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FILTERS IN COMMUTATIVE BE-ALGEBRAS

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Abstract. The notions of terminal sections of BE-algebras are introduced. Characterizations of a commutative BE-algebra are provided.

1. Introduction

Y. Imai and K. Iséki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras ([4, 5]). It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [3, 4] Q. P. Hu and X. Li introduced a wide class of abstract algebras: BCH-algebras. They have shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. J. Neggers and H. S. Kim ([13]) introduced the notion of a d-algebra which is a generalization of BCK-algebras, and also they introduced the notion of a B-algebra ([14, 15]), i.e., (I) $x * x = 0$; (II) $x * 0 = x$; (III) $(x * y) * z = x*(z*(0*y))$, for any $x, y, z \in X$, which is equivalent in some sense to the groups. Moreover, Y. B. Jun, E. H. Roh and H. S. Kim ([7]) introduced a new notion, called an *BH-algebra*, which is another generalization of $BCH/BCI/BCK$-algebras, i.e., (I); (II) and (IV) $x * y = 0$ and $y * x = 0$ imply $x = y$ for any $x, y \in X$. A. Walendziak obtained another equivalent axioms for B-algebra ([16]). H. S. Kim, Y. H. Kim and J. Neggers ([10]) introduced the notion a (pre-)Coxeter algebra and showed that a Coxeter algebra is equivalent to an abelian group all of whose elements have order 2, i.e., a Boolean group. C. B. Kim and H. S. Kim ([8]) introduced the notion of a BM-algebra which is a specialization of B-algebras. They proved that the class of BM-algebras is a proper subclass of B-algebras and also showed that a BM-algebra is equivalent to a 0-commutative B-algebra. In [9], H. S. Kim and Y. H. Kim introduced the notion of a BE-algebra as a generalization of a BCK-algebra. Using the notion of upper sets they gave an equivalent condition of the filter in BE-algebras. In [1, 2], S. S. Ahn and K. S. So introduced the notion of ideals in BE-algebras, and proved several characterizations of such ideals. Also they generalized the notion of upper sets in BE-algebras, and discussed some properties of the

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characterizations of generalized upper sets $A_n(u, v)$ related to the structure of ideals in transitive and self distributive BE-algebras.

In this paper, we introduce the notion of a terminal section of BE-algebras, and give some characterizations of commutative BE-algebras in terms of lattices order relations, and terminal sections.

2. Preliminaries

We recall some definitions and results discussed in [1, 2, 9, 11, 17].

Definition 2.1 ([9])**.** An algebra $(X; *, 1)$ of type (2, 0) is called a *BE-algebra* if

(BE1) $x * x = 1$ for all $x \in X$;
(BE2) $x * 1 = 1$ for all $x \in X$;
(BE3) $1 * x = x$ for all $x \in X$;
(BE4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$ (*exchange*).

We introduce a relation "$\leq$" on X by $x \leq y$ if and only if $x * y = 1$.

Proposition 2.2 ([9])**.** *If $(X; *, 1)$ is a BE-algebra, then $x * (y * x) = 1$ for any $x, y \in X$.*

Example 2.3 ([9])**.** Let $X := \{1, a, b, c, d, 0\}$ be a set with the following table:

$*$	1	a	b	c	d	0
1	1	a	b	c	d	0
a	1	1	a	c	c	d
b	1	1	1	c	c	c
c	1	a	b	1	a	b
d	1	1	a	1	1	a
0	1	1	1	1	1	1

Then $(X; *, 1)$ is a BE-algebra.

Definition 2.4 ([9])**.** Let $(X; *, 1)$ be a BE-algebra and let F be a non-empty subset of X. Then F is called a *filter* of X if

(F1) $1 \in F$;
(F2) $x * y \in F$ and $x \in F$ imply $y \in F$.

In Example 2.3, $F_1 := \{1, a, b\}$ is a filter of X, but $F_2 := \{1, a\}$ is not a filter of X, since $a * b \in F_2$ and $a \in F_2$, but $b \notin F_2$.

Proposition 2.5 ([9])**.** *Let X be a BE-algebra and let F be a filter of X. If $x \leq y$ and $x \in F$ for any $y \in X$, then $y \in F$.*

Definition 2.6. A BE-algebra $(X; *, 1)$ is said to be *self distributive* if $x * (y * z) = (x * y) * (x * z)$ for all $x, y, z \in X$.

Example 2.7 ([9])**.** Let $X := \{1, a, b, c, d\}$ be a set with the following table:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

Then it is easy to see that X is a self distributive BE-algebra.

Note that the BE-algebra in Example 2.3 is not self distributive, since $d * (a * 0) = d * d = 1$, while $(d * a) * (d * 0) = 1 * a = a$.

Proposition 2.8. *Let X be a self distributive BE-algebra. If $x \leq y$, then $z * x \leq z * y$ and $y * z \leq x * z$ for any $x, y, z \in X$.*

Proof. Straightforward. □

Definition 2.9 ([11])**.** A *dual BCK-algebra* is an algebra $(X; *, 1)$ of type (2,0) satisfying (BE1), (BE2), and the following axioms:
(dBCK1) $x * y = y * x = 1 \Rightarrow x = y$;
(dBCK2) $(x * y) * ((y * z) * (x * z)) = 1$;
(dBCK3) $x * ((x * y) * y) = 1$.

Proposition 2.10 ([17])**.** *Any dual BCK-algebra is a BE-algebra.*

It is known that the converse of Proposition 2.10 does not hold in general ([17]).

Definition 2.11 ([17])**.** Let X be a BE-algebra or a dual BCK-algebra. X is said to be *commutative* if the following identity holds

(C) $(x * y) * y = (y * x) * x$, i.e., $x \vee_B y = y \vee_B x$ where $x \vee_B y = (y * x) * x$

for all $x, y \in X$.

Theorem 2.12 ([17])**.** *If X is a commutative BE-algebra, then $(X; *, 1)$ is a dual BCK-algebra.*

Corollary 2.13 ([17])**.** *X is a commutative BE-algebra if and only if it is a commutative dual BCK-algebra.*

If X is a commutative BE-algebra, then the relation " $\leq$ " is a partial order on X (see [17]).

3. Filter in commutative BE-algebras

In what follows, let X be a BE-algebra unless otherwise specified.

Theorem 3.1. *For any filter F of a self-distributive BE-algebra X and any $a \in X$, the set $F_a := \{x \in X \mid a * x \in F\}$ is the least filter of X containing F and a.*

Proof. It follows from (BE2) that $a * 1 = 1$ for any $a \in X$, i.e., $1 \in F_a$. Using (BE1), we have $a * a = 1 \in F$ for any $a \in F$ and so $a \in F_a$. Let $x \in F_a$ and $x*y \in F_a$. Then $a*x \in F$ and $a*(x*y) \in F$. Since $a*(x*y) = (a*x)*(a*y) \in F$ and $a * x \in F$, we obtain $a * y \in F$. Hence $y \in F_a$. This proves that F_a is a filter of X. Let $x \in F$. Since $x * (a * x) = 1 \in F$ and F is a filter of X, we get $a * x \in F$. Hence $x \in F_a$. Let H be any filter of X containing F and a. Let $x \in F_a$. Then $a * x \in F \subseteq H$. Since $a \in H$ and H is a filter of X, we have $x \in H$. Therefore $F_a \subseteq H$. Thus F_a is the least filter of X containing F and a. □

Suppose that $\mathcal{F}(X)$ is the set of all filters of a BE-algebra X. Then $F = \cap\mathcal{F}(X)$ is also a filter of X. Let A be a non-empty subset of a BE-algebra X. The least filter containing A is called the *filter generated by* A, written $\langle A\rangle$. If $A = \{a\}$, we will denote $\langle\{a\}\rangle$, briefly by $\langle a\rangle$, and we call it a *principal filter* of X.

Theorem 3.2. *If A is a non-empty subset of a self-distributive BE-algebra X, then*

$$\langle A\rangle = \{x \in X \mid a_n * (\cdots * (a_1 * x) \cdots) = 1 \text{ for some } a_1, \ldots, a_n \in A\}.$$

Proof. We define a set B as follows.

$$B := \{x \in X \mid a_n * (\cdots * (a_1 * x) \cdots) = 1 \text{ for some } a_1, \ldots, a_n \in A\}.$$

We first prove that B is a filter of X. Clearly $1 \in B$. Let $x \in B$ and $x * y \in B$. Then there exist $a_1, \ldots, a_n \in A$ and $b_1, \ldots, b_m \in A$ such that $a_n * (\cdots * (a_1 * x) \cdots) = 1$ and $b_m * (\cdots * (b_1 * (x * y)) \cdots) = 1$. Hence $1 = b_m * (\cdots * (b_1 * (x * y)) \cdots) = b_m * (\cdots * (x * (b_1 * y)) \cdots) = \cdots = x * (b_m * (\cdots * (b_1 * y) \cdots))$ and so $x \le b_m * (\cdots * (b_1 * y) \cdots)$. It follows from Proposition 2.8 that $1 = a_n * (\cdots * (a_1 * x) \cdots) \le a_n * (\cdots * (a_1 * (b_m * (\cdots * (b_1 * y) \cdots))) \cdots)$. Since 1 is the greatest element of X, $a_n * (\cdots * (a_1 * (b_m * (\cdots * (b_1 * y) \cdots))) \cdots) = 1$. Hence $y \in B$. Thus B is a filter of X. Obviously, $A \subseteq B$.

Finally we prove that B is the least filter of X containing A. Let F be a filter of X containing A. Assume $x \in B$. Then there exist $a_1, \ldots, a_n \in A$ such that $a_n * (\cdots * (a_1 * x) \cdots) = 1 \in F$. Since $A \subseteq F$ and $a_n \in A$, we have $a_n \in F$. Applying (F2), we obtain $a_{n-1} * (\cdots * (a_1 * x)) \cdots) \in F$. Repeating the above argument, we conclude that $x \in F$. This shows that $B \subseteq F$. Therefore B is the least filter of X containing A. The proof is complete. □

For any x, y in a BE-algebra X, A. Walendziak defined $x \vee_B y$ by $(y * x) * x$. Under this definition, using (BE1), (BE2) and (BE4), we have

$$\begin{aligned} x * (x \vee_B y) &= x * ((y * x) * x) \\ &= (y * x) * (x * x) \\ &= (y * x) * 1 = 1, \end{aligned}$$

i.e., $x \le x \vee_B y$. Since $y*((y*x)*x) = 1$ for any $x, y \in X$, we obtain $y \le x \vee_B y$. Hence $x \vee_B y$ is an upper bound of x and y. As easily seen, we have

(c_1) $x \vee_B x = x$ and $x \vee_B 1 = 1 \vee_B x = 1$.

Example 3.3. Let $X := \{1, a, b, c, d\}$ be a set with the following table:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	1	1	c	d
c	1	1	1	1	d
d	1	1	b	c	1

It is easy to check that X is a self distributive BE-algebra, and $a \vee_B d = a \neq 1 = d \vee_B a$ and a is the least upper bound of a and d. In general, $x \vee_B y \neq y \vee_B x$ and $x \vee_B y$ may not be the least upper bound of x and y. In fact, $d \vee_B c = (c * d) * d = d * d = 1 \neq a = \sup\{d, c\}$.

Example 3.4. Let $X := \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	a	b	1

It is easy to show that $(X; *, 1)$ is a self-distributive commutative BE-algebra.

Proposition 3.5. *Let X be a commutative BE-algebra. If $x \leq y$, then $y * z \leq x * z$ for any $x, y, z \in X$.*

Proof. Let $x \leq y$. Then $x * y = 1$ and so we have

$$\begin{aligned}(y * z) * (x * z) &= x * ((y * z) * z) \\ &= x * ((z * y) * y) \\ &= (z * y) * (x * y) \\ &= (z * y) * 1 \\ &= 1.\end{aligned}$$

Hence $y * z \leq x * z$. □

Remark. In Example 3.3, we have seen that the notation $x \vee_B y = (y * x) * x$ is not equal to the least upper bound $\sup\{x, y\}$ of x and y in a non-commutative BE-algebra. In lattice theory, there are two different definitions of a semilattice: One is an order type and the other is an algebraic type, and it was proved that they are equivalent. A. Walendziak gave an algebraic type of the proof of Theorem 3.6. We give an order type proof of Theorem 3.6 as follows:

Theorem 3.6 ([17])**.** *If $(X; *, 1)$ is a commutative BE-algebra X, then it is a semilattice with respect to $\vee_B$.*

Proof. Assume that X is a commutative BE-algebra. As already seen, $x \vee_B y$ is an upper bound of x and y. We shall show that $x \vee_B y$ is the least upper bound of x and y. To do this, suppose that $x \leq z$ and $y \leq z$. Then $x * z = y * z = 1$. Hence by commutativity we have (i): $z = 1 * z = (x * z) * z = (z * x) * x$ and (ii): $z = 1 * z = (y * z) * z = (z * y) * y$. Using (i) and (ii), we have (iii): $z = (z * x) * x = (((z * y) * y) * x) * x$. Set $u := (z * y) * y$. Then $z = (u * x) * x$ follows from (iii). Using (BE1), (BE2), and (BE4), we have $1 = (z*y)*1 = (z*y)*(y*y) = y*((z*y)*y) = 1$ and hence $y \leq (z*y)*y = u$. It follows from Proposition 3.5 that $u * x \leq y * x$. Using Proposition 3.5, we get $(y * x) * x \leq (u * x) * x = z$. Hence we obtain $x \vee_B y \leq z$, which shows that $x \vee_B y$ is the least upper bound of x and y. Therefore we have the associative law with respect to $\vee_B$. Consequently, X is a semilattice with respect to $\vee_B$. The proof is complete. □

Example 3.7. Let $X := \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

Then X is a commutative BE-algebra which is a semilattice with respect to $\vee_B$.

The converse of Theorem 3.6 need not be true in general. See the following example.

Example 3.8. (1) Let $X := \{1, a, b, c\}$ be a set with the following table:

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	1	1	c
c	1	1	b	1

It can be seen that $(X; *, 1)$ is a self-distributive BE-algebra and $(X, \vee)$ is a semilattice where $x \vee y = \sup\{x, y\}$, but X is not commutative, since $a \vee_B b = a \neq 1 = b \vee_B a$.

(2) Consider a 4-element Boolean algebra $A := \{0, 1, a, a'\}$ with the partial order $\leq$. If we define

$$x * y := \begin{cases} 1 & \text{if } x \leq y, \\ y & \text{otherwise,} \end{cases}$$

then $(A; *, 1)$ is a self-distributive BE-algebra and $(X, \vee)$ is a semilattice where $x \vee y = \sup\{x, y\}$. But it is not commutative, since $(a*0)*0 = 1 \neq a = (0*a)*a$, i.e., $a \vee_B 0 \neq 0 \vee_B a$.

Theorem 3.9. *If a BE-algebra X is commutative, then following properties hold: for all $x, y, z \in X$,*

(c_2) $y * (x \vee_B z) = (z * x) * (y * x)$;

(c_3) $x \leq y$ *implies* $x \vee_B y = y$;

(c_4) $y \vee_B x = x \vee_B (y \vee_B x)$, *i.e.*, $(x * y) * y = (((x * y) * y) * x) * x$;

(c_5) $z \leq x$ *and* $x * z \leq y * z$ *imply* $y \leq x$.

Proof. (c_2) Using (BE4), we obtain $y*(x\vee_B z) = y*((z*x)*x) = (z*x)*(y*x)$.

(c_3) If $x \leq y$, then $y = 1*y = (x*y)*y = y\vee_B x$. Hence by the commutativity, $x \leq y$ implies $x \vee_B y = y$.

(c_4) Since $x \leq y \vee_B x$, by (c_3) we obtain $x \vee_B (y \vee_B x) = y \vee_B x$.

(c_5) If $z \leq x$ and $x * z \leq y * z$, then $z * x = 1$ and $(x * z) * (y * z) = 1$. Using (BE3), (BE4) and (C), we obtain

$$\begin{aligned} y * x =& y * (1 * x) \\ =& y * ((z * x) * x) \\ =& y * ((x * z) * z) \\ =& (x * z) * (y * z) \\ =& 1, \end{aligned}$$

which implies that $y \leq x$. □

For an element a of a BE-algebra X, we consider a set

$$\{x \in X \mid a \leq x\},$$

denoted by $H(a)$, which is called the *terminal section* of an element a. Since $1, a \in H(a)$, $H(a)$ is not an empty set. Using this notation, we can characterize a commutative BE-algebra.

Theorem 3.10. *If a BE-algebra X is commutative, then it satisfies the following:*

(c_6) $H(a) \cap H(b) = H(a \vee_B b)$

for all $a, b \in X$.

Proof. Let X be a commutative BE-algebra and let $a, b \in X$. If $x \in H(a) \cap H(b)$, then $a \leq x$ and $b \leq x$. Hence $a\vee_B b \leq x$, which implies that $x \in H(a\vee_B b)$. Hence $H(a) \cap H(b) \subseteq H(a \vee_B b)$. Now if $x \in H(a \vee_B b)$, then $a \vee_B b \leq x$. Since $a \vee_B b$ is an upper bound of a and b, it follows that $a \leq x$ and $b \leq x$, i.e., $x \in H(a)$ and $x \in H(b)$. Hence $x \in H(a) \cap H(b)$. Therefore (c_6) holds. □

Lemma 3.11. *Let X be a self-distributive commutative BE-algebra and let $a, b, p \in X$. Then the following hold.*

(1) $p \leq a$ *implies* $(a * p) * a = a$;

(2) $p \leq b$ *implies* $a * b = (a * p) \vee_B b$.

Proof. (1) Suppose that $p \leq a$. Then $p * a = 1$ and $(p * a) * a = 1 * a = a$. Since $a \vee_B p = p \vee_B a$, we have $(a * p) * p = a$. Using (BE3), since X is self-distributive, we have

$$\begin{aligned}(a * p) * a =&(a * p) * [(a * p) * p] \\ =&[(a * p) * (a * p)] * [(a * p) * p] \\ =&1 * [(a * p) * p] \\ =&1 * a = a.\end{aligned}$$

(2) If $p \leq b$, then

$$\begin{aligned}(a * p) \vee_B b =&[b * (a * p)] * (a * p) \\ =&[a * (b * p)] * (a * p) \\ =&a * [(b * p) * p] \\ =&a * (b \vee_B p) = a * b.\end{aligned}$$ □

Given $x, y \in X$, we define a set $[x, y] := \{z \in X \mid x \leq z, z \leq y\}$, which is called an *interval* in X. The following theorem describes intervals in self-distributive commutative BE-algebras.

Theorem 3.12. *Let X be a self-distributive commutative BE-algebra. For every $p \in X$, the interval $[p, 1]$ is a Boolean algebra where $a, b \in [p, 1]$ we have $a \vee_B b = (a * b) * b$ and $a \wedge_B b = [a * (b * p)] * p$, and the complement of a is $a^p = a * p$.*

Proof. The first assertion follows from Theorem 3.6. Let us prove that $a \wedge_B b = [a * (b * p)] * p$. Clearly, $[(a * (b * p)] * p \in [p, 1]$. By Lemma 3.11(2), we have $a * (b * p) = (a * p) \vee_B (b * p)$. Since $a * p \leq (a * p) \vee_B (b * p)$, by Proposition 2.8 we get

$$\begin{aligned}[a * (b * p)] * p =&[(a * p) \vee_B (b * p)] * p \\ \leq&(a * p) * p = a \vee_B p = a.\end{aligned}$$

Hence $(a * (b * p)) * p \leq a$. By a similar way, we can show $(a * (b * p)) * p \leq b$ and hence $(a * (b * p)) * p$ is a lower bound of a and b. Suppose $q \in [p, 1]$, $q \leq a$ and $q \leq b$. Then applying Proposition 2.8 again we have $a * p \leq q * p$, $b * p \leq q * p$, and hence $(a * p) \vee_B (b * p) \leq q * p$. Furthermore, this gives

$$\begin{aligned}q \leq q \vee_B p =&(q * p) * p \\ \leq&[(a * p) \vee_B (b * p)] * p \\ =&[a * (b * p)] * p.\end{aligned}$$

Thus $[a * (b * p)] * p$ is the greatest lower bound of a and b in $[p, 1]$. Let us prove that $a^p := a * p$ is a complement of $a \in [p, 1]$ in this interval. By Lemma 3.11(1) we have also

$$a \vee_B (a * p) = [(a * p) * a] * a = a * a = 1.$$

Since $p \leq a * p$, we have

$$\begin{aligned} a \wedge_B (a * p) =& [a * ((a * p) * p)] * p \\ =& [(a * p) * (a * p)] * p \\ =& 1 * p = p. \end{aligned}$$

Moreover, $a^{pp} = (a * p) * p = a \vee_B p = a$. If we prove that a^p is simultaneously a pseudocomplement of a in $[p, 1]$, then by the previous property every element of this interval is Boolean and so $[p, 1]$ is a Boolean algebra. Suppose that $b \in [p, 1]$ with $a \wedge_B b = p$, i.e., $[a * (b * p)] * p = p$. Then we have

$$\begin{aligned} a^p =& a * p = a * [(a * (b * p)) * p] \\ =& [a * (b * p)] * (a * p) \\ =& a * [(b * p) * p] \\ =& a * (b \vee_B p) = a * b, \end{aligned}$$

hence $b * a^p = b * (a * b) = 1$, and therefore $b \leq a^p$. □

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