F_{d}\Delta\tilde{\mathbf{u}}(k)+F_{o}\mathbf{u}(k-1)\leq g_{2}$$
$$\Leftrightarrow \qquad F_{d}\Delta\tilde{\mathbf{u}}(k)\leq g_{2}'=g_{2}-F_{o}\mathbf{u}(k-1), \tag{11}$$

where

$$F_d \coloneqq G_u \left( I_{N_C} - \begin{bmatrix} 0 & 0 \\ I_{(N_C-1)} & 0 \end{bmatrix} \right)^{-1}, \ F_o \coloneqq G_u \underbrace{\begin{bmatrix} 1 & \cdots & 1 \end{bmatrix}}_{N_C}^{2}$$

and

$$G_{\Delta u} \Delta \tilde{\mathbf{u}}(k) \le g_3. \tag{12}$$

Then combined constraints from (10), (11), (12) are rewritten as

$$G\Delta\tilde{\mathbf{u}}(k) = \begin{bmatrix} G_{\eta}\Theta \\ F_{d} \\ G_{\square u} \end{bmatrix} \Delta\tilde{\mathbf{u}}(k)$$
$$\leq b(k) = \begin{bmatrix} g_{1} \\ g_{2} \\ g_{3} \end{bmatrix} + \begin{bmatrix} -G_{\eta}\Xi & -G_{\eta}\Upsilon \\ 0 & -F_{\sigma} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{y}(k) \\ \mathbf{u}(k-1) \end{bmatrix}.$$
(13)

The optimization problem (9) can be treated as a QP problem of minimizing the cost function. The vector of variations of control inputs

$$\left[\Delta \mathbf{u}^{*}(k), \cdots, \Delta \mathbf{u}^{*}(k+N_{c}-1)\right]$$
(14)

is predicted at each sample time k by solving (7). The superscript \* denotes the optimized value. The resulting state feedback control law at k is given by

$$\mathbf{u}(k) = \mathbf{u}(k-1) + \Delta \mathbf{u}^*(k),$$

then the vector (14) is recalculated at the next computation cycle based on the current measured state values.

Fig. 3 shows the structure of the LKS with the MPC and multi-rate Kalman filter. The lateral offset and heading angle from the center line are measured by using the camera module, and the yaw rate are measured by



Fig. 3 Structure of the LKS with MPC and multi-rate Kalman filter

using the yaw rate sensor. We assume that the path is predefined and the controller is designed such that the vehicle should follow the path. The LKS system is implemented with the multi-rate structure, i.e., the sampling period of camera module, Tcam, is generally 6~8 times slower than that of the car ECU and inertia sensors. We thus need to design a multi-rate controller for the lateral dynamics to produce the control input at a fast rate of  $1/T_c$  using the lane detection at a slow rate of  $1/T_{cam}$ . The steering system has constraints on the steer angle and rate. The information of lateral offset with the slow period may cause the control input to violate the constraints. The MPC solves the QP problem by computing the sequence of the control input at each  $T_c$  with the optimization the future behavior of the vehicle [7]. Thus applying MPC and multi-rate Kalman filter produces the fast sampled control input.

#### 4. SIMULATION RESULTS

Performance of the proposed control method was validated via simulations implemented in MATLAB /Simulink and CarSim. We assume that the longitudinal speed is a constant of 30 [m/s]. The autonomous vehicle keeps tracking the curved lane with the minimum radius of 250[m].

The constraints on output, input and input rate are following:

$$-0.9454[\deg] \le \mathbf{u} \le 0.9454[\deg]$$
  
-0.57296[deg/s]  $\le \Delta \mathbf{u} \le 0.57296[\deg/s]$   
 $-5[m] \le y_1 \le 5[m]$   
 $-2[\deg] \le y_2 \le 2[\deg]$   
 $-15[\deg/s] \le y_3 \le 15[\deg/s].$  (15)

For comparison, we performed simulations for three different sampling periods. Sampling periods and horizons

used for the three cases are listed in Table 1. In case A, the sampling periods of the camera module is the same as that of the steering controller. It represents the case that the vehicle is equipped with the high performance vision system. In case B, we reconstruct the real system, i.e., the sampling periods of the camera is almost seven times to that of ECE sampling period. Thus control output is generated at the instance when the vision data is updated. For the comparison with the case A, we keep the other factors as the case A. In case C, we apply the proposed multi-rate Kalman filter and keep the other factors as the case B. The performance of the proposed steering control scheme is shown to be effective in improving the reference tracking performance of the multi-rate LKS.

Table 1 Simulation scenarios

Case	T <sub>cam</sub> [ms]	T <sub>c</sub> [ms]	Np	N <sub>c</sub>
А	10	10	10	8
В	70	70	10	8
С	70	10	10	8





Fig. 4 Simulation results : (a) state, (b) camera data (blue-solid : case A, red-dashed : case B, green-dashed dot : case C)



Fig. 5 (a) Steer angle, (b) rate of steer angle (blue-solid : case A, red-dashed : case B, green-dashed dot : case C)

Fig. 4(a) shows the simulation results of each state (lateral offset at look-ahead distance, time derivative of lateral offset at c.g., yaw angle error, yaw rate) and (b) shows camera data ( $C_0$ : lateral offset at c.g.,  $C_1$ : heading angle,  $C_2$ : curvature,  $C_3$ : curvature derivative) of the case A, case B and case C. And Fig. 5(a) shows the steer angle that is control u and (b) shows the input variation correspond to Fig. 4. Lateral offset at look-ahead distance and heading angle are related to the differences between the ego vehicle and the road. Curvature and curvature derivative are absolute data of the road so that it does not change.

The steer angle and yaw rate are saturated due to the constraint on the steer angle. In case A, responses show good behavior because the vehicle is equipped with the high performance and high cost vision system. In case B, response has the worst performance among the three cases as the maximum lateral offset at look-ahead distance is 4.5m because the control sampling period is 70ms. In case C, though the camera sampling period is 70ms, all of state are updated from the multi-rate estimator at Tc, and it shows good performance similar to the case A. Through this results, we demonstrate that the LKS applied by MPC with the multi-rate Kalman filter improves driving performance with a low cost vision system.

## 4. CONCLUSION

A multi-rate steering control scheme using model predictive control was developed for autonomous vehicles. The multi-rate Kalman filter was developed to produce the control input at a rate of the car ECU despite using the vision module having slow update period. The production of control input was solvable by computing the QP problem considering the tracking performance and constraints on steering system. Simulation results showed that the lane tracking is fulfilled with a stable vehicle motion while the information from the vision system is available at a slow sampling period.

## References

- C. Hatipoglu, Ü. Özgüner, and K. A. Redmill, "Automated lane change controller design," IEEE Trans. Intelligent Transportation Systems, vol. 4, no. 1, pp. 13–22, 2003.
- [2] J. Hwang, K. Huh, H. Na, H. Jung, H. Kang, and P. Yoon, "Development of a lane keeping assist system using vision sensor and DRPG algorithm," Trans. of KSAE, vol. 17, no. 1, pp.50–57, 2009.
- [3] J. Hwang, K. Huh, H. Na, H. Jung, H. Kang, and P. Yoon, "Development of a model based predictive

controller for lane keeping assistance system," Trans. of KSAE, vol. 17, no. 3, pp.54-61, 2009.

- [4] D.-J. Lee and M. Tomizuka, "Multirate optimal state estimation with sensor fusion," in Proc. of the American Control Conference, pp.2887–2982, 2003.
- [5] B.-S. Chen, C.-W. Lin, and Y.-L. Chen, "Optimal signal reconstruction in noisy filter bank systems: Multirate Kalman synthesis filtering approach," IEEE Trans. Signal Processing, vol. 43, no. 11, 1995.
- [6] C. E. Garcia, D. M. Prett, and M. Morari, "Model predictive control : theory and practice - a survey," Automatica, vol. 25, no. 3, pp.335–348, pp. 335–348, 1989.
- [7] S. J. Qin and T. A. Badgwell, "A survey of industrial model predictive control technology," Control Engineering Practice, vol. 11, no. 7, pp.733–764, 2003.
- [8] P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, and D. Hrovat, "Predictive active steering control for autonomous vehicle systems," IEEE Trans. Control Syst. Tech., vol. 15, no. 3, pp.566–580, 2007.
- [9] F. Borrelli, P. Falcone, T. Keviczky, and J. Asgari, "MPC-based approach to active steering for autonomous vehicle systems," International Journal of Vehicle Autonomous Systems, vol. 3, no. 2/3/4, pp. 265-291, 2005.
- [10] P. Falcone, F. Borrelli, H. E. Tseng, J. Asgari, and D. Hrovat, "Linear time-varying model predictive control and its application to active steering systems: Stability analysis and experimental validation," International Journal of Robust and Nonlinear Control, vol. 18, no. 8, pp. 862–875, 2008.
- [11] R. Rajamani, Vehicle dynamics and control, Springer, 2006.
- [12] S.-H. Lee, Y. O. Lee, Y. Son, and C. C. Chung, "Multirate active steering control for autonomous vehicle lane changes," IEEE Intelligent vehicles symposium, 2012, to appear.
- [13] G. F. Franklin, J. D. Powell, and M. L. Workman, Digital Control of Dynamic Systems, 3rded. Addison-Wesley,1997.
- [14] Y. O. Lee, S.-H. Lee, Y. S. Son, and C. C. Chung, "Robust Multirate state estimator for autonomous vehicles with uncertain vision processing period," in 11th International Conf. Control, Automation and Systems, pp. 424-427, 2011.
- [15] E. Camacho and C. Bordons, Model predictive control. Springer Verlag, 2004.
- [16] K. Enke, "Possibilities for improving safety within the driver vehicle environment loop," The 7th International Technical Conference on Experimental Safety Vehicles, 1979.
- [17] S. Hetrick, "Examination of driver lane change

behavior and the potential effectiveness of warning onset rules for lane change or side crash avoidance systems," M.S. thesis, Virginia Polytechnic Inst. and State Univ., Mar. 1997.

- [18] P. Leelavansuk, K. Shitamitsu, H. Mouri, and M. Nagai, "Study on cooperative control of driver and lane-keeping assistance system," in Proc. of the International Symposium on Advanced Vehicle Control (AVEC), pp. 219–224, 2002
- [19] A. M. Phillips and M. Tomizuka, "Multirate estimation and control under time-varying data sampling with applications to information storage device," in Proc. of the American Control Conference, pp.4151-4155, 1995.



## 김 보 아 (金 寶 雅)

Bo-Ah Kim was born in Seoul, Korea, on 14 May 1986. She received the B.S. degree in division of electronics and computer engineering from the Hanyang University, Seoul, Korea, in 2011. She is currently in the Master's

degree program at the same university. Her current research focuses on the automotive system.



# 이 영 옥 (李 英 玉)

Young Ok Lee was born in Seoul, Korea, on 29 September 1983. She received the B.S. degree in electrical and computer engineering from the Hanyang University, Seoul, Korea, in 2006. She is currently in the Ph.D.

program at the same university. Her current research focuses on the nonlinear control of power system and automotive system.



# 이 승 희 (李 承 熙)

Seung-Hi Lee received the B.S. degree in mechanical engineering from Korea University, Seoul, and the M.S. degree in mechanical engineering from Seoul National University, Seoul, Korea, in 1985 and 1987, respectively, and the

Ph.D. degree in mechanical engineering and applied

mechanics from the University of Michigan, Ann Arbor, in 1993. From 1988 to 1989, he was a Research Scientist with the Korea Institute of Science and Technology. Since 1994, he had been with Samsung Advanced Institute of Technology, Korea, where he advanced was a team leader responsible for servomechanical systems. In 2009, he joined Hanyang University, Seoul, Korea, as a Research Professor, where he is also teaching advanced control systems. His research interests include robust sampled-data feedback control of uncertain systems, and application to information storage, automotive, electromechanical, and manufacturing systems. Prof. Lee has served as a Member of the Board of Editors of International Journal of Control, Automation, and Systems.



# 정 정 주 (鄭 正 周)

Chung Choo Choo was born in Incheon, Korea, on September 5, 1958. He received the B.S. and M.S. degrees from Seoul National University, Seoul, Korea, in 1981 and 1983, respectively, and the Ph.D. degree from the

University of Southern California, LosAngeles, in 1993, all in electrical engineering. From 1983 to 1985, he was Research Engineer at the Central Research а Laboratory, GoldStar (currently LG Electronics), Seoul, where he was engaged in the areas of robotics and copiers. In 1985, he joined the International Procurement Office (IPO), IBM Korea, where he was a Procurement and Quality Assurance Associate Engineer until 1987. From 1993 to 1994, he was a Research Associate in electrical and computer engineering at the University of Colorado at Boulder. From 1994 to 1997, was with Samsung Advanced Institute of he Technology (SAIT), Korea, where he was a Team Leader and developed a disk drive servo development svstem. He finished the Samsung Advanced Management Program at theWharton Business School, University of Pennsylvania, Philadelphia. In 1997, he joined the faculty of Hanyang University, Seoul, where he was the Chairman of the Division of Electrical and Computer Engineering from 2004 to 2005, the Associate Dean for Planning, College of Engineering, from 2006 to 2008, and has been a Professor since 2007. He was an Associate Editor for the Asian Journal of Control from 2000 to 2002, the Director of the Editorial Board for the Transactions on Control, Automation and Systems Engineering from 2001 to 2002, and the Editor for the International Journal of Control, Automation and Systems from 2003 to 2005. His current research interests include the control areas of robotic systems,

automotive systems, power systems, lithography systems, and data storage systems, including hard disk drives, optical disk drives, holographic data storage systems, and scanning-probe-microscope-based storage systems. Prof. Chung is a member of the American Society of Mechanical Engineers (ASME), the Institute of Control, Robotics and Systems (ICROS), and the Korean Institute of Electrical Engineers (KIEE). He is the Program Co-Chair of the International Conference on Control, Automation and Systems (ICCAS) of the Society of Instrument and Control Engineers (SICE) 2009, Fukuoka, Japan. In 1996, he was selected as one of the Samsung 21st Century Leaders by the Samsung Group. He was an Associate Editor for the 2003 IEEE Conference on Decision and Control. He was an Associate Editor and the Co-Chair of Publicity of the International Federation of Automatic Control (IFAC) World Congress, Korea, 2008. He has chaired numerous sessions and was a member of a number of International Program Committees of various IEEE, ASME, ICROS, SICE, Asian Control Conference (ASCC), and IFAC conferences. Since 2000, he has been the President of the Control Theory Study Society of the ICROS, Korea.