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일반화된 삼각퍼지집합에 대한 정규퍼지확률

Normal fuzzy probability for generalized triangular fuzzy sets

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요약

확률공간 $(\Omega,\mathcal{F},P)$ 위에 정의된 퍼지집합을 퍼지이벤트라 한다. Zadeh는 확률 P를 이용하여 퍼지이벤트 A에 대한 확률을 정의하였다. 우리는 일반화된 삼각퍼지집합을 정의하고 거기에 확장된 대수적 작용소를 적용하였다. 일반화된 삼각퍼지집합은 대칭적이지만 함숫값으로 1을 갖지 않을 수 있다. 두 개의 일반화된 삼각퍼지집합 A와 B에 대하여 $A(+)B$와 $A(-)B$는 일반화된 사다리꼴퍼지집합이 되었지만, $A(\cdot)B$와 $A(/)B$는 일반화된 삼각퍼지집합도 되지 않았고 일반화된 사다리꼴퍼지집합도 되지 않았다. 그리고 정규분포를 이용하여 $\mathbb{R}$ 위에서 정규퍼지확률을 정의하였다. 그리고 일반화된 삼각퍼지집합에 대한 정규퍼지확률을 계산하였다.

Abstract

A fuzzy set A defined on a probability space $(\Omega,\mathcal{F},P)$ is called a fuzzy event. Zadeh defines the probability of the fuzzy event A using the probability P. We define the generalized triangular fuzzy set and apply the extended algebraic operations to these fuzzy sets. A generalized triangular fuzzy set is symmetric and may not have value 1. For two generalized triangular fuzzy sets A and B, $A(+)B$ and $A(-)B$ become generalized trapezoidal fuzzy sets, but $A(\cdot)B$ and $A(/)B$ need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. We define the normal fuzzy probability on $\mathbb{R}$ using the normal distribution. And we calculate the normal fuzzy probability for generalized triangular fuzzy sets.

Key Words : fuzzy event, generalized triangular fuzzy set, normal fuzzy probability

1. Introduction

We define the generalized triangular fuzzy set and calculate four operations of two generalized triangular fuzzy sets([4]). Four operations are based on the Zadeh's extension principle([6], [7], [8]). And Zadeh defines the probability of fuzzy event as follows.

Let $(\Omega,\mathcal{F},P)$ be a probability space, where Ω denotes the sample space, $\mathcal{F}$ the σ-algebra on Ω, and P a probability measure. A fuzzy set A on Ω is called a fuzzy event. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event A.
Then the probability of the fuzzy event A is defined by Zadeh([9]) as

$$\widetilde{P}(A)=\int_{\Omega}\mu_A(\omega)dP(\omega),\quad \mu_A(\cdot):\Omega\to[0,1]$$

We defined the normal fuzzy probability using the normal distribution and calculated the normal fuzzy probability for quadratic fuzzy number([2]). And then we had the explicit formula for the normal fuzzy probability for trigonometric fuzzy number. Furthermore we calculated the normal fuzzy probability for trigonometric fuzzy numbers driven by the above four operations([3]).

In this paper, we calculate the normal fuzzy probability for generalized triangular fuzzy sets.

2. Preliminaries

Let $(\Omega,\mathcal{F},P)$ be a probability space, and X be a random variable defined on it. Let g be a real-valued Borel-measurable function on $\mathbb{R}$. Then $g(X)$ is also a random variable.

Definition 2.1. We say that the mathematical

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expectation of $g(X)$ exists if

$$E[g(X)] = \int_{\Omega} g(X(\omega))dP(\omega) = \int_{\Omega} g(X)dP$$

is finite.

We note that a random variable X defined on $(\Omega, \mathcal{F}, P)$ induces a measure P_X on a Borel set $B \in \mathcal{B}$ defined by the relation $P_X(B) = P\{X^{-1}(B)\}$. Then P_X becomes a probability measure on $\mathcal{B}$ and is called the probability distribution of X. It is known that if $E[g(X)]$ exists, then g is also integrable over $\mathbb{R}$ with respect to P_X. Moreover, the relation

$$\int_{\Omega} g(X)dP = \int_{\mathbb{R}} g(t)dP_X(t) \tag{2.1}$$

holds. We note that the integral on the right-hand side of (2.1) is the Lebesgue- Stieltjes integral of g with respect to P_X. In particular, if g is continuous on $\mathbb{R}$ and $E[g(X)]$ exists, we can rewrite (2.1) as follows

$$\int_{\Omega} g(X)dP = \int_{\mathbb{R}} g dP_X = \int_{-\infty}^{\infty} g(x)dF(x), \tag{2.2}$$

where F is the distribution function corres- ponding to P_X, and the last integral is a Riemann-Stieltjes integral.

Let F be absolutely continuous on $\mathbb{R}$ with probability density function $f(x) = F'(x)$. Then $E[g(X)]$ exists if and only if the integral $\int_{-\infty}^{\infty} |g(x)| f(x)dx$ is finite and in that case we have

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)dx$$

Example 2.2. Let the random variable X (denoted $X \sim N(m, \sigma^2)$) have the normal distribution with the probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}, \quad x \in \mathbb{R}$$

where $\sigma^2 > 0$ and $m \in \mathbb{R}$. Then $E[|X|^{\gamma}] < \infty$ for every $\gamma > 0$, and we have

$$E[X] = m \text{ and } E[(X-m)^2] = \sigma^2$$

The induced measure P_X is called the normal distribution.

A fuzzy set A on Ω is called a *fuzzy event*. Let $\mu_A(\cdot)$ be the membership function of the fuzzy event A. Then the probability of the fuzzy event A is defined by Zadeh([9]) as

$$\widetilde{P}(A) = \int_{\Omega} \mu_A(\omega)dP(\omega), \quad \mu_A(\cdot) : \Omega \to [0,1]$$

Definition 2.3. The normal fuzzy probability $\widetilde{P}(A)$ of a fuzzy set A on $\mathbb{R}$ is defined by

$$\widetilde{P}(A) = \int_{\mathbb{R}} \mu_A(x)dP_X, \tag{2.3}$$

where P_X is the normal distribution.

Definition 2.4. A triangular fuzzy number is a fuzzy number A having membership function

$$\mu_A(x) = \begin{cases} 0, & x < a_1,\ a_3 \le x, \\ \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \le x < a_3. \end{cases}$$

The above triangular fuzzy number is denoted by $A = (a_1, a_2, a_3)$.

Definition 2.5. The addition, subtraction, multi-plication, and division of two fuzzy sets are defined as

1. Addition $A(+)B$:
   $\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}$
2. Subtraction $A(-)B$:
   $\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}$
3. Multiplication $A(\cdot)B$:
   $\mu_{A(\cdot)B}(z) = \sup_{z=x\cdot y} \min\{\mu_A(x), \mu_B(y)\}$
4. Division $A(/)B$:
   $\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\}$

Example 2.6. ([2]) For two triangular fuzzy number $A = (1, 2, 4)$ and $B = (2, 4, 5)$, we have

1. Addition : $A(+)B = (3, 6, 9)$.
2. Subtraction : $A(-)B = (-4, -2, 2)$.
3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2,\ 20 \le x, \\ \dfrac{-2+\sqrt{2x}}{2}, & 2 \le x < 8, \\ \dfrac{7-\sqrt{9+2x}}{2}, & 8 \le x < 20. \end{cases}$$

Note that $A(\cdot)B$ is not a triangular fuzzy number.

4. Division :

$$\mu_{A(/)B}(x)=\begin{cases}0, & x<\frac{1}{5},\ 2\le x,\\ \frac{5x-1}{x+1}, & \frac{1}{5}\le x<\frac{1}{2},\\ \frac{-x+2}{x+1}, & \frac{1}{2}\le x<2.\end{cases}$$

Note that $A(/)B$ is not a triangular fuzzy number.

3. Generalized triangular fuzzy sets

In this section, we define the generalized triangular fuzzy set and apply the extended algebraic operations to these fuzzy sets. A generalized triangular fuzzy set is symmetric and may not have value 1.

Definition 3.1. A generalized triangular fuzzy set is a symmetric fuzzy set A having membership function

$$\mu_A(x)=\begin{cases}0, & x<a_1,\ a_2\le x,\\ \frac{2c(x-a_1)}{a_2-a_1}, & a_1\le x<\frac{a_1+a_2}{2}\\ \frac{-2c(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2}\le x<a_2,\end{cases}\qquad(3.1)$$

where $a_1,a_2\in\mathbb{R}$ and $0<c<1$.

The above generalized triangular fuzzy set is denoted by $A=((a_1,c,a_2))$.

Theorem 3.2. ([4]) For two generalized triangular fuzzy sets $A=((a_1,c_1,a_2))$ and $B=((b_1,c_2,b_2))$, if $c_1\le c_2$ and $\mu_B(x)\ge c_1$ in $[k_1,k_2]$, we have the followings.

1. $A(+)B=(a_1+b_1,\frac{1}{2}(a_1+a_2)+k_1,c_1,\frac{1}{2}(a_1+a_2)+k_2,a_2+b_2)$, i.e., $A(+)B$ is a generalized trapezoidal fuzzy set.
2. $A(-)B=(a_1-b_2,\frac{1}{2}(a_1+a_2)-k_2,c_1,\frac{1}{2}(a_1+a_2)-k_1,a_2-b_1)$, i.e., $A(-)B$ is a generalized trapezoidal fuzzy set.
3. $A(\cdot)B$ is a fuzzy set on (a_1b_1,a_2b_2), but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.
4. $A(/)B$ is a fuzzy set on $(\frac{a_1}{b_2},\frac{a_2}{b_1})$, but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

Example 3.3. ([4]) Let $A=((2,\frac{1}{2},8))$ and $B=((1,\frac{4}{5},5))$ be generalized triangular fuzzy sets, i.e.,

$$\mu_A(x)=\begin{cases}0, & x<2,\ 8\le x,\\ \frac{1}{6}(x-2), & 2\le x<5\\ -\frac{1}{6}(x-8), & 5\le x<8,\end{cases}$$

and

$$\mu_B(x)=\begin{cases}0, & x<1,\ 5\le x,\\ \frac{2}{5}(x-1), & 1\le x<3\\ -\frac{2}{5}(x-5), & 3\le x<5.\end{cases}$$

Let A_α and B_α be the α-cuts of A and B, respectively. Let $A_\alpha=[a_1^{(\alpha)},a_2^{(\alpha)}]$ and $B_\alpha=[b_1^{(\alpha)},b_2^{(\alpha)}]$. Since $\alpha=\frac{1}{6}(a_1^{(\alpha)}-2)$ and $\alpha=-\frac{1}{6}(a_2^{(\alpha)}-8)$, we have $A_\alpha=[a_1^{(\alpha)},a_2^{(\alpha)}]=[6\alpha+2,-6\alpha+8]$. Since $\alpha=\frac{2}{5}(b_1^{(\alpha)}-1)$ and $\alpha=-\frac{2}{5}(b_2^{(\alpha)}-5)$, we have $B_\alpha=[b_1^{(\alpha)},b_2^{(\alpha)}]=[\frac{5}{2}\alpha+1,-\frac{5}{2}\alpha+5]$. Then we have the followings.

1. Addition :

$$\mu_{A(+)B}(x)=\begin{cases}0, & x<3,\ 13\le x,\\ \frac{2}{17}(x-3), & 3\le x<\frac{29}{4},\\ \frac{1}{2}, & \frac{29}{4}\le x<\frac{35}{4},\\ -\frac{2}{17}(x-13), & \frac{35}{4}\le x<13,\end{cases}$$

i.e., $A(+)B=(3,\frac{29}{4},\frac{1}{2},\frac{35}{4},13)$.

2. Subtraction :

$$\mu_{A(-)B}(x)=\begin{cases}0, & x<-3,\ 7\le x,\\ \frac{2}{17}(x+3), & -3\le x<\frac{5}{4},\\ \frac{1}{2}, & \frac{5}{4}\le x<\frac{11}{4},\\ -\frac{2}{17}(x-7), & \frac{11}{4}\le x<7,\end{cases}$$

i.e., $A(-)B=(-3,\frac{5}{4},\frac{1}{2},\frac{11}{4},7)$.

3. Multiplication :

$\mu_{A(\cdot)B}(x)=$

$$\begin{cases} 0, & x<2,\ 40\le x, \\ \frac{1}{30}(-11+\sqrt{121-60(2-x)}), & 2\le x<\frac{45}{4}, \\ \frac{1}{2}, & \frac{45}{4}\le x<\frac{75}{4}, \\ \frac{1}{30}(50-\sqrt{2500-60(40-x)}), & \frac{75}{4}\le x<40. \end{cases}$$

Thus $A(\cdot)B$ is a fuzzy set on $(2,40)$, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

4. Division :

$$\mu_{A(/)B}(x)=\begin{cases} 0, & x<\frac{2}{5},\ 8\le x, \\ \frac{10x-4}{5x+12}, & \frac{2}{5}\le x<\frac{4}{3}, \\ \frac{1}{2}, & \frac{4}{3}\le x<\frac{20}{9}, \\ -\frac{2(x-8)}{5x+12}, & \frac{20}{9}\le x<8. \end{cases}$$

Thus $A(/)B$ is a fuzzy set on $(\frac{2}{5},8)$, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

4. Normal fuzzy probability

In this section, we derive the explicit formula for the normal fuzzy probability for generalized triangular fuzzy sets and give some examples.

Theorem 4.1. Let $X\sim N(m,\sigma^2)$ and $A=((a_1,c,a_2))$ be generalized triangular fuzzy set. Then the normal fuzzy probability of a generalized triangular fuzzy set A is

$$\begin{aligned}\widetilde{P}(A)=&\frac{2mc}{\sqrt{2\pi}(a_2-a_1)}(2N(\frac{a_1+a_2-2m}{2\sigma}) \\ &-N(\frac{a_1-m}{\sigma})-N(\frac{a_2-m}{\sigma})) \\ &-\frac{2c(a_1+\sigma)}{\sqrt{2\pi}(a_2-a_1)}(N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma})) \\ &+\frac{2c(a_2+\sigma)}{\sqrt{2\pi}(a_2-a_1)}(N(\frac{a_2-m}{\sigma})-N(\frac{a_1+a_2-2m}{2\sigma})),\end{aligned} \tag{4.1}$$

where $N(a)$ is the standard normal distribution, that is,

$$N(a)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{a}\exp(-\frac{t^2}{2})dt.$$

Proof. Since

$$\mu_A(x)=\begin{cases} 0, & x<a_1,\ a_2\le x, \\ \frac{2c(x-a_1)}{a_2-a_1}, & a_1\le x<\frac{a_1+a_2}{2}, \\ \frac{-2c(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2}\le x<a_2, \end{cases}$$

we have

$$\begin{aligned}\widetilde{P}(A)&=\int_{-\infty}^{\infty}\mu_A(x)dP_X \\ &=\int_{a_1}^{\frac{a_1+a_2}{2}}g_1(x)f(x)dx \\ &\quad+\int_{\frac{a_1+a_2}{2}}^{a_2}g_2(x)f(x)dx,\end{aligned}$$

where $g_1(x)=\frac{2c(x-a_1)}{a_2-a_1}$, $g_2(x)=\frac{-2c(x-a_2)}{a_2-a_1}$ and $f(x)=\frac{1}{\sqrt{2\pi}\sigma}\exp(-\frac{(x-m)^2}{2\sigma^2})$. Putting $\frac{x-m}{\sigma}=t$, then

$$\begin{aligned}\widetilde{P}(A)=&\int_{a_1}^{\frac{a_1+a_2}{2}}\frac{2c(x-a_1)}{a_2-a_1}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}dx \\ &+\int_{\frac{a_1+a_2}{2}}^{a_2}\frac{-2c(x-a_2)}{a_2-a_1}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-m)^2}{2\sigma^2}}dx \\ =&\frac{2c}{\sqrt{2\pi}(a_2-a_1)}\int_{\frac{a_1-m}{\sigma}}^{\frac{a_1+a_2-2m}{2\sigma}}(m+\sigma t)e^{-\frac{t^2}{2}}dt \\ &-\frac{2ca_1}{\sqrt{2\pi}(a_2-a_1)}\int_{\frac{a_1-m}{\sigma}}^{\frac{a_1+a_2-2m}{2\sigma}}e^{-\frac{t^2}{2}}dt \\ &-\frac{2c}{\sqrt{2\pi}(a_2-a_1)}\int_{\frac{a_1+a_2-2m}{2\sigma}}^{\frac{a_2-m}{\sigma}}(m+\sigma t)e^{-\frac{t^2}{2}}dt \\ &+\frac{2ca_2}{\sqrt{2\pi}(a_2-a_1)}\int_{\frac{a_1+a_2-2m}{2\sigma}}^{\frac{a_2-m}{\sigma}}e^{-\frac{t^2}{2}}dt \\ =&\frac{2mc}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma})\right) \\ &-\frac{2mc}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_2-m}{\sigma})-N(\frac{a_1+a_2-2m}{2\sigma})\right) \\ &-\frac{2ca_1}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma})\right) \\ &+\frac{2ca_2}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_2-m}{\sigma})-N(\frac{a_1+a_2-2m}{2\sigma})\right) \\ &-\frac{2c\sigma}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma})\right) \\ &+\frac{2c\sigma}{\sqrt{2\pi}(a_2-a_1)}\left(N(\frac{a_2-m}{\sigma})-N(\frac{a_1+a_2-2m}{2\sigma})\right)\end{aligned}$$

$$= \frac{2mc}{\sqrt{2\pi}(a_2-a_1)}(2N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma}) - N(\frac{a_2-m}{\sigma}))$$
$$- \frac{2c(a_1+\sigma)}{\sqrt{2\pi}(a_2-a_1)}(N(\frac{a_1+a_2-2m}{2\sigma})-N(\frac{a_1-m}{\sigma}))$$
$$+ \frac{2c(a_2+\sigma)}{\sqrt{2\pi}(a_2-a_1)}(N(\frac{a_2-m}{\sigma})-N(\frac{a_1+a_2-2m}{2\sigma})).$$

Thus the proof is complete.

Example 4.2. 1. Let $A=((2,\frac{1}{2},8))$ be a generalized triangular fuzzy number. Then the normal fuzzy probability of A with respect to $X\sim N(3,2^2)$ is 0.1777. In fact, putting $\frac{x-3}{2}=t$, we have

$$\widetilde{P}(A) = \int_2^5 \frac{x-2}{6}\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-3)^2}{8}}dx + \int_5^8 \frac{8-x}{6}\frac{1}{2\sqrt{2\pi}}e^{-\frac{(x-3)^2}{8}}dx$$
$$= \frac{1}{6\sqrt{2\pi}}\int_{-1/2}^{1}(2t+1)e^{-\frac{t^2}{2}}dt - \frac{1}{6\sqrt{2\pi}}\int_{1}^{5/2}(2t-5)e^{-\frac{t^2}{2}}dt$$
$$= \frac{1}{3\sqrt{2\pi}}\int_{-1/2}^{1}te^{-\frac{t^2}{2}}dt + \frac{1}{6\sqrt{2\pi}}\int_{-1/2}^{1}e^{-\frac{t^2}{2}}dt - \frac{1}{3\sqrt{2\pi}}\int_{1}^{5/2}te^{-\frac{t^2}{2}}dt + \frac{5}{6\sqrt{2\pi}}\int_{1}^{5/2}e^{-\frac{t^2}{2}}dt$$
$$= \frac{1}{3\sqrt{2\pi}}\left(e^{-\frac{1}{8}}-2e^{-\frac{1}{2}}+e^{-\frac{25}{8}}\right) + \frac{1}{6\sqrt{2\pi}}\left(N(1)-N(-\frac{1}{2})\right) + \frac{5}{6\sqrt{2\pi}}\left(N(\frac{5}{2})-N(1)\right)$$
$$= 0.1777.$$

2. Let $B=((1,\frac{3}{4},5))$ be a generalized triangular fuzzy number. Then the normal fuzzy probability of B with respect to $X\sim N(3,3^2)$ is 0.1924. In fact, putting $\frac{x-3}{3}=t$, we have

$$\widetilde{P}(B) = \int_1^3 \frac{3(x-1)}{8}\frac{1}{3\sqrt{2\pi}}e^{-\frac{(x-3)^2}{18}}dx + \int_3^5 \frac{3(5-x)}{8}\frac{1}{3\sqrt{2\pi}}e^{-\frac{(x-3)^2}{18}}dx$$
$$= \frac{3}{8\sqrt{2\pi}}\int_{-2/3}^{0}(3t+2)e^{-\frac{t^2}{2}}dt - \frac{3}{8\sqrt{2\pi}}\int_{0}^{2/3}(3t-2)e^{-\frac{t^2}{2}}dt$$
$$= \frac{9}{8\sqrt{2\pi}}\int_{-2/3}^{0}te^{-\frac{t^2}{2}}dt + \frac{3}{4\sqrt{2\pi}}\int_{-2/3}^{0}e^{-\frac{t^2}{2}}dt - \frac{9}{8\sqrt{2\pi}}\int_{0}^{2/3}te^{-\frac{t^2}{2}}dt + \frac{3}{4\sqrt{2\pi}}\int_{0}^{2/3}e^{-\frac{t^2}{2}}dt$$
$$= \frac{9}{4\sqrt{2\pi}}\left(e^{-\frac{2}{3}}-1\right) + \frac{3}{4\sqrt{2\pi}}\left(N(\frac{2}{3})-N(-\frac{2}{3})\right)$$
$$= 0.1924.$$

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