A New Charge Analysis Derived From the Results of Semi-Emprical Mo-Lcao Calculation

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ABSTRACT. In this study we present a new approach for computing the partial atomic charge derived from the wavefunctions of molecules. This charge, which we call the "y_charge", was calculated by taking into account the energy level and orbital populations in each molecular orbital (MO). The charge calculations were performed in the software, which was developed by us, developed using the C# programming language. Partial atomic charges cannot be calculated directly from quantum mechanics. According to a partitioning function, the electron density of constituent molecular atoms depends on the electrostatic attraction field of the nucleus. Taking into account the Boltzmann population of each MO as a function of its energy and temperature we obtain a formula of partial charges.

Key words: Partial atomic charges, Predictions of molecular properties, Coulomb potential

INTRODUCTION

The partial atomic charge is a fundamental concept in the molecular modeling of chemical systems and defines the identity of a molecule. Partial atomic charges are considered as an extremely powerful tool to describe chemical reactivity, physical properties and the biological activity of molecules.[1-4]

There exist several different methods for deriving partial charges. Some of these methods are purely empirical, making use of experimental quantities (e.g. enthalpy of vaporization, density of liquids, or crystal properties),[5-10] whereas many current methods employ quantum mechanics. A good overview of partial atomic charges and the different procedures to derive them has been given by Wiberg and Rablen.[11] Truhlar *et al*. reported the partial atomic charges calculated with the Mulliken and Löwdin schemes for a wide variety of systems in their effort to develop the redistributed Löwdin population analysis (RLPA).[12] Although straightforward, the resultant charges from a Mulliken analysis have been shown to be highly basis set dependent, and are unsatisfactory for deriving electrostatic potentials and intermolecular interaction energies.[13] Other partitioning protocols, which are less basis set dependent, include modifications to the Mulliken approach,[14] as well as Löwdin population analysis[15] and natural population analysis.[16]

Most recently, Baerends and coworkers reported charge parameters for a wide variety of systems using the newly formulated Voronoi deformation density (VDD) method.[17] The electrons do not actually block the full Coulombic attraction of the nucleus. In multielectron atoms, the electron-to-electron repulsions reduce the strength of the nuclear charge felt by the electrons. The effective nuclear charge, Z_{eff}, is the net nuclear charge actually felt by an electron, and is calculated as follows: $Z_{eff} = Z$ (the number of protons) $-\sigma$ (the electron shielding constant), $Z_{eff} = Z - \sigma$.[18] We have studied the partial atomic charge because it is a more localized property to atomic environments. Also, for the dissection of electron distribution in a molecule and its assignment to individual atoms, the partial atomic charge is a drastic oversimplification. Since the electron density of a molecule is partitioned into atomic components, partial atomic charge is assigned to each atom in the molecule. One should clearly understand that the notion of the partial atomic charges is shrouded in uncertainty because the partial atomic charge is not a physical observable. As a result any method to obtain the partial atomic charge is non-unique and its merits can only be assessed in terms of its chemical sensibility. Thus chemists have no choice but to construct partial atomic charges that fulfill the qualitative or quantitative needs for this crucial aspect of chemical understanding. Any method to obtain charges consists of partitioning a direct

observable and requires an additional assumption about the partitioning method. To apply this method to chemical problems, some approximations have to be employed. The interpretation of the partial atomic charges obtained by empirical, semi-empirical or non-empirical calculations is completely arbitrary, and all of them coexist as useful tools for chemical reasoning. Firstly, to calculate the charge, since partial charges depend directly on "Closed-Shell MO Coefficients" (CSMC) and on their energy, the sum of the multiplication of the adjustable partitioning energy function, ξ, and the square of the coefficients of each atom was calculated for all occupied MO. Secondly, since only one electron of each of the constituent molecular atoms was used to participate on atomic charge, the value arising from the difference between one proton in the nucleus and one CSMC electron formed the partial atomic charge. If one wishes to produce the partial atomic charge by fitting î the total atomic charge in the neutral molecule will be zero. One can derive the partial atomic charges using the energy and coefficients in the occupied MO.

The shielding constant is simply a way of expressing the net outcome of nuclear attraction and the electronic repulsions in terms of a single equivalent charge at the center of the atom. Because the shielding constant in its ground state is different for each atom type, they have different levels of attraction on the electrons in the outermost shell of an atom, which are called the CSMC electrons.

If an electron is at a distance r from the nucleus, it experiences a Coulombic repulsion that can be represented by a point negative charge placed at the nucleus and is equal to the magnitude given by the integral over the electron
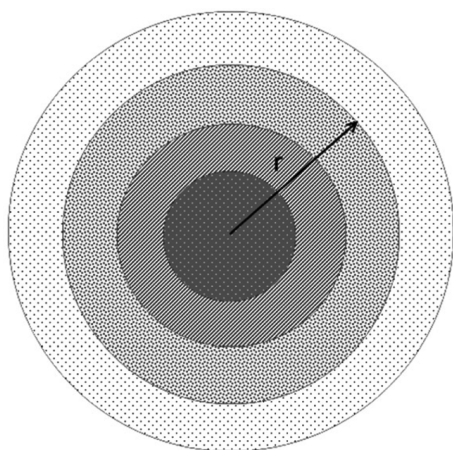

Fig. 1. An electron at a distance r from the nucleus experiences a Coulombic repulsion from the total charge of the electrons within a sphere of radius r. The total number of electrons in a closed shell system, and in the outermost shell for a multiple-electron atom, is given by the integral over the electron density.

density. The distribution of electrons around a molecule may depend upon the spatial configuration of its nuclei. An electron close to the core causes the potential to decay with distance more sharply than an electron far from the core. The shielding in an electrostatics problem is taken into account by replacing the Coulomb potential ($\phi_i = \frac{Z_i}{r}$; $Z_i = \frac{z_i e}{4\pi\varepsilon}$) by the shielded Coulomb potential, an expression of the form $\phi_i = \frac{Z_i}{r} \cdot e^{-r/r_D}$ where r_D is called the Debye length and r, is the distance of the electron from nucleus.[18] Because the electrons, inside the inner shell sphere (the core electrons), contribute to shielding for the outermost electrons of the atom, the atoms with different configurations and types apply different power on them. The coefficients at each energy level of the MOs are obtained from the power of the core attracting the outermost electrons. The attractive power of the core is defined by the difference in positive charge between the nucleus and partial negative charge in the r radius. The extent of coefficients derived from wavefunctions of molecules is responsible for the partial atomic charge and is balanced by the core's attraction. The Coulomb potential between the core and high-energy electrons in the MO is shielded by the Coulomb potential arising from low-energy electrons. According to Boltzmann population analysis, the low coefficient of the low-energy orbital in may contribute to negative charge more than a high energy large coefficient. The larger the coefficients of the low energy-orbitals, the larger the total negative charge of the atom as a whole.

The model of the partial atomic charge for the physical and chemical properties of compounds has long been an important focus of interest in chemistry. It predicts the values of experimental observables such as molecular biological activity. In general, the extent of the partial atomic charge alters chemical activities and biological activities due to the physicochemical properties of the compounds. Thus, knowledge of charge distribution is fundamental for molecular modeling of chemical systems. Because of the lack of a unique definition, we developed a novel method to compute the partial atomic charge. The developed charge models are well suited for application in many areas of molecular modeling and QSAR studies.

Here in we report an examination of the alterations in the most representative molecular structures to compute the partial atomic charges. We also concentrate our attention on Mulliken-type charge (STO-3G) defined by Mulliken Population Analysis (MPA) and on the new partial atomic charge, the y-charge. The research is especially aimed at understanding the link between the partial atomic

charge and the size of the coefficient relating to its energy level.

MATERIAL AND METHODS

Quantum chemical calculations were performed using SPARTAN (v. 5.01).[19] As output from the SPARTAN interface AM1 charges, are a set of Mulliken-type charges (STO-3G) derived from a semi-empirical quantum mechanical calculation, and are directly taken. Mulliken population analysis (MPA).[20] Which assigns the entire electron density to atom-centered orbitals and natural population analysis,[21] is recognized to be both the most widely used and the most criticized method[22,23] (Note that Mulliken and natural charges are identical at the semi-empirical level).

The y_charge was calculated from the CSMC in the MOs, derived from the valence electrons in atomic orbitals, and performed with software programs using the C# programming language.

Since the CSMC is used to generate the partial charges, it will be dealt with in the equation, taking of Boltzmann distribution into consideration.

In this study we compared our employed method to MPA method.

· Firstly, the MPA method, which simply partitions the charge density, is derived from the overlap of atomically centered orbitals distributed evenly between the two bonded atoms.

· Secondly, the y_charge takes into account the coefficients at all occupied orbitals on each of the constituent molecular atoms depending on energy level. In this approach, the partial atomic charge is defined by Boltzmann population of the atomic coefficients using an adjustable energy function, ξ.

Due to the dependence of both methods on MOs, the coefficient size of the basis set used in calculations of electronic bonding characteristics is known to often strongly influence predicted partial atomic charges. The starting data for the computation is the coefficients of the occupied MO in a molecule. The partial atomic charge is affected by the extent of the coefficients and by the energy level. In a more advanced model, the magnitude of a coefficient, according to Boltzmann population reflects the role of the energy at MO. The y_charge is presented to give an accurate description of the partial atomic charge from the coefficient distribution and the energy level in the occupied MO.

This is achieved in three steps:

First, to find the total density on each atom, we need to calculate the ξ according to Boltzmann population. The total electronic density on each atom is defined as the sum of production of both ξ and the coefficient square in the occupied MO. As regards the Boltzmann population analysis, ξ can often be used to regulate the coefficient corresponding to energy. This ξ is not constant from molecule to molecule.

For each electron in the occupied MOs the following equation can be written by ascribing it to the accumulated population, P_j, on j-atom. A potential energy on the electron in related MO (i+1) is between the attraction from the core and the repulsion from all low-energy electrons (i). This equation has the form

$$P_j = \sum_{i=1}^{n_{HOMO}} P_{ij} e^{-\xi'\{(E_{i+1}/RT)-(E_i/RT)\}} \tag{1}$$

where the energy of MOs is labeled i, and the number of atoms j; the sums on the right are the accumulated population for all occupied MOs.

$$P_{ij} = 2(s^2 + p_x^2 + p_y^2 + p_z^2)_{ij}/N_V \tag{2}$$

Eq. (2) gives the population of j-atom in i-MO, where N_V is the number of valence electrons of the j-atom. Each atom contributes to the occupied MO as one electron defined by dividing the total orbital population with N_V. In Eq. (2) represents two electrons in each occupied MO, the total of the coefficient square of j-atom in i-MO must be multiplied by 2.

Although the $e^{-(E_{i+1}/RT)}$ equation shows Boltzmann expression, the $e^{-\xi'\{(E_{i+1}/RT)-(E_i/RT)\}}$ equation provides the Boltzmann population between two different energies of MO. At this stage we need to simplify the expression to avoid any awkward exponential terms. Taking $\xi = \xi'/RT$, we may write Eq. (2) as

$$P_j = \sum_{i=1}^{n_{HOMO}} P_{ij} e^{-\xi(E_{i+1}-E_i)} \tag{3}$$

An important feature revealed by this equation is that, as the difference $|E_{i+1}-E_i|$ increases, the effect value of the atomic coefficient decreases. When the energy difference is large, the coefficients of high-energy MOs in proportion to those of the lower energy MOs contribute only slightly to the accumulated coefficient and have a reduced amount of partition.

This left us the task of finding a value for the exponential prefatory ξ, which we want to make as large as possible to use the effect of the energy. On the other hand, if we make it too large, the difference of the charge dis-

tribution will approach a high value, leading to hardly any freedom to distribute the charges over the other atoms and to numerical instabilities. Previous values, which we have varied between 0.02 and 1.0, are accepted with a value of 0.05. Therefore, inclusion of this term will involve an iterative procedure are varying until the resulting charges remain constant. We took as the "optimal" value the highest value which turned out to be 0.05. When using charges on the molecule, all calculations should result in charges of exactly zero, because they are neutral.

Next, from these populations of the j-atom in i-MO, a set of the atomic densities is defined; supposing that each valence electron in an atom occupies a MO. To find the partial atomic charge we must deal the value arising from difference between each proton in the nucleus and each valence electron.

$$q_{i,j} = 1 - P_{i,j} n_{atm}/n_{HOMO}, \; i = 1 \tag{4}$$

$$q_{i,j} = q_{i-1,j} - p_{i,j} e^{-\xi(E_{i+1}-E_i)} (n_{atm}/n_{HOMO}), \; i = 2,3...n_{HOMO} \tag{5}$$

Where n_{atm} is the number of the constituent molecular atoms. There is an electron number as well as a n_{atm}, if only one valence electron from one atom is taken into account. n_{HOMO} is the maximum number of occupied MOs. Usually; n_{HOMO} is greater than n_{atm} since any atom participating in the MO, except atoms like the H-atom, has more than one valence electron. The correction factor of (n_{atm}/n_{HOMO}) is used to take the average of all occupied MOs. Thus, any atom with more than one valence electron contributes to the occupied MO as only one electron where the charge at i-energy level can be integrated from the difference between 1-proton and the electron density for the j-atom in Eq. (4). Firstly since the coefficient with the lowest energy level is directly affected by the core, the difference in the energies is not used, and the coefficient size is not changed. Thus, in the first term, the charge can be calculated for i=1. Then from the difference ($q_{i+1,j}$) of the remaining core charge and electron density at the next energy level, the charge can be integrated using the energy levels. The equation of the partial atomic charge has the following form

$$q_j = \sum_{i=1}^{n_{HOMO}} q_{i,j} \tag{6}$$

Finally, the resulting molecular charge must be set to exactly zero by totaling the partial atomic charges over all the atoms. Thus, the well-developed formalized theory of the y_charge was applied for the computation of partial atomic charges.

RESULTS AND DISCUSSION

A comparison was made between the values obtained by our method and those calculated by MPA. MPA is the most widely used quantum mechanical method for the derivation of partial atomic charges. In spite of its known weaknesses it is still very much in use. MPA, which is ill-suited to the handling of the overlap population, is heavily dependent on the coefficients in the basis set but not on energy. For an unambiguous comparison of the results of the two methods, a set of molecules was chosen for which both theoretical and experimental data were available in the literature. As experimental data[24] C-1s core electron binding energies, obtained from Electron Spectroscopy for Chemical Analysis (ESCA) measurements, were preferred because they were known to depend directly on valence electron distribution. As theoretical data, values of the partial atomic charges of an MPA were chosen from the semi-empirical calculations.

There were many methods capable of providing the partial atomic charge, but in this article we took advantage of the way in which the partial atomic charge was calculated within the coefficient framework of the occupied MO depending on energy level. We investigated a set of 22 C-atoms with different configurations for 17 molecules to test the quality of the method, and compared with these results obtained by the MPA method. The deviation from MPA and the values we calculated for the partial atomic charge are also a little ambiguous.
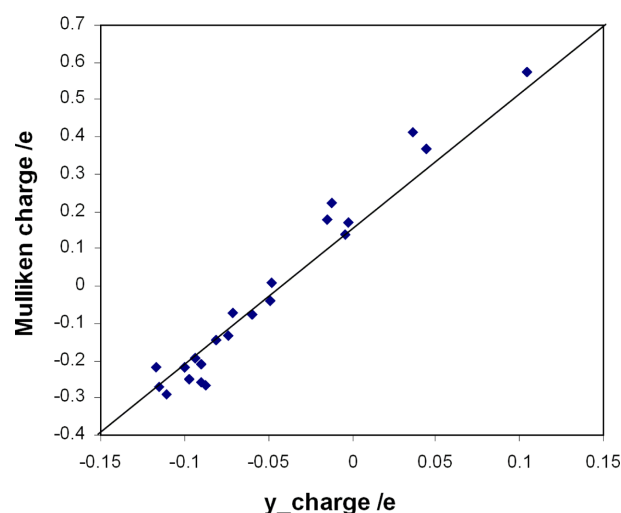

Fig. 2. Correlation of the partial atomic charges calculated with the Mulliken and y_charge.
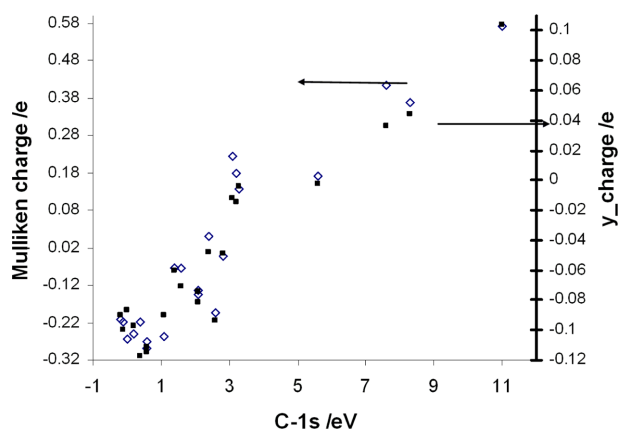
Fig. 3. The partial atomic charges calculated with the Mulliken and y_charge methods for the experimental data ^{13}C-1s core electron binding energy shifts.

The variations of these charges, which were calculated by the two different methods, are illustrated in *Figs.* 2-3. Here the semi-empirical methods give relatively larger values for the Mulliken charges on the carbon atom, whereas our method gives consistently small values. The y_charge is more consistent over Mulliken charge. The generated charges were often surprisingly accurate, when compared to those derived from the more used MPA method. Considering just the semi-empirical values, Mulliken charges still show considerable dependence on the basis set employed, whereas y_charges depend not only on changes in the basis set but also on energy levels. In *Table* 1 the partial charge on the carbon bonding the H-atom reaches a negative value relative to the other carbon atoms due to the C-atom having more electronegativity than the H-atom.

The y_charge shows a fairly strong correlation with the MPA charges (0.98). The correlation passes through the 0.173-point and has a slope of 4.21 with the MPA charges having larger values (*Fig.* 2).

$$q_{MPA} = 4.21 * q_{y\_charge} + 0.173$$

To further explore the quality of the two sets of charge values, a comparison was made with the C-1s core electron binding energy shifts for the same set of compounds. ESCA shifts were those experimental data which were probably most closely related to valence electron distribution. y_charges gave a correlation with the ESCA shifts of 0.95. The MPA charges showed a correlation coefficient of 0.93 (*Fig.* 3). This study showed that the y_charges can reproduce the experimental data of C-1s ESCA shifts better than Mulliken charges. To calculate theoretically justifiable partial atomic charges for a variety of chemical

Table 1. Gives a comparison of the y_charges (elementary charge; e) with those obtained from MPA (e) and C-1s (eV) core electron binding energy shifts

	Molecules	y_Charge/e	Mulliken Charge/e	C-1s/eV
1	CH_4	-0.087	-0.265	0
2	CH_3CH_3	-0.090	-0.211	-0.2
3	$CH_2=CH_2$	-0.100	-0.217	-0.1
4	HCCH	-0.117	-0.218	0.4
5	CH_3F	-0.049	-0.041	2.8
6	CH_2F_2	-0.002	0.171	5.6
7	CHF_3	0.044	0.370	8.28
8	CF_4	0.104	0.576	11.0
9	*CH_3CH_2F	-0.097	-0.249	0.2
10	CH_3*CH_2F	-0.048	0.009	2.4
11	*CH_3CF_3	-0.090	-0.258	1.1
12	CH_3*CF_3	0.036	0.413	7.6
13	CH_3OH	-0.071	-0.073	1.6
14	CH_3OCH_3	-0.060	-0.075	1.4
15	H_2CO	-0.004	0.138	3.3
16	*CH_3CHO	-0.111	-0.289	0.6
17	CH_3*CHO	-0.015	0.180	3.2
18	*CH_3COCH_3	-0.115	-0.272	0.6
19	CH_3*$COCH_3$	-0.012	0.224	3.1
20	HCN	-0.094	-0.192	2.6
21	*CH_3CN	-0.074	-0.135	2.1
22	CH_3*CN	-0.081	-0.144	2.1

species, y_charge model provides fast, convenient and reliable results. Finally, our partial atomic charges reproduced both the atomic and molecular charges exactly.

REFERENCES

1. Bachrach, S. M. *In Reviews in Computational Chemistry*; K.B. Lipkowitz, D.B. Boyd, Eds.; VCH: New York, 1993; Vol. 5, p 171.
2. Cramer, C.; Truhlar, D. *Theor. Chim. Acta* **1997**, *98*, 206.
3. Li, J.; Zhu, T.; Cramer, C. J.; Truhlar, D. G. New Class IV Charge Model for Extracting Accurate Partial Charges from Wave Functions. *J. Phys. Chem. A* **1998**, *102*, 1820.
4. Naray-Szabo, G.; Ferenczy, G. *Chem. Rev.* **1995**, *95*, 829.
5. Jorgensen, W. L.; Maxwell, D. S.; Tirado, R. J. *J Am Chem. Soc.* **1996**, *118*,11225.
6. Maple, J. R.; Hwang, M. J.; Stockfisch, T. P.; Dinur, U.;

Waldman, M.; Ewig, C. S.; Hagler, A. T. *J. Comput. Chem.* **1994**, *15*, 162.

7. Pearlman, D. A.; Kim, S. H. *Biopolymers.* **1985**, *24*, 327.

8. Rappé, A. K.; Goddard, W. A. III. *J. Phys. Chem.* **1991**, *95*, 3358.

9. Coppens, P. *Annu. Rev. Phys. Chem.* **1992**, *43*, 663.

10. Woods, R. J.; Chappelle, R. *J. Mol. Struct. (Theochem)* **2000**, *527*, 149.

11. Wiberg, K.; Rablen, P. *J. Comput. Chem.* **1993**, *14*, 1504.

12. Thompson, J. D.; Xidos Sonbuchner, M.; Cramer, C. J.; Truhlar, D. G. *Phys. Chem. Commun.* **2002**, *5*, 117.

13. Williams, D. E.; Yan, J. M. *In Advances in Atomic and Molecular Physics*; Academic Press: San Diego, 1987; p. 87.

14. Gussoni, M.; Ramos, M. N.; Castiglioni, C.; Zerbi, G. *Chem. Phys. Lett.* **1987**, *142*, 515.

15. Löwdin, P. O. *Adv. Quantum Chem.* **1970**, *5*, 185; p. 8.

16. Reed, A. E.; Weinstock, R. B.; Weingold, F. *J. Chem Phy.* **1985**, *83*, 735.

17. Fonseca Guerra C.; Handgraaf, J. W.; Baerends, E. J.; Bickelhaupt, F. M. *J. Comp. Chem.* **2004**, *25*, 189.

18. Atkins, P. *Physical Chemistry* 6th ed.; W. H. Freeman, Ed.; New York, 1998; Chapter 13.

19. Spartan'04 Windows, *Wavefunction*, Inc., 18401 Von Karman, Suite370 Irvine, CA 92612.

20. Mulliken, R. S. *J. Chem. Phys.* **1955**, *23*, 1833.

21. Reed, A.; Weinstock, R.; Weinhold, F. *J. Chem. Phys.* **1985**, *83*, 735.

22. Hehre, W. J.; Radom, L.; Schleyer, R. P.; Pople, J. A. *Ab initio Molecular Orbital Theory*; Wiley: New York, 1986; Chapter 4.

23. Politzer, P.; Mulliken, R. S. *J. Chem. Phys.* **1971**, *55*, 5135.

24. Gasteiger, J.; Marsili, M. *Tetrahedron* **1980**, *36*, 3219.