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APPROXIMATE CONTROLLABILITY FOR NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS†

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Abstract. In this paper, we study the control problems governed by the semilinear parabolic type equation in Hilbert spaces. Under the Lipschitz continuity condition of the nonlinear term, we can obtain the sufficient conditions for the approximate controllability of nonlinear functional equations with nonlinear monotone hemicontinuous and coercive operator. The existence, uniqueness and a variation of solutions of the system are also given.

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1. Introduction

Let H and V be real separable Hilbert spaces such that V is a dense subspace of H. We are interested in the following nonlinear functional control system on H

$$\begin{cases} x'(t) + Ax(t) = f(t, x(t)) + (Bu)(t), & 0 < t \le T, \\ x(0) = x_0, \end{cases} \tag{1.1}$$

where the nonlinear mapping f is Lipschitz continuous from $\mathbb{R} \times V$ into H. Here, the operator A is given as a single valued, hemicontinuous and monotone operator from V to V^*. Here, V^* stands for the dual space of V. The controller B is a linear bounded operator from a Banach space $L^2(0, T; U)$ to $L^2(0, T; H)$ for any $T > 0$, where U is a Banach space of control variables.

When the right hand side of (1.1) belongs to $L^2(0, T; V^*)$, it is well known as the quasi-autonomous differential equation(see Theorem 2.6 of Chapter III in [1]). In [2], Jeong, et al. have established the existence and the norm estimate of a solution of (1.1) on $L^2(0, T; V) \cap W^{1,2}(0, T; V^*)$ by using the contraction

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mapping principle, which is also applicable to optimal control problem. The control systems governed by a class of nonlinear evolution equations were developed in many references [3, 4, 5, 6, 7]. As for the semilinear control system with the linear operator A generated C_0-semigroup, Naito [5] proved the approximate controllability under the range conditions of the controller B.

The main objective of this paper is to consider the approximately controllable for (1.1) with the nonlinear principal operator A under a stronger assumption that $\{y : y(t) = Bu(t), \quad u \in L^2(0,T;U)\}$ is dense subspace of $L^2(0,T,H)$, which is reasonable and widely used in case of the nonlinear system. Generally, we can give the sufficient conditions for the approximate controllability of nonlinear functional equations with nonlinear monotone hemicontinuous and coercive operator.

In [8, 9], they studied the control problems of the semilinear equations by assuming a Lipschitz continuity of f and a range condition of the controller B with an inequality constraint. In this paper we no longer require the compactness of a solution mapping, and the inequality constraint on the range condition of the controller B, but instead we need the regularity and a variation of solutions of the given equations. For the basis of our study we construct the fundamental solution and establish variations of constant formula of solutions for the semilinear system by using those of the corresponding the linear system.

2. Solutions of nonlinear Systems

If H is identified with its dual space we may write $V \subset H \subset V^*$ densely and the corresponding injections are continuous. The norm on V, H and V^* will be denoted by $||\cdot||$, $|\cdot|$ and $||\cdot||_*$, respectively. The duality pairing between the element v_1 of V^* and the element v_2 of V is denoted by (v_1, v_2), which is the ordinary inner product in H if $v_1, v_2 \in H$. For the sake of simplicity, we may consider

$$||u||_* \leq |u| \leq ||u||, \quad u \in V.$$

In terms of the intermediate theory, we may assume that

$$(V, V^*)_{1/2,2} = H$$

where $(V, V^*)_{1/2,2}$ denotes the real interpolation space between V and V^*.

We note that a nonlinear operator A is said to be hemicontinuous on V if

$$w - \lim_{t \to 0} A(x + ty) = Ax$$

for every $x,\ y \in V$ where "$w - \lim$" indicates the weak convergence on V.

We will need the following hypotheses on the data of problem (1.1).

(A) Let $A : V \longrightarrow V^*$ be given a single valued, monotone operator and hemicontinuous from V to V^* such that

$$A(0) = 0,$$
$$(Au - Av, u - v) \geq \omega_1 ||u - v||^2 - \omega_2 |u - v|^2,$$

$$|Au| \leq \omega_3(||u|| + 1)$$

for every $u, v \in V$ where $\omega_2 \in \mathbb{R}$ and $\omega_1,\ \omega_3$ are some positive constants.

(F) Let $f : V \to H$ be Lipschitz continuous, i.e., there exists a constant $L > 0$ such that

$$|f(t, x_1) - f(t, x_2)| \leq L||x - y||, \quad \forall x, y \in V.$$

Here, we note that if $0 \neq A(0)$ we need the following assumption

$$(Au, u) \geq \omega_1||u||^2 - \omega_2|u|^2$$

for every $u \in V$. It is also known that A is maximal monotone and $R(A) = V^*$, where $R(A)$ denotes the range of A.

We are interested in the following nonlinear functional control system on H:

$$\begin{cases} x' + Ax(t) = f(t, x(t)) + (Bu)(t), & 0 < t \leq T, \\ x(0) = x_0, \end{cases}$$

Proposition 2.1. *Let the assumption (F) be satisfied. Assume that $Bu \in L^2(0, T; V^*)$ and $x_0 \in H$. Then, the system (1.1) has a unique solution*

$$x \in L^2(0, T; V) \cap C([0, T]; H) \cap W^{1,2}(0, T; V^*)$$

and there exists a constant C_1 depending on T such that

$$||x||_{L^2 \cap C \cap W^{1,2}} \leq C_1(1 + |x_0| + ||Bu||_{L^2(0,T;V^*)}).$$

The proof of Proposition 2.1 is from Jeong et al. [2; Theorem 2.1]. They used the monotonicity of A in order to prove the regularity for solutions of (1.1), so we obtain the following a perturbation result by a monotone operator

Corollary 2.1. *Let the assumption (F) be satisfied. Let the operator D be a monotone set in $H \times H$. Then for every $k \in L^2(0, T; V^*)$ and $x_0 \in H$, the Cauchy problem*

$$x'(t) = (A + D)x(t) + f(t, x(t)) + k(t),$$
$$x(0) = x_0$$

has a unique solution

$$x \in L^2(0, T; V) \cap C([0, T]; H)$$

and there exists a constant C_2 depending on T such that

$$||x||_{L^2 \cap C} \leq C_2(1 + |x_0| + ||k||_{L^2(0,T;V^*)}).$$

3. Approximate controllability

In what follows we assume that the embedding $V \subset H$ is compact. For $h \in L^2(0,T;V^*)$ and let x_h be the solution of the following equation with $B = I$:

$$\begin{cases} x'(t) + Ax(t) = f(t, x(t)) + h(t), \quad 0 < t \le T_0, \\ x(0) = x_0. \end{cases} \tag{3.1}$$

We assume that a nonlinear single valued mapping f is from $[0,\infty) \times H$ into H satisfying a Lipschitz continuity condition (F). Moreover, let us assume that f is uniformly bounded: there exists a constant M such that

$$|f(t,x)| \le M,$$

for all $x \in V$.

The following Lemma is from Brézis [10, Lemma A.5].

Lemma 3.1. *Let $m \in L^1(0,T;\mathbb{R})$ satisfying $m(t) \ge 0$ for all $t \in (0,T)$ and $a \ge 0$ be a constant. Let b be a continuous function on $[0,T] \subset \mathbb{R}$ satisfying the following inequality:*

$$\frac{1}{2}b^2(t) \le \frac{1}{2}a^2 + \int_0^t m(s)b(s)ds, \quad t \in [0,T].$$

Then,

$$|b(t)| \le a + \int_0^t m(s)ds, \quad t \in [0,T].$$

Lemma 3.2. *Let x_h be the solution of (3.1) corresponding to h in $L^2(0,T;H)$. Then we have that*

$$\frac{1}{2}|x_h(t)|^2 + \omega_1 \int_0^t ||x_h(s)||^2 ds \le \frac{e^{2\omega_2 t}}{2}|x_0|^2 + \int_0^t e^{2\omega_2(t-s)}|x_h(s)|(M + |h(s)|)ds. \tag{3.2}$$

Proof. In order to prove (3.2), taking scalar product on both sides of (3.1) by $x(t)$, we obtain

$$\frac{1}{2}\frac{d}{dt}|x_h(t)|^2 + \omega_1||x_h(t)||^2 \le \omega_2|x_h(t)|^2 + |x_h(s)|(M + |h(s)|).$$

Integrating the above equation on $[0,t]$, we get

$$\frac{1}{2}|x_h(t)|^2 + \omega_1 \int_0^t ||x_h(s)||^2 ds \le \frac{1}{2}|x_0|^2 + \omega_2 \int_0^t |x_h(s)|^2 ds + \int_0^t |x_h(s)|(M + |h(s)|)ds. \tag{3.3}$$

Since

$$\frac{d}{dt}\{e^{-2\omega_2 t}\int_0^t |x_h(s)|^2 ds\} \tag{3.4}$$

$$= 2e^{-2\omega_2 t}\{\frac{1}{2}|x_h(t)|^2 - \omega_2 \int_0^t |x_h(s)|^2 ds\}$$

$$\leq 2e^{-2\omega_2 t}\{\frac{1}{2}|x_0|^2 + \int_0^t |x_h(s)|(M + |h(s)|)ds\},$$

integrating (3.4) over $(0, t)$ we have

$$e^{-2\omega_2 t}\int_0^t |x_h(s)|^2 ds \leq 2\int_0^t e^{-2\omega_2 \tau}\int_0^\tau |x_h(s)|(M + |h(s)|)ds d\tau + \frac{1 - e^{-2\omega_2 t}}{2\omega_2}|x_0|^2$$

$$= 2\int_0^t \int_s^t e^{-2\omega_2 \tau} d\tau |x_h(s)|(M + |h(s)|)ds + \frac{1 - e^{-2\omega_2 t}}{2\omega_2}|x_0|^2$$

$$= 2\int_0^t \frac{e^{-2\omega_2 s} - e^{-2\omega_2 t}}{2\omega_2}|x_h(s)|(M + |h(s)|)ds + \frac{1 - e^{-2\omega_2 t}}{2\omega_2}|x_0|^2$$

$$= \frac{1}{\omega_2}\int_0^t (e^{-2\omega_2 s} - e^{-2\omega_2 t})|x_h(s)|(M + |h(s)|)ds + \frac{1 - e^{-2\omega_2 t}}{2\omega_2}|x_0|^2,$$

and hence,

$$\omega_2 \int_0^t |x(s)|^2 ds \leq \int_0^t (e^{2\omega_2 (t-s)} - 1)|x_h(s)|(M + |h(s)|)ds + \frac{e^{2\omega_2 t} - 1}{2}|x_0|^2.$$

From (3.3) it follows that

$$\frac{1}{2}|x_h(t)|^2 + \omega_1 \int_0^t ||x_h(s)||^2 ds \leq \frac{e^{2\omega_2 t}}{2}|x_0|^2 + \int_0^t e^{2\omega_2 (t-s)}|x_h(s)|(M + |h(s)|)ds.$$

□

Lemma 3.3. *If* $(x_0, h) \in H \times L^2(0, T; V^*)$, *then* $x \in L^2(0, T; V) \cap C([0, T]; H)$ *and the mapping*

$$H \times L^2(0, T; V^*) \ni (x_0, h) \mapsto x \in L^2(0, T; V) \cap C([0, T]; H)$$

is continuous.

Proof. By virtue of Proposition 2.1 for any $(x_0, h) \in H \times L^2(0, T; V^*)$, the solution x of (3.1) belongs to $L^2(0, T; V) \cap C([0, T]; H)$. Let $(x_{0i}, h_i) \in H \times$

$L^2(0,T;V^*)$ and x_i be the solution of (3.1) with (x_{0i},h_i) in place of (x_0,h) for $i=1,\ 2$. Multiplying on (3.1) by $x_1(t)-x_2(t)$, we have

$$\begin{aligned}&\frac{1}{2}\frac{d}{dt}|x_1(t)-x_2(t)|^2+\omega_1||x_1(t)-x_2(t)||^2\\ \leq&\omega_2|x_1(t)-x_2(t)|^2+L|x_1(t)-x_2(t)|^2\\ &+||x_1(t)-x_2(t)||\,||h_1(t)-h_2(t)||_*.\end{aligned}$$

By the similar process of the proof of Lemma 3.2 it holds

$$\begin{aligned}\frac{1}{2}|x_1(t)-x_2(t)|^2+\omega_1\int_0^t||x_1(s)-x_2(s)||^2ds\leq\frac{e^{2\omega_2 t}}{2}|x_{01}-x_{02}|^2\\ +\int_0^t Le^{2\omega_2(t-s)}|x_1(s)-x_2(s)|^2ds\\ +\int_0^t e^{2\omega_2(t-s)}||x_1(s)-x_2(s)||\,||h_1(s)-h_2(s)||_*ds.\end{aligned}\tag{3.5}$$

We can choose a constant $c>0$ such that

$$\omega_1-e^{2\omega_2 T}\frac{c}{2}>0$$

and, hence

$$\begin{aligned}&\int_0^T e^{2\omega_2(t-s)}||x_1(s)-x_2(s)||\,||h_1(s)-h_2(s)||_*ds\\ &\leq e^{2\omega_2 T}\int_0^T\{\frac{c}{2}||x_1(s)-x_2(s)||^2+\frac{1}{2c}||h_1(s)-h_2(s)||_*^2\}ds.\end{aligned}$$

We now apply Gronwall's inequality to (3.5) and obtain that there exists a constant $C>0$ such that

$$||x_1-x_2||_{L^2(0,T,V)\cap C([0,T];H)}\leq C(|x_{01}-x_{02}|+||h_1-h_2||_{L^2(0,T;V^*)}).\tag{3.6}$$

Suppose $(x_{0n},h_n)\to(x_0,h)$ in $H\times L^2(0,T;V^*)$, and let x_n and x be the solutions (3.1) with (x_{0n},h_n) and (x_0,h), respectively. Then, by virtue of (3.6), we see that $x_n\to x$ in $L^2(0,T,V)\cap C([0,T];H)$. □

We define the solution mapping S from $L^2(0,T;V^*)$ to $L^2(0,T;V)$ by

$$(Sh)(t)=x_h(t),\quad h\in L^2(0,T;V^*).$$

Let $\mathcal{A}$ and $\mathcal{F}$ be the Nemitsky operators corresponding to the maps A and f, which are defined by $\mathcal{A}(x)(\cdot)=Ax(\cdot)$ and $\mathcal{F}(h)(\cdot)=f(\cdot,x_h)$, respectively.

Then

$$x_h(t)=\int_0^t((I+\mathcal{F}-\mathcal{A}S)h)(s)ds,$$

and with the aid of Proposition 2.1

$$\begin{aligned}&||Sh||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)}=||x_h||_{L^2(0,T;V)\cap W^{1,2}(0,T;V^*)}\\ &\leq C_2(|x_0|+||h||_{L^2(0,T;V^*)}+1).\end{aligned}\tag{3.7}$$

Hence if h is bounded in $L^2(0,T;V^*)$, then so is x_h in $L^2(0,T;V)\cap W^{1,2}(0,T;V^*)$. Since V is compactly embedded in H by assumption, the embedding $L^2(0,T;V)\cap W^{1,2}(0,T;V^*) \subset L^2(0,T;H)$ is compact in view of Theorem 2 of Aubin [11]. Hence, since the embedding $L^2(0,T;H) \subset L^2(0,T;V^*)$ is continuous, the mapping $h \mapsto Sh = x_h$ is compact from $L^2(0,T;V^*)$ to itself. Therefore, $\mathcal{F}$ is a compact mapping from $L^2(0,T;V^*)$ to itself and so is $\mathcal{A}S$. The solution of (1.1) is denoted by $x(T;f,u)$ associated with the nonlinear term f and the control u at time T.

Definition 3.1. The system (1.1) is said to be approximately controllable at time T if $Cl\{x(T;f,u) : u \in L^2(0,T;U)\} = H$ where Cl denotes the closure in H.

We assume

(B) $Cl\{y : y(t) = Bu(t), \quad \text{a.e.}, \quad u \in L^2(0,T;U)\} = L^2(0,T;H)$ where Cl denotes the closure in $L^2(0,T;H)$.

Theorem 3.1. *Let the assumptions (B) and (F) be satisfied. Then we have*

$$Cl\{(l - (\mathcal{A}S - \mathcal{F}))h : h \in L^2(0,T;H)\} = L^2(0,T;H). \tag{3.8}$$

Thus the system (1.1) is approximately controllable at time T.

Proof. For the sake of simplicity we assume that $\omega_2 > 0$. Let us fix $T_0 > 0$ so that

$$\omega_1^{-1/2}\omega_3\{\frac{e^{2\omega_2 T_0}-1}{2\omega_2}|x_0| + \frac{e^{4\omega_2 T_0}-1}{8\omega_2}\}^{1/2} < 1. \tag{3.9}$$

Let U_r be the ball with radius r in $L^2(0,T_0;H)$ and $z \in U_r$. Put

$$\begin{aligned} C(t,|x_0|) =& \{\frac{e^{2\omega_2 t}}{2}|x_0|^2 + 2M|x_0|e^{2\omega_2 t}\frac{1-e^{-\omega_2 t}}{\omega_2}\}^{1/2} \\ &+ Mt^{1/2}(\frac{e^{4\omega_2 t}-1}{2\omega_2})^{1/2}. \end{aligned}$$

Take a constant $d > 0$ such that

$$\{r + M\sqrt{T}_0 + \omega_3(\omega_1^{-1/2}(C(T_0,|x_0|)^{1/2} + 1)\}(1-N)^{-1} < d, \tag{3.10}$$

with the constant

$$N = (\frac{e^{2\omega_2 T_0}-1}{2\omega_2}|x_0| + \frac{e^{4\omega_2 T_0}-1}{8\omega_2})^{1/2}.$$

From (3.2) and Lemma 3.1 it follows that

$$|x_h(t)| \le e^{\omega_2 t}|x_0| + \int_0^t e^{2\omega_2(t-s)}(M + |h(s)|)ds.$$

Thus,

$$\frac{1}{2}|x_h(t)|^2 + \omega_1\int_0^t ||x_h(s)||^2 ds$$

$$
\begin{aligned}
&\le \frac{e^{2\omega_2 t}}{2}|x_0|^2 + \int_0^t e^{2\omega_2(t-s)}|x_h(s)|(M+|h(s)|)ds \\
&\le \frac{e^{2\omega_2 t}}{2}|x_0|^2 \\
&+ \int_0^t e^{2\omega_2(t-s)}\{e^{\omega_2 s}|x_0| + \int_0^s e^{2\omega_2(s-\tau)}|(2M+|h(\tau)|)d\tau\}(M+|h(s)|)ds \\
&= \frac{e^{2\omega_2 t}}{2}|x_0|^2 + e^{2\omega_2 t}|x_0|\int_0^t e^{-\omega_2 s}(M+|h(s)|)ds \\
&+ e^{4\omega_2 t}\int_0^t e^{-2\omega_2 s}\{\int_0^s e^{-2\omega_2 \tau}(M+|h(\tau)|)d\tau\}(M+|h(s)|)ds \\
&= \frac{e^{2\omega_2 t}}{2}|x_0|^2 + M|x_0|e^{2\omega_2 t}\frac{1-e^{-\omega_2 t}}{\omega_2} + e^{2\omega_2 t}|x_0|\int_0^t e^{-\omega_2 s}|h(s)|ds \\
&+ e^{4\omega_2 t}\int_0^t \frac{1}{2}\frac{d}{ds}\{\int_0^s e^{-2\omega_2 \tau}(M+|h(\tau)|)d\tau\}^2 ds \\
&= \frac{e^{2\omega_2 t}}{2}|x_0|^2 + M|x_0|e^{2\omega_2 t}\frac{1-e^{-\omega_2 t}}{\omega_2} + \frac{e^{2\omega_2 t}-1}{2\omega_2}|x_0|\int_0^t |h(s)|^2 ds \\
&+ \frac{e^{4\omega_2 t}-1}{8\omega_2}\int_0^t (M+|h(\tau)|)^2 ds,
\end{aligned}
$$

that is,

$$
\begin{aligned}
||Sh||_{L^2(0,T_0;V)} &= ||x_h||_{L^2(0,T_0;V)} \\
&\le \omega_1^{-1/2}(C(T_0,|x_0|)^{1/2} + N||h||_{L^2(0,T_0;H)}).
\end{aligned} \tag{3.11}
$$

Let us consider the equation

$$
z = h - \lambda(\mathcal{A}S - \mathcal{F})h, \quad 0 \le \lambda \le 1. \tag{3.12}
$$

Let h be the solution of (3.12). Then, for the element $z \in U_r \subset U_d$, from (3.10), (3.11) it follows that

$$
\begin{aligned}
||h||_{L^2(0,T_0;H)} &\le ||z|| + ||\mathcal{A}Sh|| + ||\mathcal{F}h|| \\
&\le r + M\sqrt{T}_0 + \omega_3(||Sh|| + 1) \\
&\le r + M\sqrt{T}_0 \\
&\quad + \omega_3\{\omega_1^{-1/2}(C(T_0,|x_0|)^{1/2} + N||h||_{L^2(0,T_0;H)}) + 1\}
\end{aligned}
$$

and hence,

$$
\begin{aligned}
||h|| &\le \{r + M\sqrt{T}_0 + \omega_3(\omega_1^{-1/2}(C(T_0,|x_0|)^{1/2} + 1)\}(1-N)^{-1} \\
&< d,
\end{aligned}
$$

it follows that $h \notin \partial U_d$ where ∂U_d stands for the boundary of U_d. Thus the homotopy property of topological degree theory there exists $h \in L^2(0,T_0;H)$ such that the equation (3.12) holds. Let $y \in H$. We can choose $g \in W^{1,2}(0,T_0;H)$ such that $g(0) = 0$ and $g(T_0) = y$ and $h \in L^2(0, T_0; H)$ such that $g^{'} = (I-(\mathcal{A}S-\mathcal{F}))h$. Since the assumption (B), there exists a sequence $\{u_n\} \in L^2(0, T_0; U)$ such that $Bu_n \mapsto h$ in $L^2(0, T; H)$. Then by Lemma 3.3 we have that $x(\cdot; g, u_n) \mapsto x_h$ in $L^2(0, T_0; V) \cap C([0, T_0]; H)$. Therefore, $x(T_0; g, u_n) \mapsto y$. We conclude that the system (1.1) is approximately controllable at time T_0. Since the condition (3.9) is independent of initial values, we can solve the equation in $[T_0, 2T_0]$ with the initial value $x(T_0)$. By repeating this process, the approximate controllablility for (1.1) can be extended the interval $[0, nT_0]$ for the natural number n, i.e., for the initial $x(nT_0)$ in the interval $[nT_0, (n+1)T_0]$. □

Remark 3.1. If our constants condition in (2.1), (2.2) contains the following inequality: $\omega_1 > 1$ or $\omega_3 < 1$, then we can find that the approximate controllablility for (1.1) guarantees if $h \in L^2(0, T; V^*)$ by using a routine computation.

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