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위상배열레이더 추적 각도예측의 정확도 정량화

(Quantification of Angular Prediction Accuracy for Phased Array Radar Tracking)

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요 약

위상배열레이더를 이용한 추적의 성능을 지수화하기 위해서 각도예측 오차공분산행렬의 함수를 이용한 정량화 방법에 대해 검토하였다. 특히, 이 오차공분산행렬의 함수로 최대고유값과 대각합을 이용하여 정량화하는 경우에 대해 안테나 빔조향 손실을 반영한 표적탐지확률을 기준으로 정량화의 일관성을 수치실험을 통해 검토하였다. 이를 통해 추적성능을 표적탐지확률로 기준하는 경우, 대각합이 최대고유값 보다 더 일관성을 갖는 정량화 함수임을 확인하였다.

Abstract

Scalar quantification of the angular prediction error covariance matrix is considered for characterizing tracking performances in phased array radar tracking. Specifically, the maximum eigenvalue and the trace of the covariance matrix are examined in terms of consistency in parameterizing the probability of detection, taking antenna beam-pointing losses into account, and it is shown numerically that the latter is more consistent.

Keywords: angular prediction error, beam-pointing losses, probability of detection, phased array radars.

Ⅰ. Introduction

A power loss in a received target signal results when the antenna pointing angle is offset from a true target angle. The power loss has an impact on the probability of target detection, which affects tracking performances most profoundly[1~2]. The true target position is uncertain for pointing antenna beam, and it is usually predicted using a tracking filter. The angular error of the predicted target position is a primary factor resulting in the pointing angle error.

A certain level of angular prediction accuracy should be maintained to keep beam-pointing losses from being excessively large. The uncertainty of angular prediction is often represented by the error covariance matrix of the two-dimensional angular position prediction. The error ellipse defined by the covariance matrix becomes larger as the track-revisit interval increases[3~6]. The maximum eigenvalue ofthe matrix was used in [3] as a reference parameter for quantifying the angular prediction accuracy and determining the next track-revisit time that ensures a predetermined level of track accuracy[3~5].

The angular prediction error leads to the beam-pointing error in phased array radar tracking, and it affects the probability of detection which has a

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significant impact on track accuracy. Accordingly, it is reasonable to use the probability of detection as a metric in quantifying the angular prediction accuracy. Recently, it is observed in [7] that the detection probability is dependent primarily on the trace of the prediction error covariance matrix. It suggests that the angular error covariance matrix can be characterized by its trace as long as the probability of detection is a metric of interest. Note that, while either one of the maximum eigenvalue and the trace does not vary in magnitude, the other can increase two times as the covariance matrix changes. This observation motivated us to investigate which characterization of the matrix is more consistent in terms of the probability of detection. In this paper, two quantifications, the maximum eigenvalue and the trace, of the angular prediction error covariance matrix are examined in terms of the probability of target detection as a metric.

Ⅱ. Radar Detection Model

We assume the antenna beam-shape loss to be a Gaussian-shaped function[3~4]. In this case, the received signal energy is proportional to the transmitted energy, but decreases with losses due to beam-pointing error so that the signal-to-noise ratio of the received signal, denoted by SNR can be represented by

$$SNR = SNR_0 \cdot \exp\left\{-c\left[\frac{(u-u_0)^2}{B_u^2}+\frac{(v-v_0)^2}{B_v^2}\right]\right\} \tag{1}$$

where SNR_0 is the nominal value of the signal-to-noise ratio for no angular pointing error, (u, v) denote the true direction of the target, (u_0, v_0) are the direction of the beam pointing, B_u and B_v denote the two-sided half-power antenna beamwidths defined along the u and v coordinate axes, respectively, and c is a loss factor which is set to 2.77[4].

We assume that the target fluctuation has a Rayleigh distribution. This gives the relationship between the probability of detection (P_D) the probability of false alarm (P_F), and the quantity SNR for the target as

$$P_D = P_F^{\frac{1}{1+SNR}}. \tag{2}$$

Our assumptions on radar detection are summarized as follow.

Assumption 1: The antenna beam-shape loss is represented by (1), and the probability of target detection is determined by (2).

A distinctive aspect of a phased array radar is that its beam can be positioned repeatedly, if necessary, in an allocation for a track update. Suppose that, if no detection occurs on the first beam positioning, the beam is positioned repeatedly toward the predicted target position until a successful track update is made or until the repetition reaches a predetermined number of times[5]. In this case, the expected radar energy resources required in the process is a weighted sum of the expectations of all possible multiplications of detection probabilities over the two-dimensional angular plane[5], where each of the detection probabilities corresponds to a distinct one of the repeated illuminations.

The angular error of the predicted target position is a primary factor resulting in the pointing angle error. We assume that the beam is directed toward the predicted target position, and assume that the angular prediction error has a joint Gaussian distribution. Let $x_u = (u-u_0)/B_u$ and $x_v = (v-v_0)/B_v$. x_u and x_v denote beam pointing errors normalized with respect to the corresponding beamwidth in the u and v coordinates. Let $\mathbf{x} = (x_u, x_v)^T$. Denote by X the random vector with a specific realization $\mathbf{x}$. The assumptions on the distribution of the angular prediction error and the beam positioning are summarized in terms of the distribution of the normalized beam pointing error X as follows.

Assumption 2: The normalized beam pointing error X has a zero-mean Gaussian distribution with covariance matrix P.

We can represent the probabilty density function(pdf) of the position prediction error X as $f(\mathrm{x}) = (2\pi)^{-1} |\mathrm{P}|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}\mathrm{x}^T \mathrm{P}^{-1}\mathrm{x}\right)$. Obviously, $\mathrm{P} = E[XX^T]$. Also, we can represent (1) in terms of x, that is,

$$SNR(\mathrm{x}) = SNR_0 \exp(-c\mathrm{x}^T\mathrm{x}). \tag{3}$$

The beam of a phased array radar can be repositioned within a few microseconds without inertia, and we can illuminate a target repeatedly, if necessary, for a track update before the distribution of the beam pointing error changes considerably. As a consequence, the following assumption holds.

Assumption 3: The pdf of the normalized beam pointing error X does not change throughout the repeated beam illuminations.

In the sequel, we indicate an association of notations with the i-th illumination by inserting subscript i to them. To be more specific, $SNR_i(\mathrm{x})$ and SNR_{0i} denote $SNR(\mathrm{x})$ and SNR_0 for the i-th beam illumination, respectively. Similarly, we use P_{Fi} to denote the false alarm probability P_F for the i-th illumination.

The probability of target detection depends on beampointing error x. Let $P_{Di}(\mathrm{x})$ and $\overline{P}_{Di}$ denote the detection probability and its average over x for the i th beam illumination, respectively. Note that $\overline{P}_{Di} = \iint P_{Di}(\mathrm{x})\, f(\mathrm{x})\, dx_u\, dx_v$. Substituting (3) into (2) gives

$$P_{Di}(\mathrm{x}) = P_{Fi}^{1/\left[1+SNR_{0i}\exp(-c\mathrm{x}^T\mathrm{x})\right]}, \tag{4}$$

and we have

$$\overline{P}_{Di} = \frac{1}{2\pi|\mathrm{P}|^{\frac{1}{2}}} \iint P_{Fi}^{1/\left[1+SNR_{0i}\exp(-c\mathrm{x}^T\mathrm{x})\right]} \cdot \exp\left(-\frac{1}{2}\mathrm{x}^T\mathrm{P}^{-1}\mathrm{x}\right) dx_u\, dx_v\,. \tag{5}$$

Similarly, we use $\overline{P}_{D_{i1,i2,\ldots,il}}$ to represent

$$\overline{P}_{D_{i1,i2,\ldots,il}} = \frac{1}{2\pi|\mathrm{P}|^{\frac{1}{2}}} \iint P_{D_{i1}}(\mathrm{x}) P_{D_{i2}}(\mathrm{x}) \cdots P_{D_{il}}(\mathrm{x}) \cdot \exp\left(-\frac{1}{2}\mathrm{x}^T\mathrm{P}^{-1}\mathrm{x}\right) dx_u dx_v \tag{6}$$

for a collection of l distinct illumination indices $\{i_1, i_2, \ldots, i_l\}$. A weighted sum of a collection of $\overline{P}_{D_{i1,i2,\ldots,il}}$'s can represent the expectation of the radar energy resources required in a track-update process described above. More discussion on this can be found in [5].

Ⅲ. QUANTIFICATION OF ANGULAR PREDICTION ERROR COVARIANCE MATRICES

Suppose that a track was updated at t, and let $t+\tau$ denote the next track-revisit time instant. Obviously, P is a function of τ. As τ increases, the uncertainty in predicting target position increases continuously in all directions in the two-dimensional angular plane. That is, $\mathrm{P}(\tau_2) > \mathrm{P}(\tau_2)$ if $\tau_2 > \tau_1$. This also implies that the maximum eigenvalue and the trace of $\mathrm{P}(\tau)$, $\lambda_{\max}(\mathrm{P}(\tau))$ and $tr(\mathrm{P}(\tau))$, are continuous and strictly increasing in τ [8].

In [3], the square root of $\lambda_{\max}(\mathrm{P})$, denoted by V_0, was used to represent an accuracy of angular prediction, which corresponds to the standard deviation along the major axis of the error ellipse defined by P. In the work, $(\lambda_{\max}(\mathrm{P}(\tau)))^{\frac{1}{2}} = \overline{V_0}$ was solved to determine the trackrevisit interval $\bar{\tau}$ for given $\overline{V_0}$. The track should be revisited no later than $t+\bar{\tau}$ to assure track accuracy specified by $\overline{V_0}$ at the revisit. The parameter $\overline{V_0}$ and transmit signal strength were optimized to minimize the radar energy resources required for track maintenance in [3] and [5].

Let $\mathrm{P}=\begin{bmatrix}\sigma_u^2 & \rho\,\sigma_u\sigma_v \\ \rho\sigma_u\sigma_v & \sigma_v^2\end{bmatrix}$. Here, σ_u and σ_v are the prediction error standard deviations in the u and v coordinate axes, normalized with respect to the beamwidths B_u and B_v, respectively. ρ is the correlation coefficient between the prediction errors of angular positions in the u and v coordinates. The correlation almost always occurs when tracking a target in clutter using the probabilistic data association filter[4~5], and it depends on the locations of measurements in the validation gate, the target detection probability and the expected number of false measurements in the gate. Note that P is specified by σ_u^2, σ_v^2, and ρ.

Obviously, $\overline{P}_{Di}$ of (5) depends on the angular prediction error covariance matrix P. The quantification problem can be stated as follows: for given SNR_{0i} and P_{Fi}, find a scalar function μ of P such that $\overline{P}_{Di}(\mathrm{P})$ does not vary for all $\mathrm{P}\in\Lambda$, $\Lambda=\{\ \mathrm{P}:\mu(\mathrm{P})=\overline{\mu}\}$, where $\overline{\mu}$ is a real number. Unfortunately, $\overline{P}_{Di}(\mathrm{P})$ is not available in analytic form, and it appears difficult to find such a function μ. In practice, it is sufficient to find μ such that the probability of detection does not vary considerably over a set Λ which includes P's that occur typically in tracking with a phased array radar. It is obvious that $\lambda_{\max}(\mathrm{P})$ is a prominent possibility for $\mu(\mathrm{P})$.

The probability of detection should not be much affected regardless of P as long as the index μ does not vary. It has been observed in [7] that, in terms of the distribution of angular prediction errors, the probability of detection is dependent primarily on $tr(\mathrm{P})$. This suggests that $tr(\mathrm{P})$ is also a possible choice for μ. Let $\beta=\sigma_u^2/\sigma_v^2$. Then, the ratio $\lambda_{\max}(\mathrm{P})/tr(\mathrm{P})$ can be represented as a function of ρ and β:

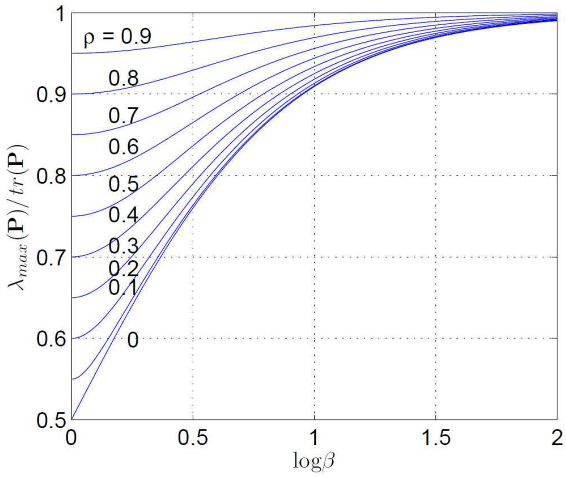

그림 1.
Fig. 1. The ratio $\lambda_{\max}(\mathrm{P})/tr(\mathrm{P})$.

$$\frac{\lambda_{\max}(\mathrm{P})}{tr(\mathrm{P})}=\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{4(1-\rho^2)\beta}{(1+\beta)^2}}\,. \tag{7}$$

Let $g(\beta,\rho)=\lambda_{\max}(\mathrm{P})/tr(\mathrm{P})$. Note that $0.5\le g(\beta,\rho)\le 1$, $g(\beta,\rho)=g(\beta^{-1},\rho)$, $g(1,0)=0.5$, and $g(\beta,1)=g(0,\rho)=g(\infty,\rho)=1$. The ratio is presented in Fig. 1. The figure indicates that $\lambda_{\max}(\mathrm{P})$ can increase two times even while $tr(\mathrm{P})$ may not vary. In the following section, we examine $tr(\mathrm{P})$ and $\lambda_{\max}(\mathrm{P})$ numerically in terms of consistency in representing $\overline{P}_{Di}$ and $\overline{P}_{D_{i1,i2,i3}}$ over a set Λ.

Ⅳ. NUMERICAL RESULTS

We evaluated numerically deviations in the probability of detection $\overline{P}_{Di}$ over a set of angular prediction error covariance matrices. The set consists of the covariance matrices with either the maximum eigenvalue or the trace pre-specified and the other varied. The results are presented in Figs. 2 and 3 for $SNR_{0i}=30\,(14.8\,\mathrm{dB})$ and $P_{Fi}=10^{-4}$. Fig. 2 presents $\overline{P}_{Di}$ in terms of typical values of $tr(\mathrm{P})$, β and ρ. The figure indicates that, for given $tr(\mathrm{P})$, $\overline{P}_{Di}$ is almost invariant to the changes of β and ρ. In Fig. 3, $\overline{P}_{Di}$ is presented as a function of typical values of $\lambda_{\max}(\mathrm{P})$, β and ρ. In contrast to the case of

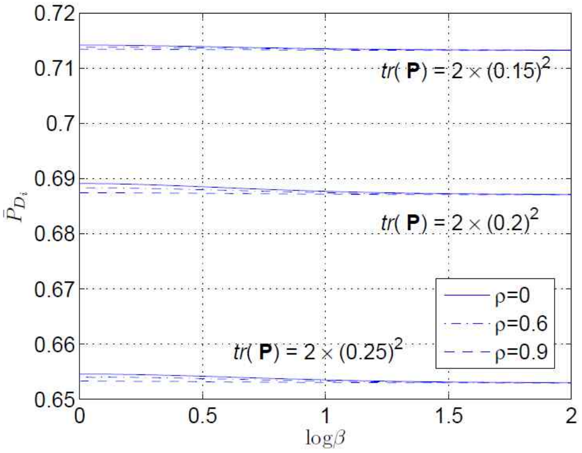

그림 2.

Fig. 2. $\overline{P}_{Di}$ as a function of $tr(\mathrm{P})$, β and ρ for $SNR_{0i} = 30$, $\overline{P}_{Fi} = 10^{-4}$

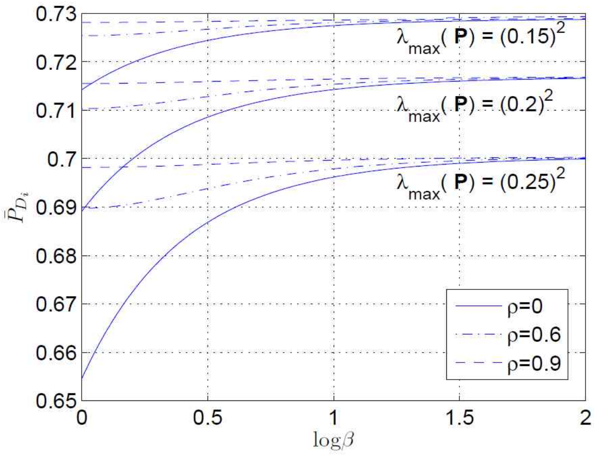

그림 3.

Fig. 3. $\overline{P}_{Di}$ as a function of $\lambda_{\max}(\mathrm{P})$, β and ρ for $SNR_{0i} = 30$, $\overline{P}_{Fi} = 10^{-4}$

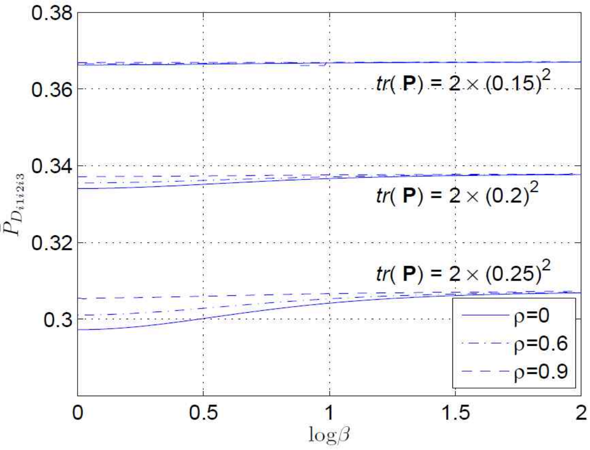

그림 4.

Fig. 4. $\overline{P}_{D_{i1,i2,i3}}$ as a function of $tr(\mathrm{P})$, β and ρ for $SNR_{0i} = 30$, $\overline{P}_{Fi} = 10^{-4}$

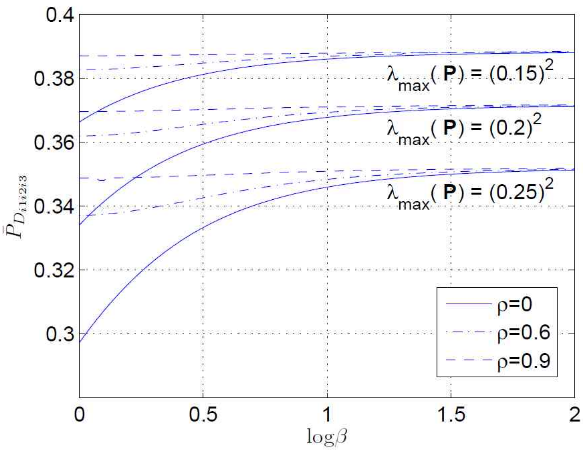

그림 5.

Fig. 5. $\overline{P}_{D_{i1,i2,i3}}$ as a function of $\lambda_{\max}(\mathrm{P})$, β and ρ for $SNR_{0i} = 30$, $\overline{P}_{Fi} = 10^{-4}$

parameterization with $tr(\mathrm{P})$ in Fig. 2, $\overline{P}_{Di}$ was affected considerably when β and ρ are varied. The deviations, however, diminishes gradually as β increases.

The probability $\overline{P}_{D_{i1,i2,i3}}$ was also evaluated under the same conditions as above. The results are presented in Figs. 4 and 5. The results indicate that the trace represents $\overline{P}_{D_{i1,i2,i3}}$ more uniformly over a set Λ. Recall that a weighted sum of a collection of $\overline{P}_{D_{i1,i2,\ldots,il}}$'s in (6) can represent the expected radar energy resources required in a track-update process described in Section Ⅱ. We obtained additional numerical results for SNR_{0i} in the interval $(13.5\,\mathrm{dB}, 18\,\mathrm{dB})$ and P_{Fi} in $(10^{-4}, 10^{-5})$, and confirmed that the above arguments also hold. This suggests that $tr(\mathrm{P})$ can represent the probability of detection as well as a tracking performance, defined by a weighted sum of a collection of

$\overline{P}_{D_{i1,i2,\ldots,il}}$'s, more uniformly over an extensive set Λ. This implies that the trace can serve better as a reference parameter for determining the next track-revisit time, which ensures consistency in the probability of detection and also in a certain tracking performance characterized in terms of radar transmit power efficiency.

V. CONCLUSIONS

Two quantifications, the maximum eigenvalue and the trace, of the angular prediction error covariance matrix were examined in terms of consistency in parameterizing the probability of detection. It was shown numerically that, compared to the maximum eigenvalue, the trace represents the probability of detection more uniformly over an extensive set of the prediction error covariance matrices. The numerical results also suggest that the trace can serve better as a reference parameter in characterizing tracking performances.

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