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ON (ϵ)-LORENTZIAN PARA-SASAKIAN MANIFOLDS

Rajendra Prasad and Vibha Srivastava

Abstract. In this paper we study (ϵ)-Lorentzian para-Sasakian manifolds and show its existence by an example. Some basic results regarding such manifolds have been deduced. Finally, we study conformally flat and Weyl-semisymmetric (ϵ)-Lorentzian para-Sasakian manifolds.

1. Introduction

In [1] Bejancu and K. L. Duggal introduced (ϵ)-Sasakian manifolds. Also Xufeng and Xiaoli [11] showed that every (ϵ)-Sasakian manifold must be a real hypersurface of some indefinite Kähler manifold. Further, in [6] R. Kumar, R. Rani and R. Nagaich study (ϵ)-Sasakian manifolds. Since Sasakian manifolds with indefinite metric play significant role in Physics [5], our natural trend is to study various contact manifolds with indefinite metric. Recently, in 2009, U. C. De, Avijit Sarkar [4] study (ϵ)-Kenmotsu manifolds. In 1989, K. Matsumoto [7] introduced the notion of Lorenzian para-Sasakian manifolds. I. Mihai and R. Rosca [9] defined the same notion independently and several authors [8, 10] studied LP-Sasakian manifolds. In this paper we like to introduce (ϵ)-Lorentzian para-Sasakian manifolds with indefinite metric which also include usual LP-Sasakian manifold. The present paper is organized as follows:

Section 1 is introductory. In Section 2, we define (ϵ)-LP-Sasakian manifolds and give an example of such a manifold. We also give some basic results of such a manifold in the same section. In Section 3, we study conformally flat (ϵ)-LP-Sasakian manifolds. Finally, we consider Weyl-semisymmetric (ϵ)-LP-Sasakian manifolds.

2. (ϵ)-Lorentzian para-Sasakian manifolds

An n-dimensional differentiable manifold is called (ϵ)-Lorentzian para-Sasakian manifold if the following conditions hold:

$$\phi^2 = I + \eta(X)\xi, \qquad \eta(\xi) = -1, \tag{2.1}$$

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$$g(\xi,\xi)=\epsilon, \qquad \eta(X)=\epsilon g(X,\xi), \tag{2.2}$$

$$g(\phi X,\phi Y)=g(X,Y)+\epsilon\eta(X)\eta(Y), \tag{2.3}$$

where ϵ is 1 or -1 according as ξ is space-like or time-like vector field. Also in (ϵ)-Lorentzian para-Sasakian manifold, we have

$$(\nabla_X\phi)Y=g(X,Y)\xi+\epsilon\eta(Y)X+2\epsilon\eta(X)\eta(Y), \tag{2.4}$$

where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g.

Definition 2.1. An (ϵ)-LP-Sasakian manifold will be called a manifold of quasi-constant curvature if the curvature tensor $\check{R}$ of type $(0,4)$ satisfies the condition

$$\begin{aligned}\check{R}(X,Y,Z,W)=&\,a[g(Y,Z)g(X,W)-g(X,Z)g(Y,W)]\\&+b[g(X,W)T(Y)T(Z)-g(X,Z)T(Y)T(W)\\&+g(Y,Z)T(X)T(W)-g(Y,W)T(X)T(Z)],\end{aligned} \tag{2.5}$$

where $\check{R}(X,Y,Z,W)=g(R(X,Y)Z,W)$, R is the curvature tensor of type $(1,3)$; a,b are scalar functions and ρ is a unit vector field defined by

$$g(X,\rho)=T(X). \tag{2.6}$$

The notion of quasi-constant curvature for Riemannian manifolds were given by Chen and Yano [2].

Definition 2.2. An (ϵ)-LP-Sasakian manifold will be called an η-Einstein manifold if the Ricci tensor S of type $(0,2)$ satisfies

$$S(X,Y)=ag(X,Y)+b\eta(X)\eta(Y),$$

where a and b are scalar functions.

Definition 2.3. A type of Riemannian manifold whose curvature tensor $\check{R}$ of type $(0,4)$ satisfies the condition

$$\check{R}(X,Y,Z,W)=F(Y,Z)F(X,W)-F(X,Z)F(Y,W), \tag{2.7}$$

where F is a symmetric tensor of type $(0,2)$ is called a special manifold with the associated symmetric tensor F and is denoted by $\psi(F)_n$.

In 1956, S. S. Chern [3] study such type of manifolds. These manifolds are important for the following reasons:

Firstly, for possessing some remarkable properties relating to curvature and characteristic classes and secondly, for containing a manifold of quasi-constant curvature [2].

Definition 2.4. An (ϵ)-LP-Sasakian manifold will be called Weyl-semisymmetric if it satisfies $(R.(X,Y).C)(Y,Z)W=0$, where $R(X,Y)$ denotes the curvature operator and $C(Y,Z)W$ is the Weyl-conformal curvature tensor.

Lemma 2.1. *An* (ϵ)*-contact metric manifold is an* (ϵ)*-LP-Sasakian manifold if and only if*

$$\nabla_X \xi = \epsilon \phi X. \tag{2.8}$$

Proof. Let the manifold be an (ϵ)-Lorentzian para-Sasakian manifold. Then from the equation (2.4) it follows that

$$\nabla_X \phi Y - \phi \nabla_X Y = g(X,Y)\xi + \epsilon\eta(Y)X + 2\epsilon\eta(X)\eta(Y)\xi.$$

Putting $Y = \xi$, we get

$$-\phi\nabla_X \xi = -\epsilon(X + \eta(X)\xi),$$

or,

$$\phi\nabla_X \xi = \epsilon\phi^2(X),$$

which implies,

$$\nabla_X \xi = \epsilon\phi(X).$$

Conversely, let the above relation holds. Now the fundamental 2-form Φ of the (ϵ)-almost contact metric structure is defined by [5]

$$\Phi(X,Y) = g(X,\phi Y)$$

for all vector fields $X, Y \in \chi(M)$. Now since $\eta \wedge \phi$ is up to a constant factor the volume element of the manifold, it is parallel with respect to ∇, i.e., $\nabla_X(\eta \wedge \phi) = 0$. Hence we have

$$\begin{aligned}&(\nabla_X\eta)(Y)\Phi(Z,W) + \eta(Y)(\nabla_X\Phi)(Z,W) + (\nabla_X\eta)(Z)\Phi(W,Y)\\&+\eta(Z)(\nabla_X\Phi)(W,Y) + (\nabla_X\eta)(W)\Phi(Y,Z) + (\nabla_X\eta)(W)\Phi(Y,Z)\\&+\eta(W)(\nabla_X\Phi)(Y,Z) = 0.\end{aligned} \tag{2.9}$$

Putting $W = \xi$, we get

$$(\nabla_X\Phi)Y = \epsilon g(\Phi\nabla_X\xi, Y)\xi + \eta(Y)\Phi\nabla_X\xi,$$

Now using the value of $\nabla_X\xi$, we have

$$(\nabla_X\phi)Y = g(X,Y)\xi + \epsilon\eta(Y)X + 2\epsilon\eta(X)\eta(Y)\xi.$$

Hence the manifold is an (ϵ)-Lorentzian para-Sasakian manifold. □

Example. Consider the 3-dimensional manifold $M = [(x,y,z)] \in \mathbb{R}^3, z \neq 0$, where (x,y,z) are the standard coordinates in $\mathbb{R}^3$. The vector fields

$$e_1 = e^z\frac{\partial}{\partial y}, e_2 = e^z\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right), e_3 = \frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the Lorentzian metric defined by

$$g(e_1, e_3) = g(e_2, e_3) = g(e_1, e_2) = 0,$$
$$g(e_1, e_1) = g(e_2, e_2) = \epsilon, \qquad g(e_3, e_3) = -\epsilon.$$

Let η be the 1-form defined by $\eta(Z) = g(Z, e_3)$ for any $Z \in \chi(M)$. Let ϕ be the $(1,1)$ tensor field defined by $\phi e_1 = -e_1$, $\phi e_2 = -e_2$, $\phi\, e_3 = 0$. Then using the linearity of ϕ and g, we have

$$\eta(e_3) = -1, \phi^2(Z) = Z + \eta(Z)\xi, \text{and } g(\phi Z, \phi W) = g(Z, W) + \epsilon\eta(Z)\eta(W)$$

for any $Z, W \in \chi(M)$. Let ∇ be the Levi-Civita connection with respect to the Lorentzian metric g. Then we have

$$[e_1, e_2] = 0,\ [e_1, e_3] = -\epsilon e_1,\ [e_2, e_3] = -\epsilon e_2.$$

The Riemannian connection ∇ of the Lorentzian metric g is given by

$$2g(\nabla_X Y, Z) = Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) - g(X, [Y, Z]) \\ - g(Y, [X, Z]) + g(Z, [X, Y])$$

which is known as Koszul's formula.

From Koszul's formula, we have

$$\nabla_{e_1} e_3 = -\epsilon e_1, \nabla_{e_1} e_2 = 0, \nabla_{e_1} e_1 = -\epsilon e_3,$$

$$\nabla_{e_2} e_3 = -\epsilon e_2, \nabla_{e_2} e_2 = -\epsilon e_3, \nabla_{e_2} e_1 = 0,$$

$$\nabla_{e_3} e_3 = 0, \nabla_{e_3} e_2 = 0, \nabla_{e_3} e_1 = 0.$$

From the above result it can be easily seen that the manifold satisfies

$$\nabla_X \xi = \epsilon\phi X$$

for $\xi = e_3$. Hence the manifold under consideration is an (ϵ)-Lorentzian para-Sasakian manifold.

Lemma 2.2. *In an (ϵ)-Lorentzian para-Sasakian manifold*

$$(\nabla_X \eta)(Y) = g(\phi X, Y). \tag{2.10}$$

Proof.

$$(\nabla_X \eta)(Y) = \nabla_X \eta(Y) - \eta(\nabla_X Y) \\ = \epsilon\nabla_X g(Y, \xi) - \epsilon g(\nabla_X Y, \xi) - \epsilon g(Y, \nabla_X \xi) + \epsilon g(Y, \nabla_X \xi).$$

Using the value of $\nabla_X \xi$, we have

$$(\nabla_X \eta)(Y) = g(\phi X, Y).$$ □

Lemma 2.3. *In an (ϵ)-Lorentzian para-Sasakian manifold*

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y. \tag{2.11}$$

Proof.

$$R(X, Y)\xi = \nabla_X \nabla_Y \xi - \nabla_Y \nabla_X \xi - \nabla_{[X,Y]}\xi \\ = \nabla_X(\epsilon\phi Y) - \nabla_Y(\epsilon\phi X) - \epsilon\phi([X, Y]).$$

The above relation after simplification gives

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y.$$ □

Note. From the equation (2.11) it follows that in an (ϵ)-Lorentzian para-Sasakian manifold,

$$R(\xi, X) Y = \epsilon g(X, Y) \xi - \eta(Y) X. \tag{2.12}$$

Also in an (ϵ)-Lorentzian para-Sasakian manifold

$$\eta(R(X, Y) Z) = \epsilon(g(Y, Z) \eta(X) - g(X, Z) \eta(Y)). \tag{2.13}$$

Lemma 2.4. *In an* (ϵ)*-Lorentzian para-Sasakian manifold*

$$S(X, \xi) = (n-1) \eta(X). \tag{2.14}$$

Proof. From the equation (2.13) we have

$$g(R(X, Y) Z, \xi) = \epsilon g(Y, Z) g(X, \xi) - \epsilon g(X, Z) g(Y, \xi).$$

Putting $Y = Z = e_i$, where $\{e_i\}$ is an orthonormal basis of the tangent space at each point of the manifold, and taking summation over i where $i = 1, 2, \ldots, n$, we get

$$S(X, \xi) = (n-1) \eta(X).$$

□

3. Conformally flat (ϵ)-Lorentzian para-Sasakian manifold

The Weyl conformal curvature tensor C of type $(1, 3)$ of an n-dimensional Riemannian manifold is given by

$$\begin{aligned} C(X, Y) Z = R(X, Y) Z - \frac{1}{(n-2)}[S(Y, Z) X - S(X, Z) Y \\ + g(Y, Z) QX - g(X, Z) QY] + \frac{r}{(n-1)(n-2)} \\ [g(Y, Z) X - g(X, Z) Y], \end{aligned} \tag{3.1}$$

where Q is the Ricci operator defined by $g(QX, Y) = S(X, Y)$ and r is the scalar curvature. Let us suppose that the manifold is conformally flat. Then from the above equation, we have

$$\begin{aligned} g(R(X, Y) Z, W) = \frac{1}{(n-2)}[S(Y, Z) g(X, W) \\ - S(X, Z) g(Y, W) + g(Y, Z) S(X, W) \\ - g(X, Z) S(Y, W)] - \frac{r}{(n-1)(n-2)} \\ [g(Y, Z) g(X, W) - g(X, Z) g(Y, W)]. \end{aligned} \tag{3.2}$$

Putting $W = \xi$ and using the equaton (2.14), the above equation gives

$$\begin{aligned} \epsilon\eta(R(X, Y) Z) = \frac{1}{(n-2)}[\epsilon S(Y, Z) \eta(X) - \epsilon S(X, Z) \eta(Y) \\ + (n-1) g(Y, Z) \eta(X) - (n-1) g(X, Z) \eta(Y)] \\ - \frac{r}{(n-1)(n-2)}[\epsilon g(Y, Z) \eta(X) - \epsilon g(X, Z) \eta(Y)]. \end{aligned} \tag{3.3}$$

In view of the equation (2.13) and $\epsilon^2 = 1$, the above equation yields

$$\begin{aligned} & S(Y,Z)\,\eta(X) \\ (3.4)\quad & = S(X,Z)\,\eta(Y) + \left(\frac{r}{n-1} - \epsilon\right)\left(g(Y,Z)\,\eta(X) - g(X,Z)\,\eta(Y)\right). \end{aligned}$$

For $X = \xi$, we get

$$(3.5)\qquad S(Y,Z) = \left(\frac{r}{n-1} - \epsilon\right) g(Y,Z) - \left(\frac{r\epsilon + n - n^2}{n-1}\right)\eta(Y)\,\eta(Z).$$

Hence we can state the following:

Theorem 3.1. *An $(2n+1)$-dimensional $(n > 1)$ coformally flat (ϵ)-Lorentzian para-Sasakian manifold is an η-Einstein manifold.*

Using the equation (3.5) in (3.2), we get

$$\begin{aligned} & g(R(X,Y)Z,W) \\ & = \frac{1}{n-2}[\left(\frac{2r}{n-1} - 2\epsilon\right) g(Y,Z)\,g(X,W) \\ & - \left(\frac{2r}{n-1} - 2\epsilon\right) g(X,Z)\,g(Y,W)] - \left(\frac{r\epsilon + n - n^2}{(n-1)(n-2)}\right) \\ & [\eta(Y)\,\eta(Z)\,g(X,W) - \eta(X)\,\eta(Z)\,g(Y,W) \\ & + \eta(X)\,\eta(W)\,g(Y,Z) - \eta(Y)\,\eta(W)\,g(X,Z)] \\ & - \frac{r}{(n-1)(n-2)}\left[g(Y,Z)\,g(X,W) - g(X,Z)\,g(Y,W)\right]. \end{aligned}$$

The above relation can be written as

$$\begin{aligned} & g(R(X,Y)Z,W) \\ & = \frac{r - 2n\epsilon + 2\epsilon}{(n-1)(n-2)}[g(Y,Z)\,g(X,W) - g(X,Z)\,g(Y,W)] \\ & - \left(\frac{r\epsilon + n - n^2}{(n-1)(n-2)}\right)[\eta(X)\,\eta(Z)\,g(Y,W) + \eta(Y)\,\eta(W)\,g(X,Z) \\ & - \eta(X)\,\eta(W)\,g(Y,Z) - \eta(Y)\,\eta(Z)\,g(X,W)]. \end{aligned}$$

In view of Definition (2.1) and the above relation we have the following:

Theorem 3.2. *An n-dimensional coformally flat (ϵ)-Lorentzian para-Sasakian manifold is of quasi-constant curvature.*

It is also proved that a $\psi(F)_n$ contains a manifold of quasi-constant curvature as a subclass:

Let

$$F(X,Y) = \sqrt{a}\,g(X,Y) + \frac{b}{\sqrt{a}}T(X)\,T(Y).$$

Now from the equation (2.5) we know that

$$\check{R}(X,Y,Z,W) = F(Y,Z)\,F(X,W) - F(X,Z)\,F(Y,W).$$

Therefore the manifold of quasi-constant curvature is a $\psi(F)_n$.

From the above condition and Theorem 3.2 we have the following:

Theorem 3.3. *A conformally flat (ϵ)-Lorentzian para-Weyl-semisymmetric Sasakian manifold is a* $\psi(F)_n$.

4. Weyl-semisymmetric (ϵ)-Lorentzian para-Sasakian manifolds

An (ϵ)-Lorentzian para-Sasakian manifold is said to be Weyl-semisymmetric if

$$R.C = 0.$$

From the equation (3.1), we get

$$\begin{aligned} g(C(X,Y)Z,\xi) = {} & g(R(X,Y)Z,\xi) - \frac{1}{n-2}[g(Y,Z)S(X,\xi) \\ & - g(X,Z)S(Y,\xi) + S(Y,Z)g(X,\xi) \\ & - S(X,Z)g(Y,\xi)] + \frac{r}{(n-1)(n-2)} \\ & [g(Y,Z)g(X,\xi) - g(X,Z)g(Y,\xi)]. \end{aligned} \tag{4.1}$$

From the above equation, we have

$$\begin{aligned} \eta(C(X,Y)Z) = {} & \frac{1}{(n-2)}[\left(\frac{r}{n-1} - \epsilon\right)(g(Y,Z)\eta(X) \\ & - g(X,Z)\eta(Y)) - S(Y,Z)\eta(X) \\ & + S(X,Z)\eta(Y)]. \end{aligned} \tag{4.2}$$

Putting $Z = \xi$, in the above equation, we have

$$\eta(C(X,Y)\xi) = 0. \tag{4.3}$$

Again putting $X = \xi$ in the equation (4.2), we get

$$\begin{aligned} \eta(C(\xi,Y)Z) = {} & \frac{1}{n-2}[\left(\frac{r}{n-1} - \epsilon\right)(g(Y,Z) - \epsilon\eta(Y)\eta(Z)) \\ & - S(Y,Z) + (n-1)\eta(Y)\eta(Z)]. \end{aligned} \tag{4.4}$$

If the manifold is Weyl-semisymmetric, then we have

$$\begin{aligned} & g[R(\xi,Y)C(U,V)W,\xi] - g[C(R(\xi,Y)U,V)W,\xi] \\ & - g[C(U,R(\xi,Y)V,W),\xi] - g[C(U,V)R(\xi,Y)W,\xi] = 0. \end{aligned} \tag{4.5}$$

From the equation (2.12), we have

$$g(R(\xi,X)Y,\xi) = g(X,Y) - \epsilon\eta(Y\eta(X)). \tag{4.6}$$

Using the equation (4.6) in (4.5), we get

$$
\begin{aligned}
& g\left(Y, C\left(U, V\right) W\right) - \epsilon\eta\left(C\left(U, V\right) W\right) \eta\left(Y\right) \\
& - g[C\left(\epsilon g\left(Y, U\right) \xi - \eta\left(U\right) Y, V\right) W, \xi] \\
& - g[C(U, \epsilon g\left(Y, V\right) \xi - \eta\left(V\right) Y)W, \xi] \\
& - g[C\left(U, V\right)\left(\epsilon g\left(Y, W\right) \xi - \eta\left(W\right) Y\right), \xi) = 0.
\end{aligned} \tag{4.7}
$$

From the above equation, we have

$$
\begin{aligned}
& - \check{C}\left(U, V, W, Y\right) + \eta\left(Y\right) \eta\left(C\left(U, V\right) W\right) \\
& - \epsilon\eta(U)\eta\left(C\left(Y, V\right) W\right) - \epsilon\eta(V)\eta\left(C\left(U, Y\right) W\right) \\
& - \epsilon\eta(W)\eta\left(C\left(U, V\right) Y\right) + g\left(Y, U\right) \eta\left(C\left(\xi, V\right) W\right) \\
& + g\left(Y, V\right) \eta\left(C\left(U, \xi\right) W\right) + g\left(Y, W\right) \eta\left(C(U, V)\xi\right) = 0,
\end{aligned} \tag{4.8}
$$

where $\check{C}\left(U, V, W, Y\right) = g\left(C\left(U, V\right) W, Y\right)$.

Putting $Y = U$, we get

$$
\begin{aligned}
& - \check{C}\left(U, V, W, U\right) + \eta\left(U\right) \eta\left(C\left(U, V\right) W\right) \\
& (V)\eta\left(C\left(U, U\right) W\right) \\
& - \epsilon\eta(W)\eta\left(C\left(U, V\right) U\right) + g\left(U, U\right) \eta\left(C\left(\xi, V\right) W\right) \\
& + g\left(U, V\right) \eta\left(C\left(U, \xi\right) W\right) + g\left(U, W\right) \eta\left(C(U, V)\xi\right) = 0.
\end{aligned} \tag{4.9}
$$

Again putting $U = e_i$, where $\{e_i\}$ is an ortonormal basis of the tangent space at each point of the manifold, and taking summation over i where $i = 1, 2, \ldots, n$, we get

$$\sum_{i=1}^{n} \check{C}\left(e_i, V, W, e_i\right) = 0$$

and using (4.3) in (4.9), we have

$$\eta\left(C\left(\xi, V\right) W\right) = 0. \tag{4.10}$$

Using the equation (4.3) and (4.10) in (4.8), we get

$$
\begin{aligned}
& - \check{C}\left(U, V, W, Y\right) + \eta\left(Y\right) \eta\left(C\left(U, V\right) W\right) \\
& - \epsilon\eta(U)\eta\left(C\left(Y, V\right) W\right) - \epsilon\eta(V)\eta\left(C\left(U, Y\right) W\right) \\
& - \epsilon\eta(W)\eta\left(C\left(U, V\right) Y\right) = 0.
\end{aligned} \tag{4.11}
$$

Using the equation (4.2) in (4.11), we get

$$
\begin{aligned}
& - \check{C}\left(U, V, W, Y\right) - \frac{\eta\left(W\right)}{n-2}[\left(\frac{\epsilon r}{n-1} - 1\right) g\left(Y, V\right) \eta\left(U\right) \\
& - g\left(U, Y\right) \eta\left(V\right) - \epsilon\left(S\left(Y, V\right) \eta\left(U\right) - S\left(Y, U\right) \eta\left(V\right)\right)] \\
& - \frac{\left(\epsilon - 1\right)}{n-2}[\left(\frac{\epsilon r}{n-1} - 1\right) \{g\left(U, W\right) \eta\left(V\right) \eta\left(Y\right) \\
& - g\left(V, W\right) \eta\left(U\right) \eta\left(Y\right) - S\left(U, W\right) \eta\left(Y\right)\left(V\right) \\
& + S\left(V, W\right) \eta\left(U\right) \eta\left(Y\right)\} = 0.
\end{aligned} \tag{4.12}
$$

From the equation (4.10), we have from (4.4)

$$S(Y,Z) = \left(\frac{r}{n-1} - \epsilon\right) g(Y,Z) - \left(\frac{r\epsilon}{n-1} - n\right) \eta(Y)\,\eta(Z). \tag{4.13}$$

Using the equation (4.13) in (4.12)

$$\check{C}(U,V,W,Y) = 0. \tag{4.14}$$

From the above equation we see that $R.C = 0$ implies that $C = 0$. Hence using this condition with the help of Theorem 3.2 we have the following:

Theorem 4.1. *A n-dimensional Weyl-semisymmetric (ϵ)-Lorentzian para-Sasakian manifold is of quasi-constant curvature.*

Theorem 3.3 and (4.14) leads the following:

Corollary 4.1. *A n-dimensional Weyl-semisymmetric (ϵ)-Lorentzian para-Sasakian manifold is a $\psi(F)_n$.*

Application. (ϵ)-Lorentzian para-Sasakian manifolds are used in the theory of Relativity and Newtons law of gravitational field.

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Rajendra Prasad
Department of Mathematics & Astronomy
University of Lucknow
Lucknow-226007, India
E-mail address: rp.manpur@rediffmail.com

Vibha Srivastava
Department of Mathematics & Astronomy
University of Lucknow
Lucknow-226007, India
E-mail address: Vibha.one22@rediffmail.com