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CERTAIN INTEGRAL REPRESENTATIONS OF EULER TYPE FOR THE EXTON FUNCTION X_8

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Abstract. Exton introduced 20 distinct triple hypergeometric functions whose names are X_i $(i = 1, \ldots, 20)$ to investigate their twenty Laplace integral representations whose kernels include the confluent hypergeometric functions ${}_0F_1$, ${}_1F_1$, a Humbert function Ψ_1, and a Humbert function Φ_2. The object of this paper is to present 18 new integral representations of Euler type for the Exton hypergeometric function X_8, whose kernels include the Exton functions (X_2, X_8) itself, the Horn's function H_4, the Gauss hypergeometric function F, and Lauricella hypergeometric function F_C. We also provide a system of partial differential equations satisfied by X_8.

1. Introduction

Exton [4] introduced 20 distinct triple hypergeometric functions whose names are X_i $(i = 1, \ldots, 20)$ to investigate their twenty Laplace integral representations which include the confluent hypergeometric functions ${}_0F_1$, ${}_1F_1$, a Humbert function Ψ_1, a Humbert function Φ_2 in their kernels. The Exton functions X_i have been studied a lot until today, for example, see [2, 5, 6, 7, 8, 9, 10]. Here, we choose to investigate the Exton function X_8 to present (presumably new) 14 integral representations of Euler type whose kernels contain the Exton function X_2 itself, the Horn's function H_4, Gauss hypergeometric function $F = {}_2F_1$, and Lauricella hypergeometric function F_C.

Exton [4] defined the function X_8 by the following triple series

(1.1)
$$X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) = \sum_{m,n,p=0}^{\infty} \frac{(a_1)_{2m+n+p}\,(a_2)_n\,(a_3)_p}{(c_1)_m\,(c_2)_n\,(c_3)_p\, m!n!p!} x^m y^n z^p,$$

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where $(\lambda)_m$ denotes the Pochhammer symbol defined by

$$(\lambda)_m := \frac{\Gamma(\lambda+m)}{\Gamma(\lambda)} \quad \left(\lambda \in \mathbb{C}\setminus\mathbb{Z}_0^-;\ m \in \mathbb{N}_0\right),$$

$\mathbb{C}$, $\mathbb{Z}_0^-$, and $\mathbb{N}_0$ being the set of complex numbers, the set of nonpositive integers, and the set of nonnegative integers, respectively. The precise three-dimensional region of convergence of (1.1) is given by Srivastava and Karlsson [10, p. 102, 44a]:

$$\left\{2\sqrt{r}+s+t<1\right\}, \quad |x|<r,\ |y|<s,\ |z|<t,$$

where the positive quantities r, s and t are associated radii of convergence. For more details about this function and many other three-variable hypergeometric functions, we also refer to Srivastava and Karlsson [10].

It may be recalled the Laplace integral representation of (1.1) (see [4]) in passing that

$$\begin{aligned}
&X_8(a,\, b_1,\, b_2;\, c_1,\, c_2,\, c_3;\, x,\, y,\, z)\\
(1.2)\qquad &= \frac{1}{\Gamma(a)}\int_0^\infty e^{-s}s^{n-1}{}_0F_1(-;\, c_1;\, xs^2){}_1F_1(b_1;\, c_2; ys){}_1F_1(b_2;\, c_3; zs)\, ds\\
&(\Re(a)>0).
\end{aligned}$$

2. Integral representations of Euler type for X_8

Theorem 1. *Each of the following integral representations for X_8 holds true.*

$$\begin{aligned}
&X_8\,(a_1,a_2,a_3;c_1,c_2,c_3;x,y,z)\\
&= \frac{\Gamma(c_3)}{\Gamma(a_3)\,\Gamma(c_3-a_3)}\int_0^1 \xi^{a_3-1}(1-\xi)^{c_3-a_3-1}(1-z\xi)^{-a_1}\\
(2.1)\qquad &\times H_4\left(a_1,a_2;c_1,c_2;\frac{x}{(1-z\xi)^2},\frac{y}{1-z\xi}\right)d\xi\\
&(\Re(c_3)>\Re(a_3)>0);
\end{aligned}$$

$$\begin{aligned}
&X_8\,(a_1,a_2,a_3;c_1,c_2,c_3;x,y,z)\\
&= \frac{\Gamma(c_3)\,(1+\lambda)^{a_3}}{\Gamma(a_3)\,\Gamma(c_3-a_3)}\times\int_0^1 \xi^{a_3-1}(1-\xi)^{c_3-a_3-1}(1+\lambda\xi)^{a_1-c_3}\\
(2.2)\qquad &\times\left[1+\lambda\xi-(1+\lambda)\,z\xi\right]^{-a_1}H_4\left(a_1,a_2;c_1,c_2;\sigma^2x,\sigma y\right)d\xi\\
&\left(\sigma=\frac{1+\lambda\xi}{1+\lambda\xi-(1+\lambda)\,z\xi},\ \Re(c_3)>\Re(a_3)>0,\ \lambda>-1\right);
\end{aligned}$$

(2.3)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{\Gamma\left(c_3\right)(\beta-\gamma)^{a_3}(\alpha-\gamma)^{c_3-a_3}}{\Gamma\left(a_3\right) \Gamma\left(c_3-a_3\right)(\beta-\alpha)^{c_3-a_1-1}} \int_\alpha^\beta(\beta-\xi)^{c_3-a_3-1} \\
& \times(\xi-\alpha)^{a_3-1}(\xi-\gamma)^{a_1-c_3}[(\beta-\alpha)(\xi-\gamma)-(\beta-\gamma)(\xi-\alpha) z]^{-a_1} \\
& \times H_4\left(a_1, a_2; c_1, c_2; \sigma^2 x, \sigma y\right) d\xi, \\
& \left(\sigma=\frac{(\beta-\alpha)(\xi-\gamma)}{(\beta-\alpha)(\xi-\gamma)-(\beta-\gamma)(\xi-\alpha) z}, \ \Re(c_3)>\Re(a_3)>0, \quad \gamma<\alpha<\beta\right);
\end{aligned}
$$

(2.4)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{\Gamma\left(c_3\right)(\gamma-\beta)^{a_3}(\gamma-\alpha)^{c_3-a_3}}{\Gamma\left(a_3\right) \Gamma\left(c_3-a_3\right)(\beta-\alpha)^{c_3-a_1-1}} \int_\alpha^\beta(\xi-\alpha)^{a_3-1} \\
& \times(\beta-\xi)^{c_3-a_3-1}(\gamma-\xi)^{a_1-c_3}[(\beta-\alpha)(\gamma-\xi)-(\gamma-\beta)(\xi-\alpha) z]^{-a_1} \\
& \times H_4\left(a_1, a_2; c_1, c_2; \sigma^2 x, \sigma y\right) d\xi, \\
& \left(\sigma=\frac{(\beta-\alpha)(\gamma-\xi)}{(\beta-\alpha)(\gamma-\xi)-(\gamma-\beta)(\xi-\alpha) z}, \ \Re(c_3)>\Re(a_3)>0, \quad \alpha<\beta<\gamma\right);
\end{aligned}
$$

(2.5)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{2\Gamma\left(c_3\right)}{\Gamma\left(a_3\right) \Gamma\left(c_3-a_3\right)} \times \int_0^{\frac{\pi}{2}}\left(\sin^2 \xi\right)^{a_3-\frac{1}{2}}\left(\cos^2 \xi\right)^{c_3-a_3-\frac{1}{2}}\left(1-z \sin^2 \xi\right)^{-a_1} \\
& \times H_4\left(a_1, a_2; c_1, c_2; \frac{x}{\left(1-z \sin^2 \xi\right)^2}, \frac{y}{1-z \sin^2 \xi}\right) d\xi \\
& \left(\Re(a_3)>0, \quad \Re(c_3-a_3)>0\right);
\end{aligned}
$$

(2.6)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{2\Gamma\left(c_3\right)(1+\lambda)^{a_3}}{\Gamma\left(a_3\right) \Gamma\left(c_3-a_3\right)} \\
& \times \int_0^{\frac{\pi}{2}} \frac{\left(\sin^2 \xi\right)^{a_3-\frac{1}{2}}\left(\cos^2 \xi\right)^{c_3-a_3-\frac{1}{2}}}{\left(1+\lambda \sin^2 \xi\right)^{c_3-a_1}}\left[1+\lambda \sin^2 \xi-(1+\lambda) z \sin^2 \xi\right]^{-a_1} \\
& \times H_4\left(a_1, a_2; c_1, c_2; \sigma^2 x, \sigma y\right) d\xi \\
& \left(\sigma=\frac{1+\lambda \sin^2 \xi}{1+\lambda \sin^2 \xi-(1+\lambda) z \sin^2 \xi}, \quad \Re(c_3)>\Re(a_3)>0, \ \lambda>-1\right);
\end{aligned}
$$

(2.7)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{2\Gamma\left(c_3\right)\lambda^{a_3}}{\Gamma\left(a_3\right)\Gamma\left(c_3-a_3\right)} \\
& \times \int_0^{\frac{\pi}{2}} \frac{\left(\sin^2\xi\right)^{a_3-\frac{1}{2}}\left(\cos^2\xi\right)^{c_3-a_3-\frac{1}{2}}}{\left(\cos^2\xi+\lambda\sin^2\xi\right)^{c_3-a_1}}\left[\cos^2\xi+\lambda\sin^2\xi-\lambda z\sin^2\xi\right]^{-a_1} \\
& \quad H_4\left(a_1, a_2; c_1, c_2; \sigma^2 x, \sigma y\right) d\xi \\
& \left(\sigma=\frac{\cos^2\xi+\lambda\sin^2\xi}{\cos^2\xi+\lambda\sin^2\xi-\lambda z\sin^2\xi},\ \ \Re(a_3)>0,\ \ \Re(c_3-a_3)>0,\ \ \lambda>0\right);
\end{aligned}
$$

(2.8)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
& = \frac{\Gamma\left(c_2\right)\Gamma\left(c_3\right)}{\Gamma\left(a_2\right)\Gamma\left(a_3\right)\Gamma\left(c_2-a_2\right)\Gamma\left(c_3-a_3\right)} \\
& \quad \times \int_0^1\int_0^1 \xi^{a_3-1}\eta^{a_2-1}(1-\xi)^{c_3-a_3-1}(1-\eta)^{c_2-a_2-1}(1-y\eta-z\xi)^{-a_1} \\
& \quad \times F\left(\frac{a_1}{2}, \frac{a_1}{2}+\frac{1}{2}; c_1; \frac{4x}{(1-y\eta-z\xi)^2}\right) d\xi d\eta \\
& \quad \left(\Re(c_2)>\Re(a_2)>0,\ \ \Re(c_3)>\Re(a_3)>0\right);
\end{aligned}
$$

(2.9)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
= & \frac{\Gamma\left(c_2\right)\Gamma\left(c_3\right)\left(1+\lambda_1\right)^{a_3}\left(1+\lambda_2\right)^{a_2}}{\Gamma\left(a_2\right)\Gamma\left(a_3\right)\Gamma\left(c_2-a_2\right)\Gamma\left(c_3-a_3\right)} \\
& \times \int_0^1\int_0^1 \xi^{a_3-1}\eta^{a_2-1}(1-\xi)^{c_3-a_3-1}(1-\eta)^{c_2-a_2-1}\left(1+\lambda_1\xi\right)^{a_1-c_3}\left(1+\lambda_2\eta\right)^{a_1-c_2} \\
& \times\left[\left(1+\lambda_1\xi\right)\left(1+\lambda_2\eta\right)-\left(1+\lambda_2\right)\left(1+\lambda_1\xi\right)y\eta-\left(1+\lambda_1\right)\left(1+\lambda_2\eta\right)z\xi\right]^{-a_1} \\
& \times F\left(\frac{a_1}{2}, \frac{a_1}{2}+\frac{1}{2}; c_1; \sigma x\right) d\xi d\eta \\
& \left(\sigma=\frac{4\left(1+\lambda_1\xi\right)^2\left(1+\lambda_2\eta\right)^2}{\left[\left(1+\lambda_1\xi\right)\left(1+\lambda_2\eta\right)-\left(1+\lambda_2\right)\left(1+\lambda_1\xi\right)y\eta-\left(1+\lambda_1\right)\left(1+\lambda_2\eta\right)z\xi\right]^2},\right. \\
& \quad \left.\Re(c_2)>\Re(a_2>0),\ \ \Re(c_3)>\Re(a_3)>0,\ \ \lambda_1>-1,\ \ \lambda_2>-1\right);
\end{aligned}
$$

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
& = \frac{\Gamma\left(a_1+a_2\right)}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)} \int_0^1 \xi^{a_1-1}(1-\xi)^{a_2-1} \\
& \times X_2\left(a_1+a_2, a_3; c_1, c_2, c_3; x\xi^2, y\xi(1-\xi), z\xi\right) d\xi \\
& \left(\Re(a_1)>0,\ \ \Re(a_2)>0\right);
\end{aligned} \tag{2.10}
$$

(2.11)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
& = \frac{\Gamma\left(a_1+a_2\right)(1+\lambda)^{a_1}}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)} \int_0^1 \xi^{a_1-1}(1-\xi)^{a_2-1}(1+\lambda\xi)^{-a_1-a_2} \\
& \times X_2\left(a_1+a_2, a_3; c_1, c_2, c_3; \frac{(1+\lambda)^2\xi^2 x}{(1+\lambda\xi)^2}, \frac{(1+\lambda)\xi(1-\xi)y}{(1+\lambda\xi)^2}, \frac{(1+\lambda)\xi z}{(1+\lambda\xi)}\right) d\xi \\
& \left(\Re(a_1)>0,\quad \Re(a_2)>0,\quad \lambda>-1\right);
\end{aligned}
$$

(2.12)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
& = \frac{\Gamma\left(a_1+a_2\right)}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)} \frac{(\beta-\gamma)^{a_1}(\alpha-\gamma)^{a_2}}{(\beta-\alpha)^{a_1+a_2-1}} \times \int_\alpha^\beta (\beta-\xi)^{a_2-1}(\xi-\alpha)^{a_1-1}(\xi-\gamma)^{-a_1-a_2} \\
& \times X_2\left(a_1+a_2, a_3; c_1, c_2, c_3; \sigma_1 x, \sigma_2 y, \sigma_3 z\right) d\xi \\
& \left(\sigma_1 = \frac{(\beta-\gamma)^2(\xi-\alpha)^2}{(\beta-\alpha)^2(\xi-\gamma)^2},\ \sigma_2 = \frac{(\alpha-\gamma)(\beta-\gamma)(\xi-\alpha)(\beta-\xi)}{(\beta-\alpha)^2(\xi-\gamma)^2},\right. \\
& \quad \left.\sigma_3 = \frac{(\beta-\gamma)(\xi-\alpha)}{(\beta-\alpha)(\xi-\gamma)} \Re(a_1)>0,\quad \Re(a_2)>0,\quad \gamma<\alpha<\beta\right);
\end{aligned}
$$

(2.13)

$$
\begin{aligned}
& X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right) \\
& = \frac{\Gamma\left(a_1+a_2\right)}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)} \frac{(\gamma-\beta)^{a_1}(\gamma-\alpha)^{a_2}}{(\beta-\alpha)^{a_1+a_2-1}} \times \int_\alpha^\beta (\beta-\xi)^{a_2-1}(\xi-\alpha)^{a_1-1}(\gamma-\xi)^{-a_1-a_2} \\
& \times X_2\left(a_1+a_2, a_3; c_1, c_2, c_3; \sigma_1 x, \sigma_2 y, \sigma_3 z\right) d\xi \\
& \left(\sigma_1 = \frac{(\gamma-\beta)^2(\xi-\alpha)^2}{(\beta-\alpha)^2(\gamma-\xi)^2},\ \sigma_2 = \frac{(\gamma-\alpha)(\gamma-\beta)(\xi-\alpha)(\beta-\xi)}{(\beta-\alpha)^2(\gamma-\xi)^2},\right. \\
& \quad \left.\sigma_3 = \frac{(\gamma-\beta)(\xi-\alpha)}{(\beta-\alpha)(\gamma-\xi)} \Re(a_1)>0,\quad \Re(a_2)>0,\quad \alpha<\beta<\gamma\right);
\end{aligned}
$$

$$
\begin{aligned}
&X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right)\\
&= \frac{\Gamma\left(a_1+a_2+a_3\right)}{\Gamma\left(a_1\right)\Gamma\left(a_2\right)\Gamma\left(a_3\right)} \int_0^1 \int_0^1 \xi^{a_1-1} \eta^{a_1+a_2-1} \left(1-\xi\right)^{a_2-1} \left(1-\eta\right)^{a_3-1}\\
&\times F_C\Big[\frac{a_1+a_2+a_3}{2}, \frac{a_1+a_2+a_3}{2}\\
&\qquad + \frac{1}{2}; c_1, c_2, c_3; 4x\xi^2\eta, 4y\xi\eta^2\left(1-\xi\right), 4z\xi\eta\left(1-\eta\right)\Big] d\xi d\eta\\
&\left(\Re(a_1)>0,\ \ \Re(a_2)>0,\ \ \Re(a_3)>0\right);
\end{aligned} \tag{2.14}
$$

$$
\begin{aligned}
&X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right)\\
&= \frac{\Gamma\left(a_2+a_3\right)}{\Gamma\left(a_2\right)\Gamma\left(a_3\right)} \int_0^1 \xi^{a_2-1} \left(1-\xi\right)^{a_3-1}\\
&\times X_4\left[a_1, a_2+a_3; c_1, c_2, c_3; x, y\xi, z\left(1-\xi\right)\right] d\xi\\
&\left(\Re(a_2)>0,\ \ \Re(a_3)>0\right);
\end{aligned} \tag{2.15}
$$

$$
\begin{aligned}
&X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right)\\
&= \frac{\Gamma\left(s\right)}{\Gamma\left(a_1\right)\Gamma\left(s-a_1\right)}\\
&\times \int_0^1 \xi^{a_1-1} \left(1-\xi\right)^{s-a_1-1} X_8\left(s, a_2, a_3; c_1, c_2, c_3; x\xi^2, y\xi, z\xi\right) d\xi\\
&\left(\Re(s)>\Re(a_1)>0\right);
\end{aligned} \tag{2.16}
$$

$$
\begin{aligned}
&X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right)\\
&= \frac{\Gamma\left(s\right)}{\Gamma\left(a_2\right)\Gamma\left(s-a_2\right)}\\
&\times \int_0^1 \xi^{a_2-1} \left(1-\xi\right)^{s-a_2-1} X_8\left(a_1, s, a_3; c_1, c_2, c_3; x, \xi y, z\right) d\xi\\
&\left(\Re(s)>\Re(a_2)>0\right);
\end{aligned} \tag{2.17}
$$

$$
\begin{aligned}
&X_8\left(a_1, a_2, a_3; c_1, c_2, c_3; x, y, z\right)\\
&= \frac{\Gamma\left(c_1\right)}{\Gamma\left(c_1-s\right)\Gamma\left(s\right)}\\
&\times \int_0^1 \xi^{s-1} \left(1-\xi\right)^{c_1-s-1} X_8\left(a_1, a_2, a_3; s, c_2, c_3; x\xi, y, z\right) d\xi\\
&\left(\Re(c_1)>\Re(s)>0\right),
\end{aligned} \tag{2.18}
$$

where $F = {}_2F_1$ *is the Gauss hypergeometric function,* H_4, $(X_2,\ X_8)$ *and* F_C *denote the Horn's, Exton and Lauricella functions defined, respectively, by*

$$H_4\left(\alpha,\beta;\gamma,\varepsilon;x,y\right)=\sum_{m,n=0}^{\infty}\frac{(\alpha)_{2m+n}\,(\beta)_n}{(\gamma)_m\,(\varepsilon)_n}\,\frac{x^m\,y^n}{m!\,n!},$$

$$X_2\left(a_1,a_2;c_1,c_2,c_3;x,y,z\right)=\sum_{m,n,p=0}^{\infty}\frac{(a_1)_{2m+2n+p}\,(a_2)_p}{(c_1)_m\,(c_2)_n\,(c_3)_p\,m!n!p!}x^m y^n z^p,$$

$$F_C\left(a,b;c_1,c_2,c_3;x,y,z\right)=\sum_{m,n,p=0}^{\infty}\frac{(a)_{m+n+p}\,(b)_{m+n+p}}{(c_1)_m\,(c_2)_n\,(c_3)_p\,m!n!p!}x^m y^n z^p.$$

3. Proof of results

It is noted that each of the integral representations in Section 2 can be proved mainly by expressing the series definition of the involved special function in each integrand and changing the order of the integral sign and the summation, and finally using the following well-known relationship between the Beta function $B(\alpha,\beta)$ and the Gamma function Γ:

$$B(\alpha,\,\beta):=\begin{cases}\displaystyle\int_0^1 t^{\alpha-1}(1-t)^{\beta-1}\,dt & (\Re(\alpha)>0;\ \Re(\beta)>0)\\[2mm] \dfrac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\alpha,\,\beta\in\mathbb{C}\setminus\mathbb{Z}_0^-).\end{cases} \tag{3.1}$$

4. A system of partial differential equations

Here we just provide a system of partial differential equations satisfied by Exton function X_8.

Theorem 2. $u = X_8$ *satisfies the following system of partial differential equations*:

$$\begin{cases} x(1-4x)u_{xx}-4xyu_{xy}-4xzu_{xz}-y^2u_{yy}++[c_1-2(2a_1+3)x]u_x\\ \quad -2(a_1+1)yu_y-2(a_1+1)zu_z-a_1(a_1+1)u=0,\\ y(1-y)u_{yy}-2xyu_{xy}-yzu_{yz}-2a_2xu_x+[c_2-(a_1+a_2+1)y]u_y\\ \quad -a_2zu_z-a_1a_2u=0,\\ z(1-z)u_{zz}-2xzu_{xz}-yzu_{yz}-2a_3xu_x-2a_3yu_y+[c_3(a_1+a_3+1)z]u_z\\ \quad -a_1a_3u=0.\end{cases}$$

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