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여러 가지 뉴스벤더모델의 기대값 사이의 관계에 대한 견고한 추측

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A Robust Conjecture on the Relationship among the Expected Profits of Various Newsvendor Models

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■ Abstract ■

The present study provides some extensions over a recent work in Won (2011) which investigates properties of the static newsvendor problem under a schedule involving progressive multiple discounts under the assumption that demand is given exogenously. Khouja (1995, 1996) formulated the extended versions over the classical newsvendor model with various discount policies including all-units discount and/or multiple discounts and found that the extended newsvendor models with discount schedules yield higher optimal expected profits than the classical newsvendor model with no-discounts. In this study, we establish a robust conjecture as a stronger statement than Khouja's findings with regard to the general relationship among the expected profits of newsvendor models in the sense that the conjecture holds for every order quantity as well as the optimal order quantity. The conjecture encourages the newsvendor facing quantity discounts to safely implement her own discounts policy to customer or accept quantity discounts offered by the supplier even if the optimal order quantity cannot be ordered due to additional restrictions such as budget or warehouse capacity constraints because the newsvendor models with quantity discounts always yield higher expected profit than the classic newsvendor model without quantity discounts regardless of the order quantity. Results from wide experiments with various probability distributions of demand strongly support our conjecture.

Keyword : Newsvendor Problem, Riskless Profit, All-Units Discount, Progressive Multiple Discounts

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1. Introduction

Khouja [5, 6] formulated the extended versions over the classical newsvendor model with various discount policies under the assumption that demand is given exogenously and found that extended newsvendor models with discount schedules yield higher optimal expected profits than the classical newsvendor model. He considered two kinds of discounts policy offered by the supplier and retailer : all-units quantity discounts which the supplier offers to the retailer and progressive multiple discounts which the retailer offers to the customers. Different discount policies can be offered by the supplier [12]. Depending on which party of the supplier and retailer offers discounts policy, four types of newsvendor model can be formulated:

(i) type I newsvendor model with no supplier all-units quantity discounts and with no retailer progressive multiple discounts,
(ii) type II newsvendor model with no supplier all-units quantity discounts but with retailer progressive multiple discounts,
(iii) type III newsvendor model with supplier all-units quantity discounts but with no retailer progressive multiple discounts, and
(iv) type IV newsvendor model with all-units supplier quantity discounts and with retailer progressive multiple discounts.

The purpose of this paper is to establish a much stronger statement than Khouja's findings with regard to the general relationship among the expected profits of various static newsvendor models in which demand is given exogenously. The dynamic newsvendor models [2, 8, 10, 11] in which demand depends on price are not considered in the present study. The general relationship among the expected profits will be established as a robust conjecture. Since the conjecture holds for *every* order quantity as well as the optimal order quantity, the newsvendor facing quantity discounts can safely implement her own discounts policy to customer or accept quantity discounts offered by the supplier regardless of the order quantity even if the optimal order quantity cannot be ordered due to additional restrictions such as budget or warehouse capacity constraints. Results from wide experiments with various probability distributions of demand strongly support our conjecture.

In order to establish conjecture on the general relationship among the expected profits, we use riskless profit which is the sum of expected profit and cost. The riskless profit has received little attention in the literature since most of the previous studies revealed little implication for determining the optimal order quantity from the information provided by the riskless profit.

However, Won [14] uses riskless profit-based approach to investigate properties of the newsvendor problem under a schedule involving progressive multiple discounts compared with the standard newsvendor problem under a no-discounts schedule. Unlike the conventional derivative-based approach which focuses in finding the optimal values of decision variables, the riskless profit-based approach seeks to find the decision rule that holds for *every* order quantity as well as the optimal order quantity. Therefore, the results from the analysis of riskless profit can naturally lead to the establishment of a stronger rule than the ones from the analysis of the optimal expected profit alone.

The paper is organized as follows. Section 2 describes Khouja's findngs briefly. Section 3 establishes the closed-form formulas for the riskless profits of various newsvendor models and reveals the general relationship among the riskless profits of various newsvendor models. From the analysis of riskless profits, section 4 establishes a certain conjecture on the general relationship among the expected profits of various newsvendor models. Section 5 provides extended computational experiments that strongly support our conjecture. Finally, the paper concludes with a summary of the present research and a suggestion for future research.

2. Khouja's Findings

To formulate the extended newsvendor models, Khouja [5, 6] defines the following notation：

t_i = fraction of realized demand at the regular price that can be additionally sold by discounting the product to price a_i during the ith discount period offered by the retailer, $i=0,\ 1,\ \cdots,\ n$

$a_{i,j}=a_i-b_j$ = cost of underestimating demand during the ith discount period offered by the retailer when the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$, $j=0,\ 1,\ \cdots,\ m$

$\beta_{i,j}=b_j-a_i$ = cost of overestimating demand during the ith discount period offered by the retailer when the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$,

X = random variable denoting the demand, $0 \le X < \infty$

$f(X)$ = probability density function of X

$F(X)$ = cumulative distribution function of X

$E(X)$ = expected demand of X

In addition, we define the following notation：

$R_0^I(X, Q)$ = profit for order quantity Q under demand X in the type I newsvendor model

$R_0^{II}(X, Q)$ = profit for order quantity Q under demand X in the type II newsvendor model

$R_j^{III}(X, Q)$ = profit for order quantity Q under demand X when the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$ in the type III newsvendor model

$R_j^{IV}(X, Q)$ = profit for order quantity Q under demand X when the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$ in the type IV newsvendor model

$ER_0^I(Q)$ = expected profit for order quantity Q in the type I newsvendor model, i.e., type I expected profit

$ER_0^{II}(Q)$ = type II expected profit for order quantity Q

$ER_j^{III}(Q)$ = type III expected profit for order quantity Q such that $q_j \le Q < q_{j+1}$

$ER_j^{IV}(Q)$ = type IV expected profit for order quantity Q such that $q_j \le Q < q_{j+1}$

$EC_0^I(Q)$ = expected cost of underestimating and overestimating demand for order quantity Q in the type I newsvendor model, i.e., type I expected cost

$EC_0^{II}(Q)$ = type II expected cost for order quantity Q

$EC_j^{III}(Q)$ = type III expected cost for order quantity Q such that $q_j \le Q < q_{j+1}$

$EC_j^{IV}(Q)$ = type IV expected cost for order quantity Q such that $q_j \le Q < q_{j+1}$

$ES_0^I(Q)$ = sum of expected profit and cost for order quantity Q in the type I newsvendor model, i.e., type I riskless profit

$ES_0^{II}(Q)$ = type II riskless profit for order quantity Q

$ES_j^{III}(Q)$ = type III riskless profit for order quantity Q such that $q_j \le Q < q_{j+1}$

$ES_j^{IV}(Q)$ = type IV riskless profit for order quantity Q such that $q_j \le Q < q_{j+1}$

For the extended newsvendor models, we assume the followings :

(i) The supplier offers an all-units price discounts schedule with price beaks at quantities [4] :

$$0 = q_0 < q_1 < q_2 < \cdots < q_m = \propto.$$

For an order quantity Q such that $q_j \le Q < q_{j+1}$, the cost per unit of b_j is offered and

$$b_0 > b_1 > b_2 > \cdots > b_m.$$

(ii) When the cost per unit of b_j is offered by the supplier, the retailer can offer a progressive multiple discounts schedule with the selling prices

$$a_0 > a_1 > a_2 > \cdots > a_n.$$

Then, the type I newsvendor model reduces to the case with $m = n = 0$, the type II newsvendor model reduces to the case with $m = 0$ and $n > 0$, the type III newsvendor model reduces to the case with $m > 0$ and $n = 0$, and the type IV newsvendor model reduces to the case with $m > 0$ and $n > 0$. Therefore, the type I, II, and III newsvendor models can be treated as special cases of type IV newsvendor model.

For convenience of analysis and comparative purpose among different newsvendor models, salvage value and shortage penalty cost will not be considered without loss of generality. From the definition of the nth partial moment with an upper limit Q of a random variable X given a probability density function $f(X)$, we can let $E_Q(X) = \int_0^Q Xf(X)dX$ for a specific order quantity Q [7].

Suppose when the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$, the first $(r_j - 1)$ discounts are profitable and the rest are not. Then, for the cost of underestimating demand, $\alpha_{i,j} \ge 0$, $k = 0, 1, \cdots, r_j - 1$ $\alpha_{i,j} = 0$, $k = r_j, r_j + 1, \cdots, n$ and for the cost of overestimating demand, $\beta_{i,j} = 0$, $k = 0, 1, \cdots, r_j - 1$ and $\beta_{i,j} \ge 0$, $k = r_j, r_{j+1}, \cdots, n$. We assume that $t_0 = 1$ and $t_n = \propto$ [5, 6].

Let $V_k = \sum_{i=0}^{k} t_i$, $h_k = Q/V_k$, $W_{k,j} = \sum_{i=0}^{k} \alpha_{i,j} t_i$, and $S_{k,j} = \sum_{i=r_j}^{k} \beta_{i,j} t_i$. $W_{k,j}$ is the profit for the total quantity demanded if the product is offered at all prices until the kth discount is reached before the

(r_j-1)th discount period when the cost per unit of b_j is offered by the supplier. $S_{k,j}$ is the profit for the total quantity demanded if the product is offered at all prices until the kth discount is reached after the discount period when the cost per unit of b_j is offered by the supplier. Note that since $V_0=1$ and $V_n=\propto$, $h_0=Q$, $h_n=0$, $E_{h_0}(X)=E_Q(X)$, and $E_{h_n}(X)=0$. Let $E_{h_{-1}}(X)=E(X)$ and $V_{-1}=W_{-1,j}=S_{-1,j}=0$.

To state Khouja's findings, let $Q^*_{III,j}$ and $Q^*_{IV,j}$, $j=0, 1, 2, \cdots, m$, be the local optimal order quantities of the type III and IV newsvendor models, respectively, for the price break interval $q_j \le Q^*_{III,j}$, $Q^*_{IV,j}<q_{j+1}$, and Q^*_I, Q^*_{II}, Q^*_{III}, and Q^*_{IV} be the global optimal order quantities for four newsvendor models, respectively. Khouja [6] proved :

(i) The expected profit functions $ER^I_0(Q)$, $ER^{II}_0(Q)$, $ER^{III}_j(Q)$, and $ER^{IV}_j(Q)$ are concave.

(ii) The local optimal order quantities $Q^*_{III,j}$ and $Q^*_{IV,j}$ of $ER^{III}_j(Q^{III}_j)$ and $ER^{IV}_j(Q^{IV}_j)$, respectively, satisfy $Q^*_{III,0}<Q^*_{III,1}<\cdots<Q^*_{III,j}<Q^*_{III,j+1}<\cdots<Q^*_{III,m}$, and $Q^*_{IV,0}<Q^*_{IV,1}<\cdots<Q^*_{IV,j}<Q^*_{IV,j+1}<\cdots<Q^*_{IV,m}$

(iii) The local optimal expected profits for each price beak interval satisfy
$$ER^{IV}_0(Q^*_{IV,0})<ER^{IV}_1(Q^*_{IV,1})<\cdots<ER^{IV}_j(Q^*_{IV,j})<ER^{IV}_{j+1}(Q^*_{IV,j+1})<\cdots<ER^{IV}_m(Q^*_{IV,m}).$$

However, it should be noted that, unlike the type IV local optimal expected profits for each price break interval, for the type III local optimal expected profits for each price break interval we cannot say that

$$ER^{III}_0(Q^*_{III,0})<ER^{III}_1(Q^*_{III,1})<\cdots<ER^{III}_j(Q^*_{III,j})<ER^{III}_{j+1}(Q^*_{III,j+1})<\cdots<ER^{III}_m(Q^*_{III,m}).$$

Based on the above properties regarding the expected profits and optimal order quantities of newsvendor models, Khouja [5, 6] argured the followings from the result with a numerical example :

(iv) $Q^*_{II}\ge Q^*_I$ and $ER^{II}_0(Q^*_{II})\ge ER^I_0(Q^*_I)$.

(v) $Q^*_{IV}\ge Q^*_{III}$ and $ER^{IV}_j(Q^*_{IV})\ge ER^{III}_j(Q^*_{III})$.

However, he did not provide rigorous proofs for (iv) and (v). Won [14] provides a robust conjecture for (iv) based on wide experiments with various probability distributions of demand. In this study, from the analysis of the riskless profit of each type of newsvendor model and some experiments with various demand distributions, a robust conjecture for the general relationship among the expected profits of extended newsvendor models including (iv) and (v) will be established.

3. Riskless Profits of Newsvendor Models

The type I expected profit for a specific order quantity Q [7] is given by

$$ER^I_0(Q)=a_0E_Q(X)+\alpha_{0,0}\,Q-a_0QF(Q) \tag{1}$$

and the type I expected cost associated with underestimating and overestimating demand is given by

$$EC^I_0(Q)=W_{0,0}E(X)-a_0E_Q(X)-\alpha_{0,0}Q+a_0QF(Q). \tag{2}$$

From equations (1) and (2), we immediately obtain the following well-known formula for a basic balance equation for the sum of expected profit and cost for a specific order quantity Q in the type I newsvendor model :

$$ES_0^I(Q) = ER_0^I(Q) + EC_0^I(Q) = W_{0,0}\, E(X). \quad (3)$$

The expectation $ES_0^I(Q)$ in equation (3) is known as the riskless profit [11] or maximum profit [1].

To establish the general relationship among the expected profits of the extended newsvendor models, we will focus on the analysis of the expected profit of the type IV newsvendor model and cost because the remaining three newsvendor models can be treated as special cases of the type IV newsvendor model. The type IV expected retailer profit over all discount periods for a specific order quantity Q when the cost per unit of b_j is offered by the supplier for order for order quantity Q such that $q_j \le Q < q_{j+1}$ is given by

$$\begin{aligned} ER_j^{IV}(Q) = &\sum_{k=0}^{r_j-1}(W_{k-1,j} - \alpha_{k,j}V_{k-1}) \\ &[E_{h_{k-1}}(X) - E_{h_k}(X)] \\ &+ Q\left(\sum_{k=0}^{r_j-1}\alpha_{k,j}[F(h_{k-1}) - F(h_k)]\right) \\ &+ \sum_{k=r_j}^{n}(W_{r_j-1,j} - S_{k-1,j} + \beta_{k,j}V_{k-1}) \\ &[E_{h_{k-1}}(X) - E_{h_k}(x)] \\ &- Q\left(\sum_{k=r_j}^{n}\beta_{k,j}[F(h_{k-1}) - F(h_k)]\right) \end{aligned} \quad (4)$$

and the type IV expected cost of underestimating demand and overestimating demand is given by

$$\begin{aligned} EC_j^{IV}(Q) = &\sum_{k=0}^{r_j-1}\left[W_{k-1,j} + \sum_{i=0}^{k}(\alpha_{k,j} - \alpha_{i,j})t_i\right] \\ &[E_{h_{k-1}}(X) - E_{h_k}(X)] \\ &- Q\left(\sum_{k=0}^{r_j-1}\alpha_{k,j}[F(h_{k-1}) - F(h_k)]\right) \\ &+ \sum_{k=r_j}^{n}(S_{k-1,j} - \beta_{k,j}V_{k-1}) \\ &[E_{h_{k-1}}(X) - E_{h_k}(X)] \\ &+ Q\left(\sum_{k=r_j}^{n}\beta_{k,j}[F(h_{k-1}) - F(h_k)]\right). \end{aligned} \quad (5)$$

Mathematical details for equations (4) and (5) are provided in Appendix. The purpose of the paper is to reveal the general relationship among the expected profits of the extended newsvendor models. However, it can be noticed from equations (4) and (5) that it is very difficult to try to reveal the relationship among the expected profits of the extended newsvendor models by dealing with $ER_j^{IV}(Q)$ directly. This is the reason why we seek to raise a strong conjecture on the relationship among the expected profits of the extended newsvendor models through the analysis of the riskless profits instead of the expected profits and wide experiments with various probability distributions of demand.

From equations (4) and (5), we have the following observation regarding the riskless profits of the extended newsvendor models :

Observation 1 : When the cost per unit of b_j is offered by the supplier for order quantity Q such that $q_j \le Q < q_{j+1}$, the riskless profits of the newsvendor model are constant for every order quantity Q which is valid for $q_j \le Q < q_{j+1}$ and are given by

(i) $ES_0^{II}(Q) = W_{r_0-1,0}E(X)$

(ii) $ES_j^{III}(Q) = W_{0,j}E(X)$

(iii) $ES_j^{IV}(Q) = W_{r_j-1,j}E(X)$.

Proof : Since the first two cases can be treated as special cases of case (iii), it would suffice to show only the result for case (iii). Combining equations (1) and (2) and simplifying gives

$$
\begin{aligned}
ES_j^{IV}(Q) &= ER_j^{IV}(Q) + EC_j^{IV}(Q) \\
&= W_{r_j-1,j}\left(\sum_{k=0}^{n}\left[E_{h_{k-1}}(X) - E_{h_k}(X)\right]\right) \\
&\quad + \sum_{k=0}^{r_j-1}\left[\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})t_i + W_{k-1,j} - \alpha_{k,j}V_{k-1}\right] \\
&\quad \left[E_{h_{k-1}}(X) - E_{h_k}(X)\right] \\
&= W_{r_j-1,\ j}\left[E_{h_{-1}}(X) - E_{h_n}(X)\right] \\
&\quad + \sum_{k=0}^{r_j-1}\left[\sum_{i=0}^{k}\left(\alpha_{k,j}-\alpha_{i,j}\right)t_i + W_{k-1,j} - \alpha_{k,j}V_{k-1}\right] \\
&\quad \left[E_{h_{k-1}}(X) - E_{h_k}(X)\right] \\
&= W_{r_j-1,\ j}E(X) + \\
&\quad + \sum_{k=0}^{r_j-1}\left[\sum_{i=0}^{k}\left(\alpha_{k,j}-\alpha_{i,j}\right)t_i + W_{k-1,j} - \alpha_{k,j}V_{k-1}\right] \\
&\quad \left[E_{h_{k-1}}(X) - E_{h_k}(X)\right] \\
&= W_{r_j-1,\ j}E(X) + \sum_{k=0}^{r_j-1} \\
&\quad \left[\alpha_{k,j}V_{k-1} - W_{k-1,j} + W_{k-1,j} - \alpha_{k,j}V_{k-1}\right] \\
&\quad \left[E_{h_{k-1}}(X) - E_{h_k}(X)\right] \\
&= W_{r_j-1,\ j}E(X)
\end{aligned}
$$

since $E_{h_n}(X) = 0$ and $E_{h_{-1}}(X) = E(X)$. Because $W_{r_j-1,j}$ and $E(X)$ are independent of order quantity Q, the proof is complete. ■

From observation 1, we immediately have the following observation for the relationship among the riskless profits if an identical probability distribution of demand and an identical pricing schedule are assumed :

Observation 2 : For any order quantity Q which falls within the price break interval offered by the supplier, we have :

(i) $ES_j^{IV}(Q) \geq ES_j^{III}(Q) \geq ES_0^{I}(Q)$

(ii) $ES_j^{IV}(Q) \geq ES_0^{II}(Q) \geq ES_0^{I}(Q)$.

Proof : From the definition of $W_{r_j-1,j} = \sum_{i=0}^{r_j-1}\alpha_{i,j}t_i$, $W_{r_j-1,j}$ is nondecreasing for indices j and (r_j-1). Therefore, for any order quantity Q which falls within the price break interval offered by the supplier, (i) holds true because $W_{r_j-1,j} \geq W_{0,j} \geq W_{0,0}$ and (ii) also holds true because $W_{r_j-1,j} \geq W_{r_0-1,0} \geq W_{0,0}$. The equality holds for $j=0$ or $Q=0$. ■

4. Conjecture on the Expected Profits

The observations from the analysis of the riskless profits of extended newsvendor models can lead to a certain conjecture regarding the general relationship among the expected profits of newsvendor models that holds for *every* order quantity as well as the optimal order quantities. Such a conjecture involves Khouja's findings and argument as special cases.

Observation 2 naturally leads to the following conjecture :

Conjecture : For any order quantity Q and the global optimal order quantities for four newsvendor models Q_I^*, Q_{II}^*, Q_{III}^*, and Q_{IV}^* which are valid between the price break intervals offered by the supplier, we have :

(i) $Q_{IV}^* \geq Q_{III}^* \geq Q_I^*$ and $ER_j^{IV}(Q) \geq ER_j^{III}(Q) \geq ER_0^{I}(Q)$.

(ii) $Q_{IV}^* \geq Q_{II}^* \geq Q_I^*$ and $ER_j^{IV}(Q) \geq ER_0^{II}(Q) \geq ER_0^{I}(Q)$.

The conjecture above addresses the relationship among the expected profits of newsvendor models for every order quantity as well as the optimal order quantities. As compared with Khouja's findings (iv) and (v), our conjecture is valuable even for the newsvendor model that includes budget or warehouse capacity constraint, for which solution algorithms are developed in the literature [3, 15], due to the following reasons if it is proved :

- As addressed in Khouja [6], type IV newsvendor model yields the greatest optimal expected profit of four types of newsvendor models. But our conjecture guarantees consistent advantage of type IV newsvendor model over other newsvendor models regardless of the order quantity. Therefore, if there is no restriction in relation to the implementation of discounts policy, the retailer can safely select the type IV newsvendor model regardless of the order quantity.
- Even if there are restrictions for the selection of newsvendor model and the retailer faces one of type II or type III newsvendor model, the retailer can expect consistently greater profit than the standard newsvendor model under a no-discounts schedule regardless of the order quantity.

5. Insights from Computational Experiments

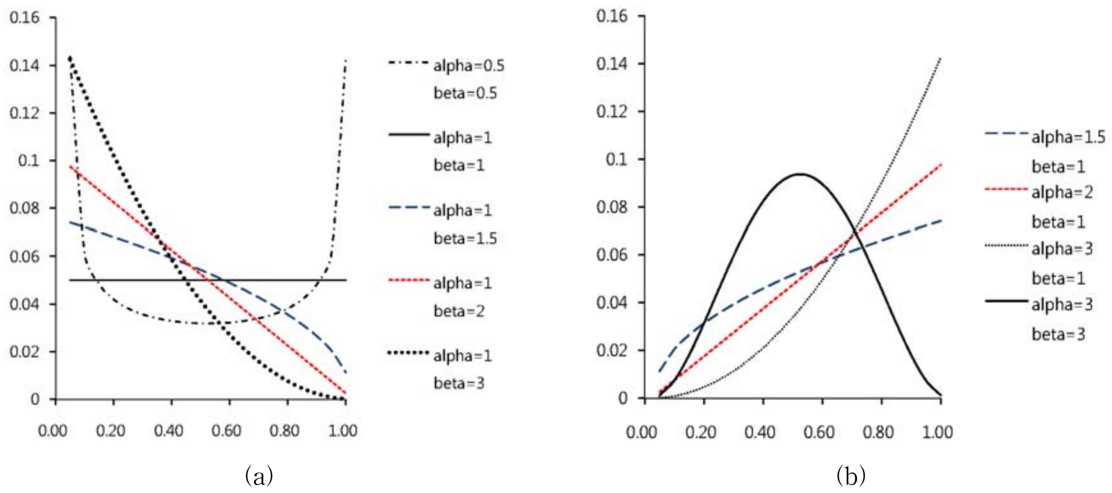
The purpose of computational experiments provided in this section is two-fold : one is to provide experiences that strongly support our conjecture for realistic situations and the other is to obtain valuable insights that may be useful for the newsvendor from the experiments.

To show that our conjecture can hold well for realistic situations, we select a variety of probability distributions of demand. Basically, two types of probability distributions are considered in our experiment : discrete probability distributions and continuous probability distributions. In Won [14] just three kinds of binomial distribution and a uniform distribution have been considered.

As the discrete probability distributions of demand, binomial distribution, uniform distribution, geometric distribution, and negative binomial distribution are considered. Ward et al. [13] pointed out that specifying the demand distribution is difficult and suggested working with an approximate discrete distribution. Like in the experiment in Won [14], we selected three kinds of binomial distribution with parameters $p=0.3$, $p=0.5$ and $p=0.7$, where p denotes the probability of success in a binomial experiment that featured 20 trials.

We assume that the demand varies in increments of 100 from 0 to 2,000 for all distributions. A uniform distribution discretized with the same increments as the binomial distribution is considered. To apply with the same increments as the binomial distribution, the geometric distributions with parameters of $p=0.4$, $p=0.5$, and $p=0.7$ are considered since the sum of the probabilities for 20 trials gives 1. In a similar way, the negative binomial distributions with parameters $p=0.8$, $p=0.85$, and $p=0.9$ are considered.

As the continuous probability distribution of demand, beta distribution is selected since the beta distribution shows a variety of demand pattern as the two positive shape parameters, denoted by α and β, of distribution vary. [Figure 1(a)] and [Figure 1(b)] show the shapes of the



[Figure 1] Probability Density Function of Beta Distribution with Different Values of Shape Parameters α and β

beta distribution due to different values of two parameters. To reflect various demand patterns, the beta distributions for with the following nine combinations of two parameters are considered :

〈Table 1〉 Scenarios of Parameters of the Beta Distribution Considered in Computational Experiment

α	0.5	1	1	1	1	1.5	2	3	3
β	0.5	1	1.5	2	3	1	1	1	3

Even if the experiments are conducted with a limited set of probability distribution of demand, such diverse patterns of demand due to the different shapes of probability distribution function of demand are very close to the representative types of demand that frequently occur in reality [14].

<Table 2> shows the price schedules offered by the retailer and supplier, respectively. For each extended newsvendor model, we consider an identical discounting schedule with the same unit selling prices and costs as the standard newsvendor model. <Table 3> shows the global optimal order quantities for each newsvendor model for three kinds of binomial distribution and a uniform distribution with the parameters assigned above.

〈Table 2〉 Price Schedules Offered by the Retailer and Supplier

Retailer			Supplier		
Discount stage i	t_i	Unit price ($)	Price-break level j	Quantity	Unit price ($)
0	0	150	0	$Q < 700$	80
1	0.2	120	1	$700 \le Q < 1400$	50
2	0.2	70	2	$1400 \le Q$	30
3	0.2	40			
4	∞	10			

〈Table 3〉 Global Optimal Order Quantities for Each Newsvendor Model

Newsvendor model	Binomial distribution with parameter						Uniform distribution	
	$p=0.3$		$p=0.5$		$p=0.7$			
	Q^*	$ER(Q^*)$	Q^*	$ER(Q^*)$	Q^*	$ER(Q^*)$	Q^*	$ER(Q^*)$
Type I	600	\$29,927	1,000	\$56,785	1,400	\$85,927	900	\$30,857
Type II	700	\$37,603	1,300	\$69,858	1,800	\$103,565	1,300	\$45,943
Type III	700	\$48,807	1,400	\$107,578	1,600	\$159,735	1,600	\$94,857
Type IV	900	\$62,430	1,700	\$142,024	2,000	\$200,006	2,000	\$128,638

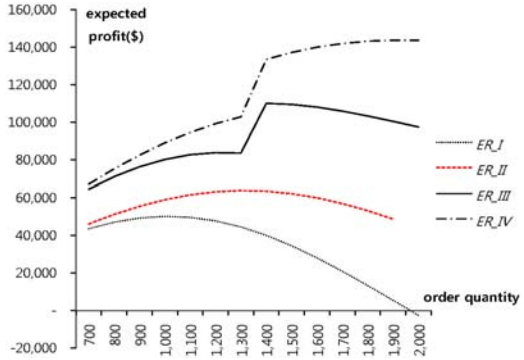
[Figure 2] through [Figure 19] show plots of the expected profits computed for the probability distributions of demand considered above. To observe the relationship among the expected profits after the supplier all-units discount is offered, only the plots of the expected profits for the order quantity larger than the first price-break quantity.

As can be seen in table 2, the inspection of the global optimal order quantities for each newsvendor model seems to strongly support our conjecture on the relationship among the global optimal order quantities for each newsvendor model. In addition, as can be seen in [Figure 3] through [Figure 20], the plots of the expected profits for each newsvendor model seem to strongly support our conjecture on the relationship among the expected profits of each newsvendor model for every order quantity.

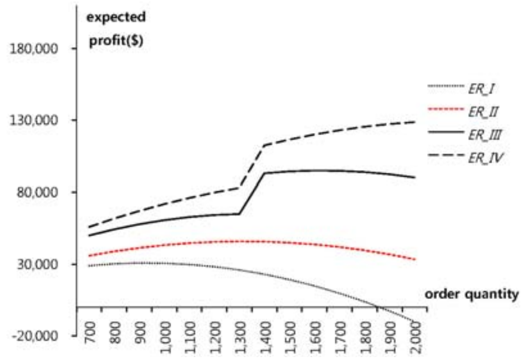
Some remarks need to be given from the computational experiments :

- As the order quantity increases, the retailer facing a type IV newsvendor model can expect consistently greater or equal expected profit for every order quantity and larger or equal optimal order quantity than one facing a type III newsvendor model. Similarly, the retailer facing a type III newsvendor model can expect consistently greater or equal expected profit for every order quantity and larger or equal optimal order quantity than one facing a type I newsvendor model.
- As the order quantity increases, the retailer facing a type IV newsvendor model can expect consistently greater or equal expected profit for every order quantity and larger or equal optimal order quantity than one facing a type II newsvendor model. Similarly, the retailer facing a type II newsvendor model can expect consistently greater or equal expected profit for every order quantity and larger or equal optimal order quantity than one facing a type I newsvendor model.
- But we can say nothing about the consistent advantage of a type III newsvendor model over a type II newsvendor model in terms of the optimal order quantity and the expected profit, as can be seen in [Figure 6], [Figure 7] and [Figure 8].

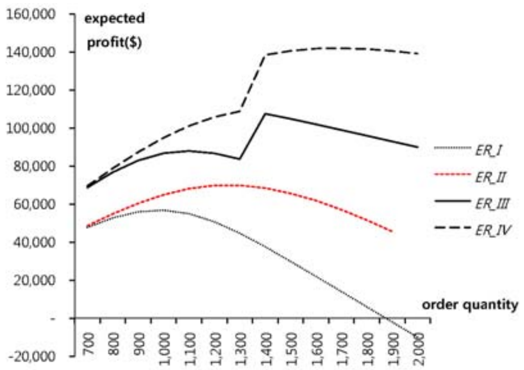
In conclusion, our conjecture encourages newsvendors to offer a multiple discounting schedule to the customer even if she may not order the optimal order quantity due to the restrictions such as budget or warehouse capacity limit when she is offered an all-units schedule by the supplier.



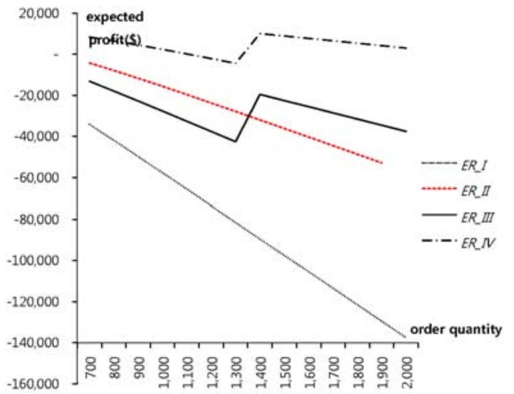
[Figure 2] Plots of the Expected Profits under the Binomial Distribution with $p=0.3$



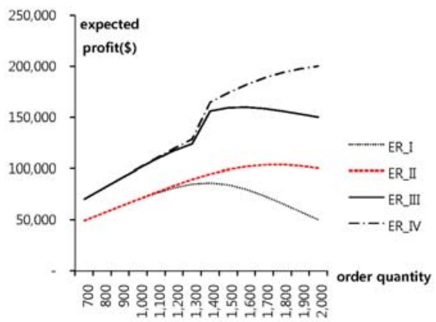
[Figure 5] Plots of the Expected Profits under the Uniform Distribution



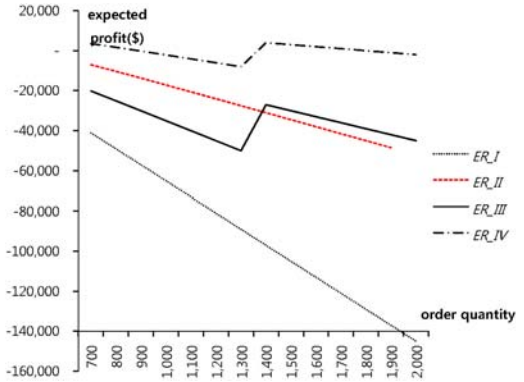
[Figure 3] Plots of the Expected Profits under the Binomial Distribution with $p=0.5$



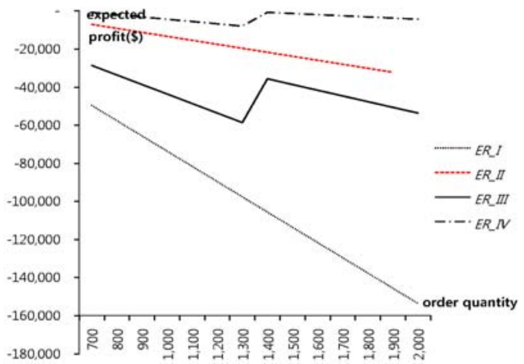
[Figure 6] Plots of the Expected Profits under the Geometric Distribution with $p=0.4$



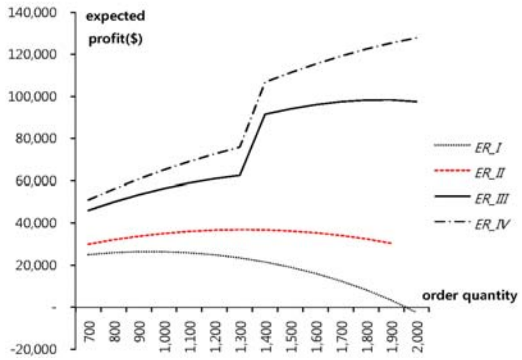
[Figure 4] Plots of the Expected Profits under the Binomial Distribution with $p=0.7$



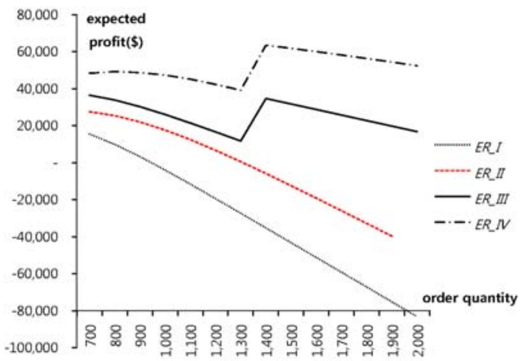
[Figure 7] Plots of the Expected Profits under the Geometric Distribution with $p=0.5$



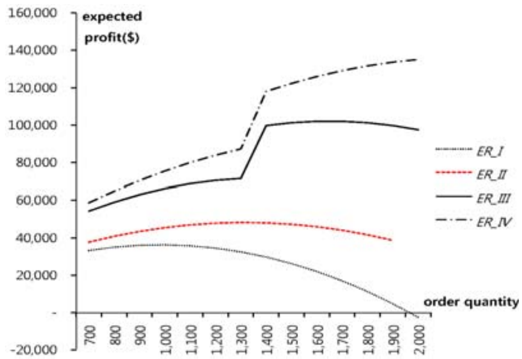
[Figure 8] Plots of the Expected Profits under the Geometric Distribution with $p=0.7$



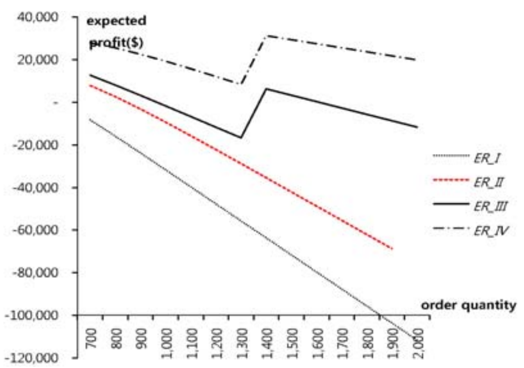
[Figure 11] Plots of the Expected Profits under the Beta Distribution with $\alpha=0.5$ and $\beta=0.5$



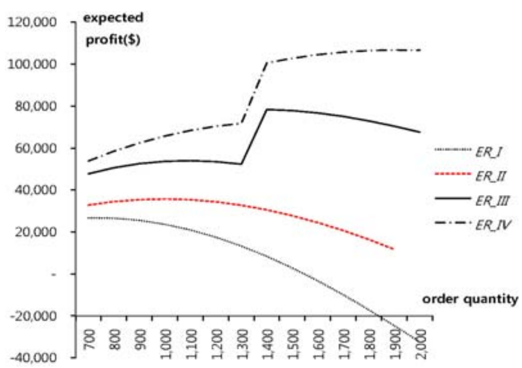
[Figure 9] Plots of the Expected Profits under the Negative Binomial Distribution with $p=0.85$



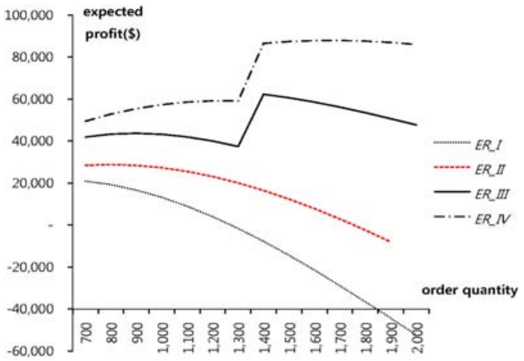
[Figure 12] Plots of the Expected Profits under the Beta Distribution with $\alpha=1$ and $\beta=1$



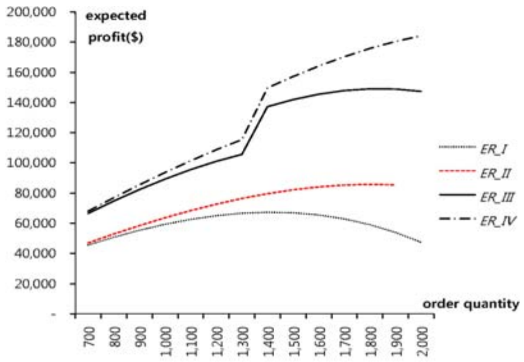
[Figure 10] Plots of the Expected Profits under the Negative Binomial Distribution with $p=0.9$



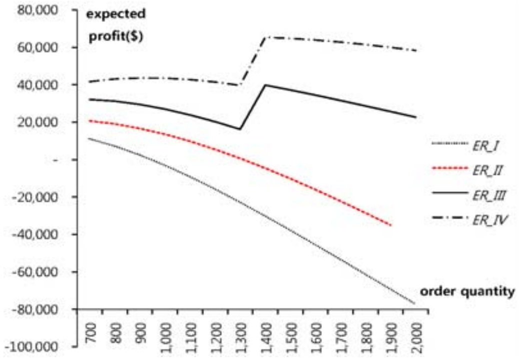
[Figure 13] Plots of the Expected Profits under the Beta Distribution with $\alpha=1$ and $\beta=1.5$



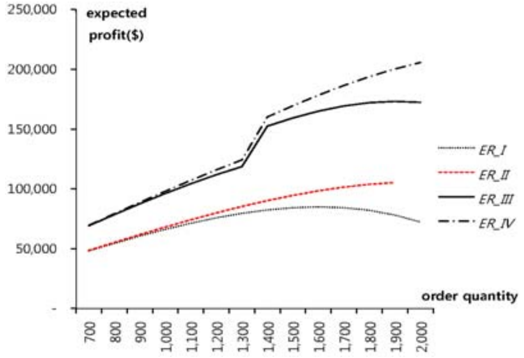
[Figure 14] Plots of the Expected Profits under the Beta Distribution with $\alpha=1$ and $\beta=2$



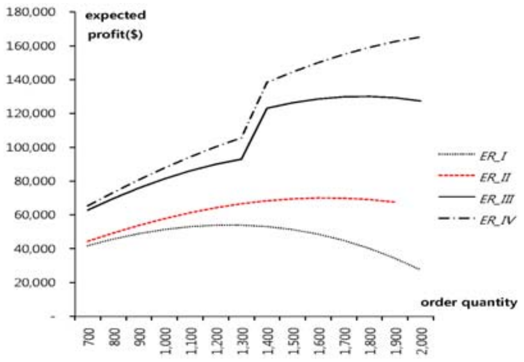
[Figure 17] Plots of the Expected Profits under the Beta Distribution with $\alpha=2$ and $\beta=1$



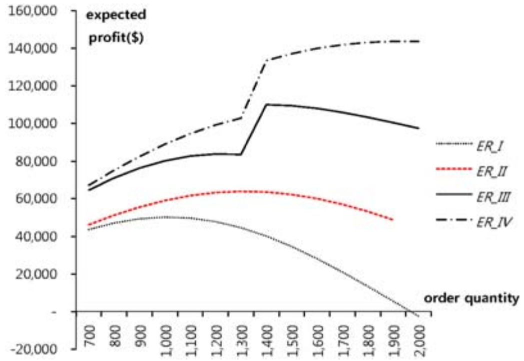
[Figure 15] Plots of the Expected Profits under the Beta Distribution with $\alpha=1$ and $\beta=3$



[Figure 18] Plots of the Expected Profits under the Beta Distribution with $\alpha=3$ and $\beta=1$



[Figure 16] Plots of the Expected Profits under the Beta Distribution with $\alpha=1.5$ and $\beta=1$



[Figure 19] Plots of the Expected Profits under the Beta Distribution with $\alpha=3$ and $\beta=3$

6. Concluding Remarks

In this study, we use riskless profit which is the sum of the expected profit and cost to investigate structural properties of various newsvendor models featured with retailer progressive multiple discounts and/or supplier all-units discounts. From the general relationship among the riskless profits of newsvendor models, we established a certain conjecture on the general relationship among the expected profits of various newsvendor models that holds for every order quantity. Our approach helps newsvendors select a newsvendor model with a more advantageous discounts policy among competitive newsvendor models even if she may not order the optimal order quantity due to the restriction such as budget or warehouse capacity limitations because the newsvendor models with quantity discounts always yield higher expected profit than the classic newsvendor model without quantity discounts regardless of the order quantity.

Even if the numerical experiments are provided with a limited set of probability distribution of demand, the experimental results show robust relationship among the expected profits of newsvendor models. Therefore, the rigorous proof of the conjecture raised in this paper is a challenging work for future research. In addition, one more interesting issue has remained unanswered. As can be seen in [Figure 6], [Figure 7], and [Figure 8], no consistent relationship between the expected profits of type II and type III newsvendor problems has been noticed. Then, finding a proper answer to the following question is another future work : "What are the conditions that guarantee the consistent relationship between the expected profits of type II and type III newsvendor problems?." Finally, the present study does not consider dynamic newsvendor models in which demand depends on price. Revealing the relationship among the expected profits of dynamic newsvendor models is also a research frontier that attracts researchers to explore.

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〈Appendix〉

Derivation of equations (4) and (5)

To establish equations (4) and (5), let

$R^{IV}_{k,j}(X,Q)$ = retailer profit for order quantity Q under demand X in the $(k-1)$th discount period when the cost per unit of b_j is offered by the supplier

$ER^{IV}_{k,j}(Q)$ = expected value of $R^{IV}_{k,j}(X,Q)$

$CU^{IV}_{k,j}(X,Q)$ = retailer cost of underestimating demand for order quantity Q under demand X in the $(k-1)$th discount period when the cost per unit of b_j is offered by the supplier

$ECU^{IV}_{k,j}(Q)$ = expected value of $CU^{IV}_{k,j}(X,Q)$

$ECU^{IV}_{j}(Q)$ = expected cost of underestimating demand for order quantity Q over all discount periods when the cost per unit of b_j is offered by the supplier

$CO^{IV}_{k,j}(X,Q)$ = retailer cost of overestimating demand for order quantity Q under demand X in the $(k-1)$th discount period when the cost per unit of b_j is offered by the supplier

$ECO^{IV}_{k,j}(Q)$ = expected value of $CO^{IV}_{k,j}(X,Q)$

$ECO^{IV}_{j}(Q)$ = expected cost of overestimating demand for order quantity Q over all discount periods when the cost per unit of b_j is offered by the supplier

Then, the retailer profit for order quantity Q under demand X in the kth discount period $(k=0, 1, \cdots, r_j-1)$ when the cost per unit of b_j is offered by supplier is

$$R^{IV}_{k,j}(X,Q)=\left(\sum_{i=0}^{k-1}\alpha_{i,j}t_i\right)X+\alpha_{k,j}(Q-V_{k-1}X)=(W_{k-1,j}-\alpha_{k,j}V_{k-1})X+\alpha_{k,j}Q.$$

.

The expected value of $R^{IV}_{k,j}(X,Q)$ is

$$\begin{aligned}ER^{IV}_{k,j}(Q)&=\int_{h_k}^{h_{k-1}}(W_{k,j-1}-\alpha_{k,j}V_{k-1})Xf(X)dX+\int_{h_k}^{h_{k-1}}\alpha_{k,j}Qf(X)dX \\ &=(W_{k,j-1}-\alpha_{k,j}V_{k-1})[E_{h_{k-1}}(X)-E_{h_k}(X)]+\alpha_{k,j}Q[F(h_{k-1})-F(h_k)].\end{aligned} \tag{6}$$

The retailer profit for order quantity Q under demand X in the kth discount period $(k=r_j, r_{j+1}, \cdots, n)$ when the cost per unit of b_j is offered by supplier is

$$R^{IV}_{k,j}(X,Q)=\left(\sum_{i=0}^{r_j-1}\alpha_{k,i}t_i-\sum_{i=r_j}^{k-1}\beta_i t_i\right)X-\beta_{k,j}(Q-V_{k-1}X)=(W_{r_j-1,j}-S_{k-1,j}+\beta_{k,j}V_{k-1})X-\beta_{k,j}Q.$$

The expected value of $R^{IV}_{k,j}(X,Q)$ is

$$ER^{IV}_{k,j}(Q)=\int_{h_k}^{h_{k-1}}\left(W_{r_j-1,j}-S_{k-1,j}+\beta_{k,j}V_{k-1}\right)Xf(X)dX-\int_{h_k}^{h_{k-1}}\beta_{k,j}Qf(X)dX \tag{7}$$
$$=\left(W_{r_j-1,j}-S_{k-1,j}+\beta_{k,j}V_{k-1}\right)\left[E_{h_{k-1}}(X)-E_{h_k}(X)\right]-\beta_{k,j}Q\left[F(h_{k-1})-F(h_k)\right].$$

Summing equations (6) and (7) over each discount period and combining the results yields equation (4) for the expected profit.

The retailer cost of underestimating demand for order quantity Q given a demand X in the kth discount period ($k=0,\ 1,\ \cdots,\ r_j-1$) when the cost per unit of b_j is offered by supplier is

$$CU^{IV}_{k,j}(X,Q)=\alpha_{k,j}(V_kX-Q)+\left(\sum_{i=k+1}^{r_j-1}\alpha_{i,j}t_i\right)X=\left(\alpha_{k,j}V_k+\sum_{i=k+1}^{r_j-1}\alpha_{i,j}t_i\right)X-\alpha_{k,j}Q$$
$$=\left[\sum_{i=0}^{r_j-1}\alpha_{i,j}t_i+\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})t_i\right]X-\alpha_{k,j}Q=\left[W_{r_j-1,j}+\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})i_i\right]X-\alpha_{k,j}Q.$$

The expected cost of $CU^{IV}_{k,j}(X,Q)$ is

$$ECU^{IV}_{k,j}(Q)=\int_{h_k}^{h_{k-1}}\left[W_{r_j-1,j}+\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})t_i\right]Xf(X)dx-\int_{h_k}^{h_{k-1}}\alpha_{k,j}Qf(X)dX$$
$$=\left[W_{r_j-1,j}+\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})t_i\right]\left[E_{h_{k-1}}(X)-E_{h_k}(X)\right]-\alpha_{k,j}Q\left[F(h_{k-1})-F(h_k)\right]$$

The expected cost of underestimating demand for order quantity Q over the discount periods $k=0,\ 1,\ \cdots,\ r_j-1$ is

$$ECU^{IV}_{j}(Q)=\sum_{k=0}^{r_j-1}\left[W_{r_j-1,j}+\sum_{i=0}^{k}(\alpha_{k,j}-\alpha_{i,j})t_i\right]\left[E_{h_{k-1}}(X)-E_{h_k}(X)\right]-Q\left(\sum_{k=0}^{r_j-1}\alpha_{k,j}\left[F(h_{k-1})-F(h_k)\right]\right). \tag{8}$$

The retailer cost of overestimating demand for order quantity Q under demand X in the kth discount period ($k=r_{j},\ r_{j+1},\ \cdots,\ n$) is

$$CO^{IV}_{k,j}(X,Q)=\beta_{k,j}(Q-V_{k-1}X)+\left(\sum_{i=r_j}^{k-1}\beta_{i,j}t_i\right)X=\left(S_{k-1,j}-\beta_{k,j}V_{k-1}\right)X+\beta_{k,j}Q.$$

The expected value of $CO^{IV}_{k,j}(X,Q)$ is

$$ECO^{IV}_{k,j}(Q)=\int_{h_k}^{h_{k-1}}(S_{k-1,j}-\beta_{k,j}V_{k-1})Xf(X)dx+\int_{h_k}^{h_{k-1}}\beta_{k,j}Qf(X)dX$$

$$= (S_{k-1,j} - \beta_{k,j} V_{k-1}) \left[E_{h_{k-1}}(X) - E_{h_k}(X) \right] + \beta_{k,j} Q \left[F(h_{k-1}) - F(h_k) \right].$$

Then, the expected cost of overestimating demand for order quantity Q over the discount periods $k = r_j, r_j + 1, \cdots, n$ is

$$ECO_j^{IV}(Q) = \sum_{k=r_j}^{n} (S_{k-1,j} - \beta_{k,j} V_{k-1}) \left[E_{h_{k-1}}(X) - E_{h_k}(X) \right] + Q \left(\sum_{k=r_j}^{n} \beta_{k,j} [F(h_{k-1}) - F(h_k)] \right). \tag{9}$$

Summing equations (8) and (9) over each discount period and combining the results yields equation (5) for the expected total cost.