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# A LOCAL FIXED POINT THEOREM ON FUZZY METRIC SPACES

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Abstract. In this paper, we present a common fixed point theorem for multivalued maps on $M$-complete fuzzy metric spaces. Also, the single valued case and an illustrative example are given.

## 1. Introduction

There exists considerable literature of fixed point theory dealing with results on fixed or common fixed points in fuzzy metric space (e.g. [1]-[13], [15], [18]-[20]). Almost all of these papers, the results have been given for single valued mappings. In the present paper, we give a local common fixed point theorem for multivalued and single valued maps on $M$-complete fuzzy metric spaces. In addition, we give an illustrative example.

For the sake of completeness, we briefly recall some notions from the theory of fuzzy metric spaces.

**Definition 1** ([17])**.** A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous $t$-norm if $([0,1], *)$ is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Two typical examples of continuous $t$-norms are $a * b = ab$ and $a * b = \min\{a, b\}$.

**Definition 2** ([6])**.** A fuzzy metric space (in the sense of George and Veeramani) is a triple $(X, M, *)$, where $X$ is a nonempty set, $*$ is a continuous $t$-norm and $M$ is a fuzzy set on $X^2 \times (0, \infty)$, satisfying the following properties:

(GV-1) $M(x, y, t) > 0, \forall t > 0$,
(GV-2) $M(x, x, t) = 1, \forall t > 0$ and if $M(x, y, t) = 1$ for some $t > 0$, then $x = y$,
(GV-3) $M(x, y, t) = M(y, x, t), \forall x, y \in X$ and $t > 0$,
(GV-4) $M(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous $\forall x, y \in X$,
(GV-5) $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s), \forall x, y, z \in X, \forall t, s > 0$.

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**Definition 3** ([6])**.** Let $(X, M, *)$ be a fuzzy metric space. A sequence $\{x_n\}$ in $X$ is called a $M$-Cauchy sequence, if for each $r \in (0,1)$ and $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - r$ for all $m, n \geq n_0$. A sequence $\{x_n\}$ in a fuzzy metric space $(X, M, *)$ is said to be convergent to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$ for all $t > 0$. A fuzzy metric space $(X, M, *)$ is called $M$-complete if every $M$-Cauchy sequence is convergent.

Let $(X, M, *)$ be a fuzzy metric space. For $t > 0$, the open ball $B(x, r, t)$ with center $x \in X$ and radius $r$ $(0 < r < 1)$ is defined by

$$B(x, r, t) = \{y \in X : M(x, y, t) > 1 - r\}$$

and the closed ball $B[x, r, t]$ is defined by

$$B[x, r, t] = \{y \in X : M(x, y, t) \geq 1 - r\}.$$

Let $\tau$ be the set of all $A \subset X$ with $x \in A$ if and only if there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Then $\tau$ is a topology on $X$ (induced by the fuzzy metric $M$). This topology is Hausdorff and first countable. A subset $A$ of $X$ is said to be $F$-bounded if there exist $t > 0$ and $0 < r < 1$ such that $M(x, y, t) > 1 - r$ for all $x, y \in A$.

**Example 1.** Let $X = \mathbb{R}$, $a * b = ab$ for all $a, b \in [0,1]$ and

$$M(x, y, t) = \frac{t}{t + |x - y|}$$

for all $x, y \in X$ and $t \in (0, \infty)$, then $(X, M, *)$ is a fuzzy metric space.

**Lemma 1** ([7])**.** *Let $(X, M, *)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to $t$, for all $x, y$ in $X$.*

**Definition 4.** Let $(X, M, *)$ be a fuzzy metric space. $M$ is said to be continuous on $X^2 \times (0, \infty)$ if

$$\lim_{n\to\infty} M(x_n, y_n, t_n) = M(x, y, t),$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X^2 \times (0, \infty)$ converges to a point $(x, y, t) \in X^2 \times (0, \infty)$, i.e.,

$$\lim_{n\to\infty} M(x_n, x, t) = \lim_{n\to\infty} M(y_n, y, t) = 1$$

and

$$\lim_{n\to\infty} M(x, y, t_n) = M(x, y, t).$$

**Lemma 2.** *Let $(X, M, *)$ be a fuzzy metric space. Then $M$ is a continuous function on $X^2 \times (0, \infty)$.*

*Proof.* See [7] and Proposition 1 of [16]. $\square$

Let $\Phi$ be the set of all strictly increasing and continuous functions $\phi : (0, 1] \longrightarrow (0, 1]$ such that $\phi(s) > s$ for every $s \in (0, 1)$.

**Example 2.** Let $\phi : (0,1] \longrightarrow (0,1]$ be a function defined by $\phi(t) = \sqrt{t}$. Then it is clear that $\phi \in \Phi$.

Let $(X, M, *)$ be a fuzzy metric space and let $A, B \subseteq X$ be two nonempty sets. Define

$$\delta_M(A, B, t) = \inf\{M(x, y, t) : x \in A, y \in B\}$$

and

$$\mathcal{M}(A, B, t) = \sup\{M(x, y, t) : x \in A, y \in B\}$$

for all $t > 0$. In particular, if $B = \{b\}$, then

$$\mathcal{M}(A, B, t) = \sup\{M(x, b, t) : x \in A\}.$$

## 2. Main result

**Theorem 3.** *Let $(X, M, *)$ be a $M$-complete fuzzy metric space such that $a*b \geq ab$ for every $a, b \in [0,1]$. Let $x_0 \in X, 0 < r < 1$ with $F, G : B[x_0, 1-r, t] \to C(X)$ (the family of all nonempty closed subsets of $X$). Suppose, for all $x, y \in B[x_0, 1-r, t]$, $Fx, Gy$ are $F$-bounded and*

$$\delta_M(Fx, Gy, t) \geq \phi(\min\{M(x, y, t), \mathcal{M}(x, Fx, t), \mathcal{M}(y, Gy, t)\}). \tag{2.1}$$

*Also we assume that the following conditions are satisfied: for all $t > 0$*

$$\mathcal{M}(x_0, Fx_0, t) > \frac{r}{\phi(r)} \tag{2.2}$$

*and*

$$\prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) \geq \phi^n(r) \tag{2.3}$$

*for $n \in \mathbb{N}$, where $\phi \in \Phi$.*

*Then there exists $x \in B[x_0, 1-r, t]$ with $x \in Fx$ and $x \in Gx$.*

*Proof.* From (2.2) we can choose, for all $t > 0$, $x_1 \in Fx_0$ with

$$M(x_0, x_1, t) > \frac{r}{\phi(r)}. \tag{2.4}$$

Thus $M(x_0, x_1, t) > \frac{r}{\phi(r)} > r = 1 - (1 - r)$, hence $x_1 \in B[x_0, 1-r, t]$. Now we have

$$\phi(M(x_0, x_1, t)) > \phi\left(\frac{r}{\phi(r)}\right) \tag{2.5}$$

(this is possible from (2.4) and the fact that $\phi$ is strictly increasing). Since $x_0, x_1 \in B[x_0, 1-r, t]$ by (2.1) we have

$$\delta_M(Fx_0, Gx_1, t) \geq \phi(\min\{M(x_0, x_1, t), \mathcal{M}(x_0, Fx_0, t), \mathcal{M}(x_1, Gx_1, t)\}).$$

Now, choose $x_2 \in Gx_1$ so that

$$\begin{aligned} M(x_1, x_2, t) &\geq \delta_M(Fx_0, Gx_1, t) \\ &\geq \phi(\min\{M(x_0, x_1, t), \mathcal{M}(x_0, Fx_0, t), \mathcal{M}(x_1, Gx_1, t)\}) \\ &\geq \phi(\min\{M(x_0, x_1, t), M(x_1, x_2, t)\}). \end{aligned}$$

Hence for all $t > 0$, we have

$$\begin{aligned} M(x_1, x_2, t) &\geq \phi\left(M(x_0, x_1, t)\right) \\ &\geq \phi\left(\frac{r}{\phi(r)}\right). \end{aligned}$$

Therefore we have

$$\phi(M(x_1, x_2, t)) > \phi^2\left(\frac{r}{\phi(r)}\right). \tag{2.6}$$

Since

$$\begin{aligned} M(x_0, x_2, t) &\geq M\left(x_0, x_1, \frac{t}{2}\right) * M\left(x_1, x_2, \frac{t}{2}\right) \\ &\geq \frac{r}{\phi(r)} * \phi\left(\frac{r}{\phi(r)}\right) \\ &\geq \frac{r}{\phi(r)}\phi(r) \\ &= r \end{aligned}$$

then we have $x_2 \in B[x_0, 1 - r, t]$. Again, choose $x_3 \in Fx_2$ so that (since $x_1, x_2 \in B[x_0, 1 - r, t]$ we can use the inequality (2.1))

$$\begin{aligned} M(x_3, x_2, t) &\geq \delta_M(Fx_2, Gx_1, t) \\ &\geq \phi(\min\{M(x_2, x_1, t), \mathcal{M}(x_2, Fx_2, t), \mathcal{M}(x_1, Gx_1, t)\}) \\ &\geq \phi(\min\{M(x_2, x_1, t), M(x_2, x_3, t)\}). \end{aligned}$$

Now, from (2.6), we have

$$M(x_3, x_2, t) \geq \phi(M(x_1, x_2, t)) \geq \phi^2\left(\frac{r}{\phi(r)}\right)$$

and so we have

$$\phi(M(x_3, x_2, t)) \geq \phi^3\left(\frac{r}{\phi(r)}\right).$$

Also, for all $t > 0$, since

$$\begin{aligned} M(x_0, x_3, t) &\geq M\left(x_0, x_1, \frac{t}{3}\right) * M\left(x_1, x_2, \frac{t}{3}\right) * M\left(x_2, x_3, \frac{t}{3}\right) \\ &\geq \frac{r}{\phi(r)}\phi\left(\frac{r}{\phi(r)}\right)\phi^2\left(\frac{r}{\phi(r)}\right) \\ &\geq \frac{r}{\phi(r)}\prod_{i=1}^{\infty}\phi^i\left(\frac{r}{\phi(r)}\right) \\ &\geq \frac{r}{\phi(r)}\phi(r) = r \end{aligned}$$

we have $x_3 \in B[x_0, 1 - r, t]$.

Continuing this way we can obtain a sequence $\{x_n\} \subseteq B[x_0, 1-r, t]$ such that $x_{2n+2} \in Gx_{2n+1}$ and $x_{2n+1} \in Fx_{2n}$ for $n \in \{0, 1, \ldots\}$ and

$$M(x_n, x_{n+1}, t) > \phi^n\left(\frac{r}{\phi(r)}\right). \tag{2.7}$$

Next we show that $\{x_n\}$ is a $M$-Cauchy sequence. Notice for all $t > 0$ and $0 < k < 1$ we can choose $t_0 > 0$ such that $t = \sum_{i=0}^{\infty} k^i t_0$. Hence using (2.3) and (2.7) we have

$$\begin{aligned}
& M(x_n, x_m, t) \\
= \;& M\left(x_n, x_m, \sum_{i=0}^{\infty} k^i t_0\right) \\
\geq \;& M\left(x_n, x_m, \sum_{i=n}^{m-1} k^i t_0\right) \\
\geq \;& M\left(x_n, x_{n+1}, k^n t_0\right) * M(x_n, x_{n+2}, k^{n+1} t_0) * \cdots * M(x_n, x_m, k^{m-1} t_0) \\
\geq \;& \phi^n\left(\frac{r}{\phi(r)}\right) \phi^{n+1}\left(\frac{r}{\phi(r)}\right) \cdots \phi^{m-1}\left(\frac{r}{\phi(r)}\right) \\
\geq \;& \prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) \geq \phi^n(r) \longrightarrow 1
\end{aligned}$$

and so $\{x_n\}$ is a $M$-Cauchy sequence. Since $X$ is $M$-complete, then there exists $x \in B[x_0, 1-r, t]$ with $x_n \to x$. It remains to show $x \in Fx$ and $x \in Gx$.

Let $\mathcal{M}(x, Fx, t) < 1$ for all $t > 0$. Since $x_{2n-1}, x \in B[x_0, 1-r, t]$ we can use the inequality (2.1), then we have

$$\begin{aligned}
& \delta_M(Fx, Gx_{2n-1}, t) \\
\geq \;& \phi(\min\{M(x, x_{2n-1}, t), \mathcal{M}(x, Fx, t), \mathcal{M}(x_{2n-1}, Gx_{2n-1}, t)) \\
\geq \;& \phi(\min\{M(x, x_{2n-1}, t), \mathcal{M}(x, Fx, t), t), M(x_{2n-1}, x_{2n}, t)).
\end{aligned}$$

Now taking limit inferior to $n \to \infty$ we have, for all $t > 0$,

$$\liminf_{n\to\infty} \delta_M(Fx, Gx_{2n-1}, t) \geq \phi(\mathcal{M}(x, Fx, t)). \tag{2.8}$$

On the other hand, for $0 < \varepsilon < t$, we have

$$\begin{aligned}
\mathcal{M}(x, Fx, t) &\geq M(x, x_{2n}, \varepsilon) * \mathcal{M}(x_{2n}, Fx, t-\varepsilon) \\
&\geq M(x, x_n, \varepsilon) * \delta_M(Gx_{2n-1}, Fx, t-\varepsilon)
\end{aligned}$$

and so, taking limit inferior to $n \to \infty$ and using (2.8), we have

$$\begin{aligned}
\mathcal{M}(x, Fx, t) &\geq \liminf_{n\to\infty} \delta_M(Fx, Gx_{2n-1}, t-\varepsilon) \\
&\geq \phi(\mathcal{M}(x, Fx, t-\varepsilon)).
\end{aligned}$$

Taking $\varepsilon \to 0$ we have

$$\mathcal{M}(x, Fx, t) \geq \phi(\mathcal{M}(x, Fx, t)),$$

which is a contradiction (since $\phi(s) > s$ for $0 < s < 1$ and $\mathcal{M}(x, Fx, t) > 0$). Thus $\mathcal{M}(x, Fx, t) = 1$ for some $t > 0$. Then $x \in \overline{Fx} = Fx$.

Similarly, we obtain $\mathcal{M}(x, Gx, t) = 1$ for some $t > 0$, so $x \in Gx$. □

Now we give an illustrative example.

**Example 4.** Let $X = (0, \infty)$, $a * b = ab$ and

$$M(x, y, t) = \frac{\min\{x, y\}}{\max\{x, y\}}, \forall t > 0.$$

Then $(X, M, *)$ is a $M$-complete fuzzy metric space (see [14]). Let $x_0 = \frac{1}{2}$ and $r = \frac{1}{3}$, then

$$\begin{aligned} B[x_0, 1 - r, t] &= B\left[\frac{1}{2}, \frac{2}{3}, t\right] \\ &= \left\{y \in X : M\left(\frac{1}{2}, y, t\right) \geq \frac{1}{3}\right\} \\ &= \left[\frac{1}{6}, \frac{3}{2}\right]. \end{aligned}$$

Now let $F, G : B[x_0, 1 - r, t] \to C(X)$,

$$Fx = Gx = \begin{cases} [1, \sqrt{x}] & , \quad x \geq 1 \\ \sqrt{x} & , \quad x < 1 \end{cases}$$

and let $\phi(t) = \sqrt{t}$, then it is clear that $\phi \in \Phi$. Also $Fx$ and $Gy$ are $F$-bounded,

$$\begin{aligned} \mathcal{M}(x_0, Fx_0, t) &= \mathcal{M}\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, t\right) \\ &= M\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, t\right) = \frac{\sqrt{2}}{2} > \frac{\sqrt{3}}{3} = \frac{\frac{1}{3}}{\phi\left(\frac{1}{3}\right)} = \frac{r}{\phi(r)} \end{aligned}$$

and

$$\prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) = \prod_{i=n}^{\infty} r^{\frac{1}{2^{i+1}}} = r^{\sum\limits_{i=n}^{\infty} \frac{1}{2^{i+1}}} = r^{\frac{1}{2^n}} = \phi^n(r).$$

Finally, we show that (2.1) is satisfied for all $x, y \in B[x_0, 1 - r, t] = [\frac{1}{6}, \frac{3}{2}]$. For this we consider the following cases:

Case 1. if $x, y \in [1, \frac{3}{2}]$ and $x > y$, then

$$\begin{aligned}
\delta_M(Fx, Gy, t) &= \inf\left\{M(a,b,t) : a \in Fx = [1, \sqrt{x}], b \in Gy = [1, \sqrt{y}]\right\} \\
&= \inf\left\{\frac{\min\{a,b\}}{\max\{a,b\}} : a \in Fx = [1, \sqrt{x}], b \in Gy = [1, \sqrt{y}]\right\} \\
&= \frac{1}{\sqrt{x}} \\
&= \phi\left(\inf\left\{\frac{\min\{x,c\}}{\max\{x,c\}} : c \in Fx = [1, \sqrt{x}]\right\}\right) \\
&= \phi\left(\inf\{M(x,c,t) : c \in Fx = [1, \sqrt{x}]\}\right) \\
&= \phi\left(\mathcal{M}(x, Fx, t)\right) \\
&\geq \phi\left(\min\{M(x,y,t), \mathcal{M}(x,Fx,t), \mathcal{M}(y,Gy,t)\}\right).
\end{aligned}$$

Case 2. if $x \in [1, \frac{3}{2}]$ and $y \in [\frac{1}{2}, 1)$, then

$$\begin{aligned}
\delta_M(Fx, Gy, t) &= \inf\left\{M(a,b,t) : a \in Fx = [1, \sqrt{x}], b \in Gy = \sqrt{y}\right\} \\
&= \inf\left\{\frac{\min\{a,b\}}{\max\{a,b\}} : a \in Fx = [1, \sqrt{x}], b \in Gy = \sqrt{y}\right\} \\
&= \frac{\sqrt{y}}{\sqrt{x}} \\
&= \sqrt{\frac{\min\{x,y\}}{\max\{x,y\}}} \\
&= \phi(M(x,y,t)) \\
&\geq \phi(\min\{M(x,y,t), \mathcal{M}(x,Fx,t), \mathcal{M}(y,Gy,t)\}).
\end{aligned}$$

Case 3. if $x, y \in [\frac{1}{2}, 1)$ and $x > y$, then

$$\begin{aligned}
\delta_M(Fx, Gy, t) &= \inf\{M(a,b,t) : a \in Fx = \sqrt{x}, b \in Gy = \sqrt{y}\} \\
&= \frac{\sqrt{y}}{\sqrt{x}} \\
&= \sqrt{\frac{\min\{x,y\}}{\max\{x,y\}}} \\
&= \phi(M(x,y,t)) \\
&\geq \phi(\min\{M(x,y,t), \mathcal{M}(x,Fx,t), \mathcal{M}(y,Gy,t)\}).
\end{aligned}$$

Therefore conditions of Theorem 3 are satisfied, thus $F$ and $G$ have a common fixed point in $B[x_0, 1-r, t] = [\frac{1}{6}, \frac{3}{2}]$.

We can have the following corollaries.

**Corollary 1.** *Let $(X, M, *)$ be a $M$-complete fuzzy metric space such that $a * b \geq ab$ for every $a, b \in [0,1]$. Let $x_0 \in X, 0 < r < 1$ with $F, G : B[x_0, 1-$*

*$r,t] \to X$. Suppose, for all $x,y \in B[x_0, 1-r, t]$*

$$M(Fx, Gy, t) \geq \phi(\min\{M(x,y,t), M(x,Fx,t), M(y,Gy,t)\}).$$

*Also we assume that the following conditions are satisfied: for all $t > 0$*

$$M(x_0, Fx_0, t) > \frac{r}{\phi(r)}$$

*and*

$$\prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) \geq \phi^n(r)$$

*for $n \in \mathbb{N}$, where $\phi \in \Phi$.*

*Then there exists a unique $x \in B[x_0, 1-r, t]$ with $x = Fx = Gx$.*

*Proof.* By Theorem 3, it is enough prove that $x$ is unique.

Let $y$ be another common fixed point of $F$ and $G$, that is $y = Fy = Gy$, then we have

$$\begin{aligned} M(Fx, Gy, t) &\geq \phi(\min\{M(x,y,t), M(x,Fx,t), M(y,Gy,t)\}) \\ &= \phi(M(x,y,t)) > M(x,y,t), \end{aligned}$$

which is a contradiction. Therefore $F$ and $G$ have a unique common fixed point in $B[x_0, 1-r, t]$. □

**Corollary 2.** *Let $(X, M, *)$ be a $M$-complete fuzzy metric space such that $a * b \geq ab$ for every $a, b \in [0,1]$. Let $x_0 \in X, 0 < r < 1$ with $F : B[x_0, 1-r, t] \to X$. Suppose, for all $x, y \in B[x_0, 1-r, t]$*

$$M(Fx, Fy, t) \geq \phi(\min\{M(x,y,t), M(x,Fx,t), M(y,Fy,t)\}).$$

*Also we assume that the following conditions are satisfied: for all $t > 0$*

$$M(x_0, Fx_0, t) > \frac{r}{\phi(r)}$$

*and*

$$\prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) \geq \phi^n(r)$$

*for $n \in \mathbb{N}$, where $\phi \in \Phi$.*

*Then there exists a unique $x \in B[x_0, 1-r, t]$ with $x = Fx$.*

**Example 5.** Let $X, *, M, x_0, r$ and $\phi$ be as in Example 4. Let $F : B[x_0, 1-r, t] \to X$, $Fx = \sqrt{x}$, then it is clear that

$$M(x_0, Fx_0, t) = M\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, t\right) = \frac{\sqrt{2}}{2} > \frac{\sqrt{3}}{3} = \frac{\frac{1}{3}}{\phi\left(\frac{1}{3}\right)} = \frac{r}{\phi(r)}$$

and

$$\prod_{i=n}^{\infty} \phi^i\left(\frac{r}{\phi(r)}\right) = \prod_{i=n}^{\infty} r^{\frac{1}{2^{i+1}}} = r^{\sum\limits_{i=n}^{\infty} \frac{1}{2^{i+1}}} = r^{\frac{1}{2^n}} = \phi^n(r).$$

Also, for all $x, y \in B[x_0, 1-r, t]$, we have

$$\begin{aligned} M(Fx, Fy, t) &= \frac{\min\{\sqrt{x}, \sqrt{y}\}}{\max\{\sqrt{x}, \sqrt{y}\}} \\ &= \sqrt{\frac{\min\{x, y\}}{\max\{x, y\}}} \\ &= \phi(M(x, y, t)) \\ &\geq \phi(\min\{M(x, y, t), M(x, Fx, t), M(y, Fy, t)\}). \end{aligned}$$

Therefore conditions of Corollary 2 are satisfied, thus $F$ has a unique fixed point in $B[x_0, 1-r, t] = [\frac{1}{6}, \frac{3}{2}]$.

Using recent ideas in the literature [2], [3], it is possible to extend our theorem to a fuzzy metric space endowed with a partial order induced by an appropriate function.

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