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Two-Faults Detection and Isolation Using Extended Parity Space Approach

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Abstract – This paper proposes a new FDI(Fault Detection and Isolation) method, which is called EPSA(Extended Parity Space Approach). This method is particularly suitable for fault detection and isolation of the system with one faulty sensor or two faulty sensors. In the system with two faulty sensors, the fault detection and isolation probability may be decreased when two faults are occurred between the sensors related to the large fault direction angle. Nonetheless, the previously suggested FDI methods to treat the two-faults problem do not consider the effect of the large fault direction angle. In order to solve this problem, this paper analyzes the effect of the large fault direction angle and proposes how to increase the fault detection and isolation probability. For the increase the detection probability, this paper additionally considers the fault type that is not detected because of the cancellation of the fault biases by the large fault direction angle. Also for the increase the isolation probability, this paper suggests the additional isolation procedure in case of two-faults. EPSA helps that the user can know the exact fault situation. The proposed FDI method is verified through Monte Carlo simulation.

Keywords: Fault detection, Sensor failure, Two faults, Hardware redundancy

1. Introduction

INS(Inertial Navigation System) is a precision instrumentation system, which provides the geographical position of a vehicle (e.g., aircraft, spacecraft, ship and missile) using inertial sensors, such as gyroscopes and accelerometers. In these vehicles, the required reliability of INS exceeds the reliability obtained by using a single string system of inertial sensors. In such a system, a failure of any sensor will cause the entire system to fail. Thus, the system with hardware redundancy has been used in order to achieve the desired levels of reliability. Additionally, FDI(Fault Detection and Isolation) method is needed in order to verify the availability of sensor signals. The several FDI methods are suggested through the extensive literatures. For example, PSA(Parity Space Approach), GLT(Generalized Likelihood ratio Test) and OPT(Optimal Parity vector Test) are suggested [1-7]. However, these FDI methods have the restriction with respect to application because of an assumption that the system has one faulty sensor. In order to solve this limitation, FDI method to consider the two-faults is needed. Recently, RAIM (Receiver Autonomous Integrity Monitoring) methods to consider the two-faults problem in GPS are reported but these are not applicable to the FDI problem of sensor because the special characteristics of satellite like SLOPEmax, the largest slope calculated from the geometry of satellite, are used [8, 9]. As a result, in sensor level, there are a few papers to treat the two-faults as compared with papers to treat the one-fault. In 1983, Ray proposed a new FDI method using the consistency of subset and this method may be applicable to the two-faults problem [3]. But he only considered not vector variable but scalar variable in order to apply to the powerplant. In order to apply to vector variable, the consistency of subset must be newly defined. Also the method suggested by Yoo may be applicable to the two-faults problem of sensor although it is suggested for RAIM [10, 11]. And two faults detection research using 7 inertial sensors was introduced by Yang [12-14], Kim [15]. Yang used a reduced-order parity vector(RPV) method, and Kim considered the same problem to detect and isolate. These methods to treat the two-faults problem are developed on the basis of the parity space concept.

However, these two-faults FDI methods have some problems although these generally show a good performance. Sometimes these show a bad performance when the fault direction angle between faulty sensors is large because these do not consider the effect of fault direction angle and fault type. In this case, especially the fault isolation probability is rapidly decreased and also a fault may be not detected by the effect of fault direction angle. These problems are explained in Appendix-B. In order to solve these problems, this paper analyzes the

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situations that two-faults occur and proposes a new FDI method based on these analysis results. This FDI method, which is called EPSA(Extended Parity Space Approach), is particularly suitable for fault detection and isolation of the system with one faulty sensor or two faulty sensors. Thus this paper presents how to improve the fault detection and isolation probability when the fault direction angle between two faulty sensors is large.

2. Extended Parity Space Approach

2.1 The concept of parity space

Assume that a set of redundant inertial sensors yields l measurements. The measurement equation of the system with l sensors is defined as follows, $1 \le n < l$.

$$m = Hx + \varepsilon + f \tag{1}$$

where m is the $l \times 1$ measurement output vector of sensor, H is the $l \times n$ observation matrix of rank n to be determined by sensor configuration, the state x is the $n \times 1$ true value of the measured variable and ε is the $l \times 1$ Gaussian measurement noise vector with $E[\varepsilon] = 0$ and $COV[\varepsilon] = I_l$. Here, $E[\cdot]$ and $COV[\cdot]$ mean an expectation and a covariance, respectively. I_l is an identity matrix with l dimensions. In this paper, f is the fault signal vector and the type of fault is modeled as constant bias. Although this assumption may have some problems, this is a valid and common assumption in that the bias type fault has a great effect on the system.

Definition 2.1 : The matrix V is a trapezoidal matrix to satisfy the following conditions:

$$VH = 0 \tag{2}$$

$$VV^T = I_{l-n} \tag{3}$$

$$\begin{aligned} V &= \begin{bmatrix} v_1 & v_2 & \cdots & v_{l-n} \end{bmatrix}^T \\ &= \begin{bmatrix} v_{c1} & v_{c2} & \cdots & v_{cl} \end{bmatrix} \end{aligned} \tag{4}$$

where I_{l-n} is an identity matrix with $l-n$ dimensions and the dimension of V is $(l-n) \times l$. Also v_i^T is the i^{th} row vector of V and v_{ck} is the k^{th} column vector of V .

Definition 2.2 : The column space of matrix V is defined as the "parity space" of the observation matrix H .

Definition 2.3 : The parity vector is defined by

$$p = Vm \tag{5}$$

$$p = \begin{bmatrix} p_1 & p_2 & \cdots & p_{l-n} \end{bmatrix}^T \tag{6}$$

where p is the $l-n$ dimensional vector. The parity vector means the projection of the measurement m onto the parity space and is independent of the state variables but dependent on the system fault. In (6), p_i is called a parity equation [1].

Definition 2.4 : The columns of V are projections of the measurement directions onto the parity space and they are called the fault directions since the fault of the k^{th} measurement m_k implies the growth of the parity vector p in the fault direction of v_{ck} . Also a vector $v_{ck} m_k$ on the fault direction of v_{ck} is called a fault direction vector [3]. The fault direction angle is defined as an angle between two fault direction vectors.

2.2 One-fault detection and isolation using parity space approach

Parity space approach uses a parity vector that is independent of the state variables but dependent on the system fault. Generally, the magnitude of p is very small when all sensors are normal. So the magnitude of parity vector p increases along the fault direction of v_{ck} in the parity space if the fault occurs at k^{th} sensor [3]. The FDI procedure of PSA using the parity space concept is as follows.

2.2.1 FaultdDetection

$p^T p$ has χ^2 distribution with $l-n$ DOF(Degrees Of Freedom) and is used as the fault detection function, i.e., $FD = \chi^2 = p^T p$. If the probability of false alarm is α , T to satisfy $P(\chi^2 > T) = \alpha$ is determined as the threshold value from χ^2 distribution table, where P means the probability function. How to detect a fault is simple. After checking the value of $p^T p$, if $p^T p \le T$, we conclude that a fault does not occur. If $p^T p > T$, we conclude that a fault occurs.

2.2.2 Fault isolation

The fault isolation function is defined as $FI_k = v_{ck}^T p / \|v_{ck}\|$. This function shows the value to be obtained as projecting a parity vector along the fault direction of each sensor. The number of FI function is l . The sensor related a maximum FI value is considered as the faulty sensor. For example, if FI_k is maximum, the k^{th} sensor is isolated as the faulty sensor. The detail information about PSA can be easily obtained [3, 7].

Lemma 2.1 : Assume that it exists l sensors to measure the n dimensional variables, $1 \le n < l$. When PSA is used as FDI method for one-fault problem, the condition for fault detection and isolation is $l - n \ge 2$.

(Proof) In PSA, the parity vector $p = Vm$ is used and the dimension of a trapezoidal matrix V is $(l-n) \times l$. Here, the row V is two or more because V is a trapezoidal and the fault effect is not shown in the parity vector p when the row of V exists only one. Therefore when PSA is used as FDI method for one-fault problem, the condition for fault detection and isolation is $l - n \ge 2$.

2.3 One-fault or Two-faults Detection and Isolation Using Extended Parity Space Approach

In this paper, we assume that the system has one faulty sensor or two faulty sensors. Generally, the fault is defined as an unpermitted deviation of at least one characteristic property or parameter of the system from the standard condition. In normal mode, the measurement noise of normal sensor is very smaller than the fault bias of the faulty sensor. To propose a new FDI method, several variables are defined as follows.

Definition 2.5 : Let S be the set of all sensor measurements and S^i be a subset of $l-1$ measurements where $m_i \notin S^i$. And let S^{ij} be a subset of $l-2$ measurements where $m_i \notin S^{ij}$ and $m_j \notin S^{ij}$ $(i \neq j)$.

$$S = \{m_1, m_2, \cdots, m_l\} \tag{7}$$

$$S^i = \{m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_l\} \quad , \quad m_i \notin S^i \tag{8}$$

$$\begin{gathered} S^{ij} = \{m_1, \cdots, m_{i-1}, m_{i+1}, \cdots m_{j-1}, m_{j+1}, \cdots, m_l\} \\ m_i \notin S^{ij} \ , \ m_j \notin S^{ij} \ (i \neq j) \end{gathered} \tag{9}$$

where m_i is the measurement of i^{th} sensor.

Definition 2.6 : Let $\overline{m}_i$ be a measurement vector obtained from elements of S^i and $\overline{m}_{ij}$ be a measurement vector obtained from elements of S^{ij}

$$\overline{m}_i = [m_1 \ \cdots \ m_{i-1} \ m_{i+1} \cdots \ m_l]^T \tag{10}$$

$$\overline{m}_{ij} = [m_1 \ \cdots \ m_{i-1} \ m_{i+1} \ \cdots \ m_{j-1} \ m_{j+1} \ \cdots \ m_l]^T \tag{11}$$

Definition 2.7 : The matrix $\overline{V}_i$ is a trapezoidal matrix to satisfy $\overline{V}_i \overline{H}_i = 0$ and $\overline{V}_i \overline{V}_i^T = I_{l-n-1}$, where $\overline{H}_i$ is the observation matrix for the subset of sensors excluding i^{th} sensor.

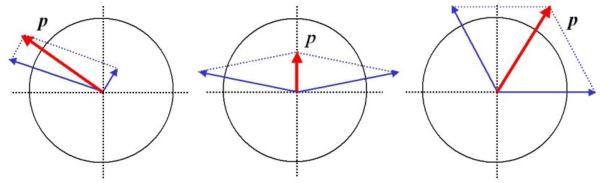
Definition 2.8 : The matrix $\overline{V}_{ij}$ is a trapezoidal matrix to satisfy $\overline{V}_{ij} \overline{H}_{ij} = 0$ and $\overline{V}_{ij} \overline{V}_{ij}^T = I_{l-n-2}$, where $\overline{H}_{ij}$ is the observation matrix for the subset of sensors excluding i^{th} and j^{th} sensor.

Definition 2.9 : Let $\overline{p}_i$ be the corresponding parity vector generated from all measurements in S^i where $m_i \notin S^i$ and $\overline{p}_{ij}$ be the corresponding parity vector generated from all measurements in S^{ij} where $m_i \notin S^{ij}$ and $m_j \notin S^{ij}$. Then the chi-square variable $\overline{\chi}_i^2$ and $\overline{\chi}_{ij}^2$ are defined as (13).

$$\overline{p}_i = \overline{V}_i \overline{m}_i \quad , \quad \overline{p}_{ij} = \overline{V}_{ij} \overline{m}_{ij} \tag{12}$$

$$\overline{\chi}_i^2 \equiv \overline{p}_i^T \overline{p}_i \quad , \quad \overline{\chi}_{ij}^2 \equiv \overline{p}_{ij}^T \overline{p}_{ij} \tag{13}$$

Definition 2.10 : A subset S^i is defined to be consistent if PSA does not detect a fault from sensor measurements in S^i . If a fault is detected, S^i is defined to be inconsistent.



(a) Fault-A (b) Fault-B (c) Fault-C

Fig. 1. The fault situations

To design a new FDI method to be applied to the system with one faulty sensor or two faulty sensors, first the fault situations must be analyzed. To explain the fault situations, the parity vector in two-dimensional parity space is shown as Fig. 1. The circle means a square root of threshold. The direction of arrow indicates the fault direction of each sensor in the parity space. The magnitude of arrow means the magnitude of noise of normal sensor or the magnitude of noise of faulty sensor plus fault bias to be shown along the fault direction. It is assumed that the system has l sensors. Under these assumptions, the three situations are considered as shown Fig. 1: Fault-A, Fault-B and Fault-C.

Definition 2.11 : *FDN* (Fault Detection Number) is defined to be the number of inconsistent subsets S^i where S^i contains all measurements except m_i . This variable provides the information of fault detection and isolation.

In this paper, the FDI method to detect and isolate the fault with respect to *FDN* is called EPSA(Extended Parity Space Approach). The fault detection procedure in EPSA is performed by *FDN* and the fault isolation procedure is performed by different methods according to the fault situations, respectively. For example, when Fault-A was occurred, the number of fault detection flags of subset S^i $(1 \le i \le 7)$ is $l-1$, because the number of subsets which have a faulty sensor measurement is $l-1$. And Fault-B also can be detected 2 subsets. Sum of two fault direction vectors is small. Therefore subset which has one fault sensor measurement can be detected. Fault-C detected always. Thus when Fault-C type two faults are generated, all subset make the fault detection flag. In Fig. 2, Fault classification and isolation process by type of fault is shown.

2.3.1 Fault isolation

The FDI procedure of Fault-A is simple. Fig. 1(a) shows the situation that the system has one faulty sensor. This situation is detected by fact that *FDN* is $l-1$. If S^i is decided as the one consistent subset, i^{th} sensor is isolated as the faulty sensor.

2.3.2 Detection and isolation of fault-B

On the other hand, Fig. 1(b) and Fig. 1(c) are the situations that the system has two faulty sensors. In case of Fault-B, the faults occur at two sensors but the resultant value of $p^T p$ is less than the value of threshold because the fault direction angle between two faulty sensors is larger than 120^0. The fault type like Fig. 1(b) is not detected by the previous FDI methods because the effect of large fault direction angle is discarded in the previous FDI methods. Therefore, for the improvement of fault detection probability, the situation of Fault-B must be additionally considered. This is the answer about question Q1. With respect to EPSA, Fault-B is the case that FDN is 2. So if S^i and S^j were decided as the inconsistent subsets, i^{th} sensor and j^{th} sensor are isolated as the faulty sensors.

2.3.3 Detection and isolation of fault-C

Finally, the Fault-C situation is the case that FDN is l. It means that all subsets are inconsistent. The Fault-C situation is also easily detected by previous FDI methods because the situation of Fault-C is generally considered in two-faults problem. Until now, how to isolate the faulty sensors in the situation of Fault-C has been proposed by two methods, which are Ray's method and Yoo's method. Appendix-A gives the additional information about these methods to you. To say shortly, Ray's method seeks the minimum $\overline{p}_i^T \overline{p}_i$ in twice. On the other hand, Yoo's method seeks the minimum $\overline{p}_{ij}^T \overline{p}_{ij}$ because the magnitude of parity vector generated from all measurements without the measurements of two faulty sensors is minimum. The latter has been widely used. It regards i^{th} sensor and j^{th} sensor related the minimum $\overline{p}_{ij}^T \overline{p}_{ij}$ as faulty sensors. Here, with respect to the fault isolation probability, the situation of Fault-C is mainly affected by the large fault direction angle. In case of the large fault direction angle, the fault isolation probability of Fault-C may be rapidly decreased as the result shown in Appendix-B. Therefore additionally, new method must be suggested in order to solve this problem. This is the question Q2 that is "what is how to improve the fault isolation probability when the fault direction angle is large?". Why the fault isolation probability is rapidly decreased?

The fault biases of two faulty sensors are combined in a manner that they cancel each other out in given parity space. This cancellation within parity space depends on the fault direction angle between two faulty sensors. Sometimes the result of this cancellation leads a false detection or an incorrect isolation. The worst case with respect to the performance of FDI is defined that the fault direction angle between two failed sensors is the largest and the best case is defined that the fault direction angle is the smallest.

Remark 2.1 : In parity space, the fault direction angle between two faulty sensors of the cone configuration sensor module is the largest when these are located in sequence. For example, the worst case is when i^{th} sensor and $i+1^{th}$ sensor are the faulty sensors. When the fault direction angle between two faulty sensors is large, the large cancellation happens because two faulty biases largely cancel each other out in parity space.

Remark 2.2 : Although the magnitude of parity vector is the minimum when two faulty sensors are excluded among the entire sensors, sometimes the exception exists by this cancellation. That is, the magnitude of parity vector can be also the minimum when two normal sensors are excluded among the entire sensors. This can lead a false isolation. These exceptions mainly happen in case of the large fault direction angle, i.e., worst case. Here, the important characteristic is the fact that the sensors located near the true faulty sensors almost falsely isolated as the faulty sensor. These falsely isolated sensors are sensors located in sequence when the true faulty sensors are excluded among the entire sensors. For example, if i^{th} sensor and $i+1^{th}$ sensor are the faulty sensors, then $\overline{p}_{i,i+1}^T \overline{p}_{i,i+1}$ is zero without noise. Therefore this value is the first smallest. Next, $\overline{p}_{i-1,i+2}^T \overline{p}_{i-1,i+2}$ is the second smallest without noise because two faulty biases largely cancel each other out in parity space. In cases with noise, sometimes the previous FDI methods determine that the minimum is not $\overline{p}_{i,i+1}^T \overline{p}_{i,i+1}$ but $\overline{p}_{i-1,i+2}^T \overline{p}_{i-1,i+2}$. This is the main reason why the fault isolation probability is rapidly decreased. Appendix-C gives the additional information to you.

How to solve this problem is as follows. When Fault-C is detected, first let's seek not one minimum $\overline{p}_{ij}^T \overline{p}_{ij}$ but the first and second smallest $\overline{p}_{ij}^T \overline{p}_{ij}$, where $i, j = 1, 2, \cdots, l$ and $i \neq j$. If $\overline{p}_{ab}^T \overline{p}_{ab}$ and $\overline{p}_{cd}^T \overline{p}_{cd}$ are selected as the first and second smallest, let the a^{th} and c^{th} sensor exclude from the entire sensors. Let the consistency of subset S^{ac} check whether a fault is detected or not through PSA. If a fault is detected in b th sensor, a^{th} sensor and b^{th} sensor are isolated as two faulty sensors. On the other hand, if a fault is detected in d^{th} sensor, c^{th} sensor and d^{th} sensor are isolated as two faulty sensors. Fig. 2 shows the flow chart of EPSA.

Lemma 2.2 : Assume that l sensors measure the n dimensional variables, $1 \le n < l$. When EPSA is used as FDI method for two-faults problem, the condition for fault detection is $l-n \ge 3$ and the condition for fault detection and isolation is $l-n \ge 4$.

(Proof) In EPSA, $\overline{p}_i = \overline{V}_i\ \overline{m}_i$ is used for calculation of FDN, where $\overline{m}_i$ is the measurement vector of subset S^i. The dimension of a trapezoidal matrix $\overline{V}_i$ is $(l-n-1)\times(l-1)$. Here, the row $\overline{V}_i$ is two or more because $\overline{V}_i$ is a trapezoidal matrix. Therefore the condition $l-n-1 \ge 2$, i.e. $l-n \ge 3$, must be satisfied. If this condition is satisfied, the fault detection is possible by using FDN. Secondly, when Fault-C occurs, for isolation of the faulty sensors, the calculation of $\overline{p}_{ij} = \overline{V}_{ij}\overline{m}_{ij}$ is needed, where $\overline{m}_{ij}$ is the measurement vector of subset S^{ij}. The dimension of $\overline{V}_{ij}$ is $(l-n-2)\times(l-2)$. Because $\overline{V}_{ij}$ is a trapezoidal matrix, $l-n-2 \ge 2$, i.e. $l-n \ge 4$, must be satisfied. When EPSA is used as FDI method for two-faults problem, the condition for fault detection is $l-n \ge 3$ and the condition for fault detection and isolation is $l-n \ge 4$.

For example, in case of two-faults problem, the inertial navigation system, $n = 3$, must have at least 6 sensors for fault detection and have at least 7 sensors for fault detection and isolation.

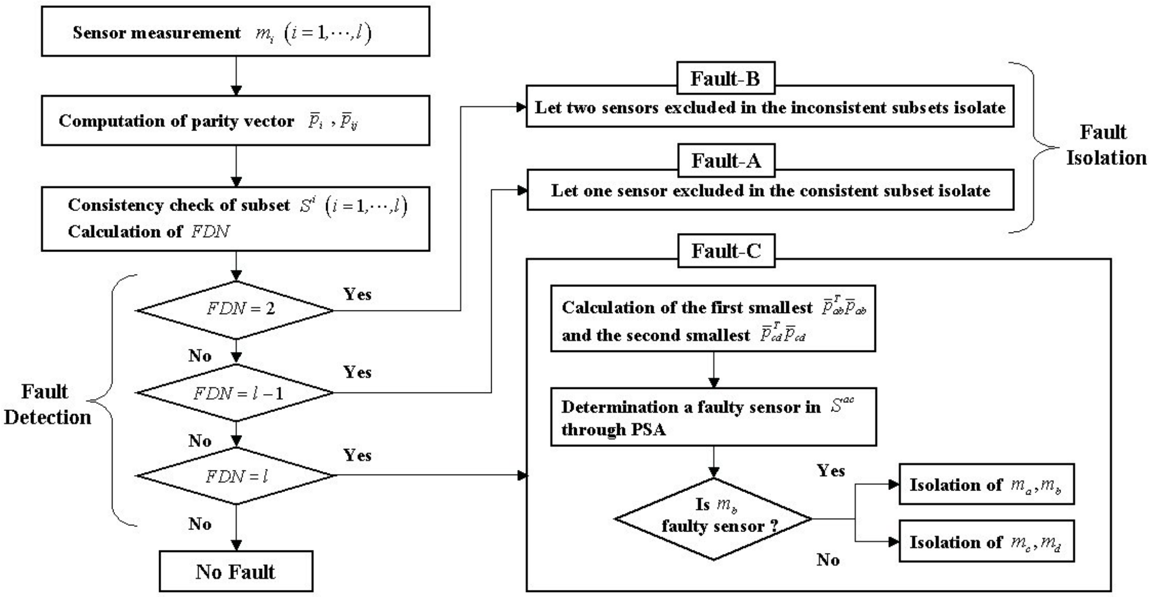


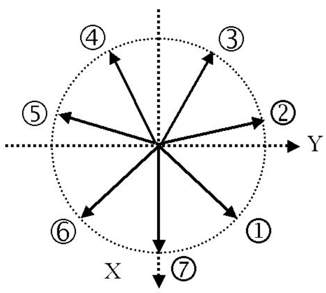
Fig. 2. Flow chart of EPSA

3. Simulation

The performance of EPSA is verified through Monte Carlo simulation. The number of performed simulation is 10000. Fig. 3 shows seven sensors to be located as cone configuration around a central pivot, z-axis. The angle between each sensor and z-axis is 54.74^0. Each sensor is equally distributed. The number of sensors is determined as seven because seven is the minimum number for FDI in two-faults problem. The information about the minimum sensor number for FDI in PSA and EPSA is given through Lemma 2.1 and Lemma 2.2. The matrix V is as follows.

$$V=\begin{bmatrix} 0.7559 & -0.4246 & -0.1049 & 0.1516 & 0.1516 & -0.1049 & -0.4246 \\ 0 & 0.6254 & -0.5845 & -0.0239 & 0.2861 & 0.1120 & -0.4151 \\ 0 & 0 & 0.4678 & -0.6821 & 0.2220 & 0.3613 & -0.3690 \\ 0 & 0 & 0 & 0.2875 & -0.6460 & 0.6460 & -0.2875 \end{bmatrix}$$

For realization of the worst case of two-faults, we insert a fault bias to 1st sensor and 2nd sensor. Fig. 4 shows the results of three FDI methods: EPSA, Ray's method [3] and Yoo's method [10, 11]. As the increase of F/N(Fault to Noise) Ratio, Fig. 4(a) shows the detection probability. From more 90%, the gradient of detection graph is slowly decreased and converges to 100%. The detection performance of EPSA is better than the other methods. The difference is small in detection probability because Fault-B happens when fault biases are near the threshold so the occurrence number is small. On the other hand, Fig. 4(b) shows the isolation probability. Ray's method shows the very low isolation probability. Ray's method almost isolates not 1^{st} sensor and 2^{nd} sensor but 3^{rd} sensor and 7^{th} sensor. Obviously, the isolation performance of EPSA is superior to that of the other methods.

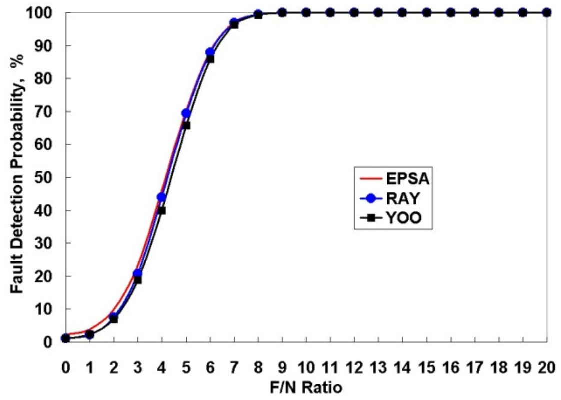


$$H=\begin{bmatrix} 0.5091 & 0.6384 & 0.5774 \\ -0.1817 & 0.7960 & 0.5774 \\ -0.7356 & 0.3543 & 0.5774 \\ -0.7356 & -0.3543 & 0.5774 \\ -0.1817 & -0.7960 & 0.5774 \\ 0.5091 & -0.6384 & 0.5774 \\ 0.8165 & 0.0000 & 0.5774 \end{bmatrix}$$

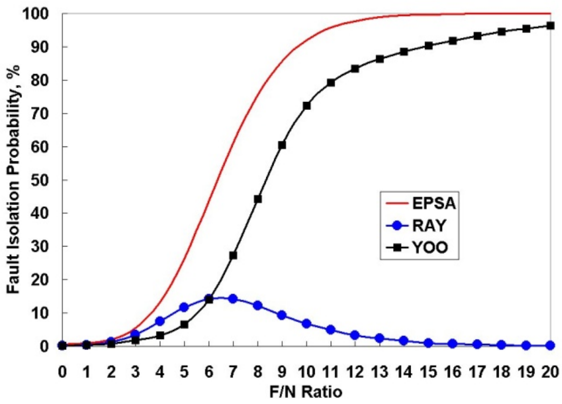
Fig. 3. Sensor configuration and the observation matrix

4. Conclusion

Sometimes, the previously suggested FDI methods are not suitable for two-faults detection and isolation because these do not consider the effect of large fault direction angle. This is a common disadvantage of the previously suggested FDI methods. In order to solve this problem, this paper analyzes the effect of the large fault direction angle and proposes how to increase the detection probability and the isolation probability. For the increase the detection probability, this paper additionally considers the fault type



(a) Detection probability



(b) Isolation probability

Fig. 4. The probability of three FDI methods

that is not detected because a fault biases cancel each other out by the large fault direction angle. For the increase the isolation probability, this paper proposes the additional isolation procedure in case of two-faults. EPSA helps that the user can know the exact fault situation. Also with respect to the performance, the performance of EPSA is better than that of the other methods.

Acknowledgements

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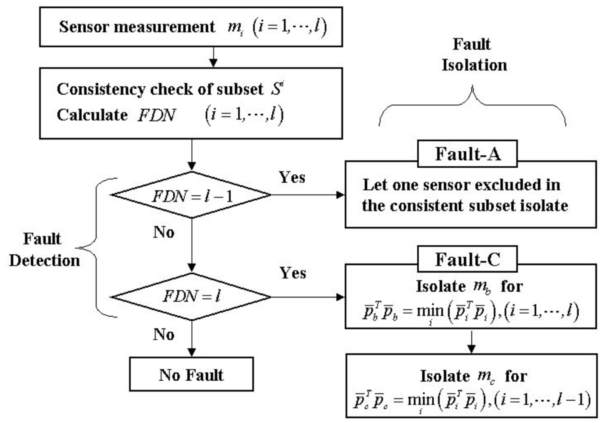
Appendix-A:Conventional two-faults FDI mehtods

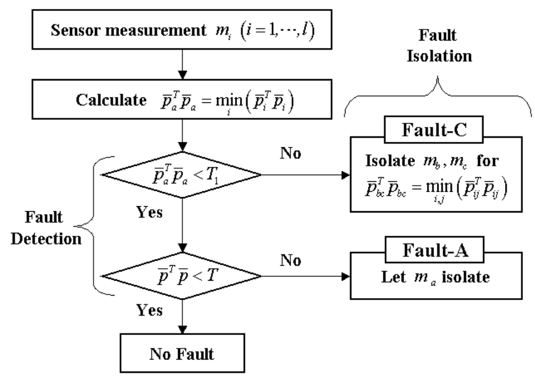
Two-faults FDI methods suggested until now can be briefly summarized as follows. From Definition 2.4 to Definition 2.9, let $S=\{m_1,m_2,\cdots,m_l\}$ be the set of measurements and S^i be a subset of $l-1$ measurements where $m_i \notin S^i$. Also let $\bar{p}_i$ be the corresponding parity vector generated from all measurements in S^i and $\bar{p}_{ij}$ be the corresponding parity vector generated from all measurements in S^{ij} where $m_i \notin S^{ij}$ and $m_j \notin S^{ij}$.

The Ray's method is formulated on the basis of consistencies of appropriate subsets of the full set of measurements like EPSA [3]. However Ray's method only considers the fault situations of Fault-A and Fault-C. The detection procedure is similar to that of EPSA. But the isolation procedure of Fault-C is different from that of EPSA. For example, when Fault-C is detected, the measurement m_i corresponding to the inconsistent subset S^i, i.e., for which $\bar{p}_i^T\bar{p}_i$ is the minimum, is most likely to have failed. Therefore the aforesaid measurement m_i is isolated and the subset S^i that does not contain m_i is considered. This step is once more again repeated in order to isolate the other faulty sensor. That is, in case of two-faults isolation, Ray's method seeks the minimum $\bar{p}_i^T\bar{p}_i$ in twice.

As Ray's method, Yoo's method only considers the fault situations of Fault-A and Fault-C [10, 11]. First let the measurement m_i corresponding to the minimum $\bar{p}_i^T\bar{p}_i$ exclude. As next step, fault detection is performed through checking $\bar{p}_i^T\bar{p}_i < T_i$ and $\bar{p}^T\bar{p} < T$. If Fault-C is detected, let two sensors related to the minimum $\bar{p}_{ij}^T\bar{p}_{ij}$ isolate. The main idea of the Yoo's method in isolation of Fault-C is to



(a) Ray's method



(b) Yoo's method

Fig. A1. Flow chart of Ray's and Yoo's method seek the minimum of $\overline{p}_{ij}^T \overline{p}_{ij}$ because the magnitude of parity vector generated from all measurements without the measurements of two faulty sensors is minimum. This idea has been widely used.

Although these algorithms give a good isolation performance in case of a best case defined in Definition 2.12, these have the problem to give a bad isolation performance in case of a worst case defined in Definition 2.12. See Appendix-B.

Appendix-B: The effect of large fault direction angle

Fault direction angle

In order to show the effect of large fault direction angle, we assume that the system with seven sensors exists. Under the simulation condition of section 3, the each column of matrix V indicates the each fault direction and for example, the angles between the fault direction of 1^{st} sensor and that of the other sensors are arranged in Table A1. In Table A1, symbol $1^{st}-2^{nd}$ means an angle between the fault direction of 1^{st} sensor and that of 2^{nd} sensor. In Table A1, the fault direction angles between 1^{st} sensor and sensors located near 1^{st} sensor, i.e., $1^{st}-2^{nd}$ and $1^{st}-7^{th}$, are relatively large and that between 1^{st} sensor and sensors located far away from 1^{st} sensor i.e., $1^{st}-4^{th}$ and $1^{st}-5^{th}$, are relatively small.

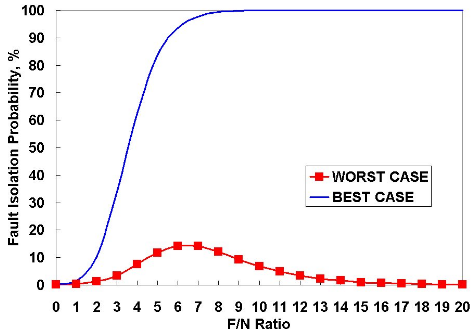
The effect of the large fault direction angle

As mentioned in Section 2, the fault biases of two failed sensors are combined in a manner that they are canceled each other out in given parity space. This cancellation within parity space depends on the fault direction angle between two faulty sensors and the result of this cancellation leads a false detection or an incorrect isolation. The worst case with respect to the performance of FDI is defined that the fault direction angle between two failed sensors is the largest and the best case is that the fault direction angle is the smallest. For example, from Table A1, the worst is the case that the faulty sensors are 1^{st} sensor and 2^{nd} sensor or 1^{st} sensor and 7^{th} sensor and the best is the case that the faulty sensors are 1^{st} sensor and 4^{th} sensor or 1^{st} sensor and 5^{th} sensor. Fig. A2(a) shows the isolation results of the best case and the worst case when Ray's method is used. In the worst case of Ray's method, the fault isolation probability is very low. Fig. A2(b) shows the isolation results of the best case and the worst case when Yoo's method is used. In the worst case of Yoo's method, the fault isolation probability is very lower than that of the best case.

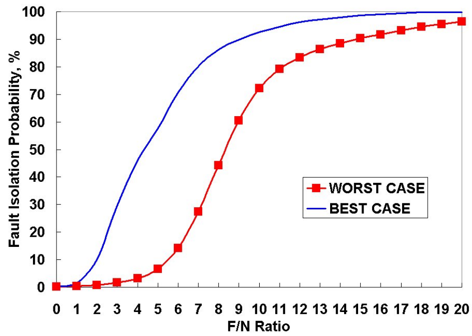
As a result, Ray's method cannot be used when the fault direction angle between two faulty sensors is large. In case of Fault-C isolation, Yoo's method is to seek the minimum $\overline{p}_{ij}^T \overline{p}_{ij}$. Yoo's method can be used when the fault direction angle between two faulty sensors is large. But the isolation performance is very decreased like Fig. A2. On the other hand, EPSA detects the fault through checking the relative consistencies of subsets of the full set of measurements and also additionally considers the type of Fault-B. Therefore EPSA can expect that the detection probability is increased. In case of two-faults isolation of Fault-C, EPSA is to seek the first smallest and the second smallest $\overline{p}_{ij}^T \overline{p}_{ij}$ through additional procedure. The isolation probability is increased because of this procedure.

Table A1. The angle between the fault direction of 1st sensor and that of the other sensors

Sensor Number	$1^{st}-2^{nd}$	$1^{st}-3^{rd}$	$1^{st}-4^{th}$	$1^{st}-5^{th}$	$1^{st}-6^{th}$	$1^{st}-7^{th}$
Fault Direction Angle	124.18^0	97.97^0	78.43^0	78.43^0	97.97^0	124.18^0



(a) The result of (Ray, 1983)



(b) The result of (Yoo, 2003)

Fig. A2. The fault isolation result of the previously suggested FDI method

Appendix-C

We assume that the number of used sensors is seven, $l=7$, under the simulation condition of Section 3. Let $\overline{p}_{ij}$ be the corresponding parity vector generated from all measurements in S^{ij} where $m_i \notin S^{ij}$ and $m_j \notin S^{ij}$. The matrix $\overline{V}_{ij}$ of twenty-one, from $\overline{V}_{12}$ to $\overline{V}_{67}$, exist because the number of cases to select two sensors among 7 sensors is ${}_7C_2$, where $i, j = 1, 2, \cdots, 7$ and $i \neq j$. From (1) and (3), $p = Vm = V\left(Hx + \varepsilon + f\right) = V\left(\varepsilon + f\right)$. In EPSA, the isolation method of Fault-C is to seek the first smallest and the second smallest $\overline{\chi}_{ij}^2$ where $\overline{\chi}_{ij}^2 \equiv \overline{p}_{ij}^T \overline{p}_{ij}$. Assume that 1^{st} sensor and 2^{nd} sensor are the faulty sensors and F/N Ratio is 10 such as $f_1 = 10$ and $f_2 = 10$.

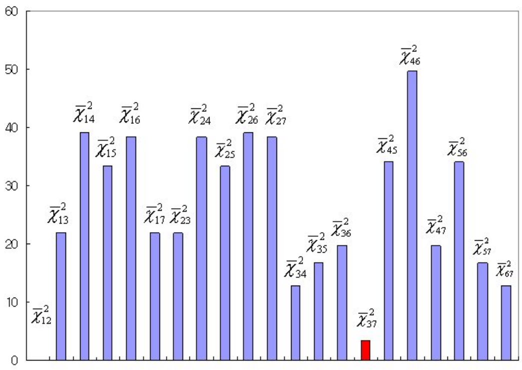
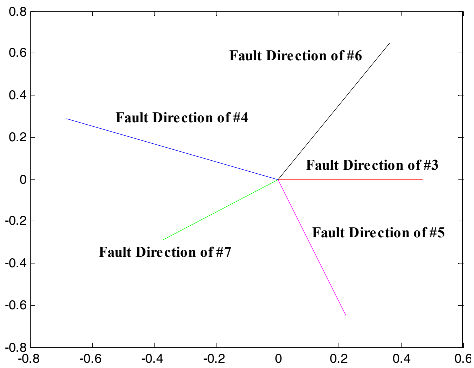
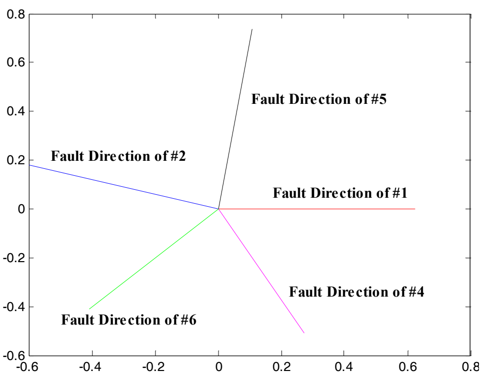


Fig. A3. The magnitude of twenty-one $\overline{\chi}_{ij}^2$



(a) $\overline{V}_{12}$



(b) $\overline{V}_{37}$

Fig. A4. The parity space of $\overline{V}_{12}$ and $\overline{V}_{37}$

If noise is, the $\overline{\chi}_{ij}^2$ of twenty-one is obtained like Fig. A3. The first smallest is $\overline{\chi}_{12}^2$ and the second smallest is $\overline{\chi}_{37}^2$ among the $\overline{\chi}_{ij}^2$ of twenty-one. The magnitude of $\overline{\chi}_{12}^2$ is zero since two faulty sensors are excluded. That is, $\overline{V}_{12} f_{12}$ is zero because the fault directions of 1^{st} sensor and 2^{nd} sensor do not exist in parity space, where f_{12} is the vector to eliminate 1^{st} row and 2^{nd} row in f. What is the reason that the magnitude of $\overline{\chi}_{37}^2$ is very small? Fig. A4(b) shows the parity space of $\overline{V}_{37}$. The magnitude of $\overline{V}_{37} f_{37}$ is that smallest except $\overline{V}_{12} f_{12}$ because the fault directions angle between 1^{st} sensor and 2^{nd} sensor is that largest among all $\overline{V}_{ij} f_{ij}$, where $i, j = 1, 2, \cdots, 7$ and $i \neq j$. Thus, sometimes $\overline{\chi}_{37}^2$ can be selected as the minimum instead of $\overline{\chi}_{12}^2$ because of the effect of noise.

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