무선인지시스템을 위한 Kullback-Leiber Divergence 기반의 스펙트럼 센싱 기법☆

A Kullback-Leiber Divergence-based Spectrum Sensing for Cognitive Radio Systems

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요 약

본 논문에서는 무선인지시스템에서 효율적으로 스펙트럼 센싱을 수행하기 위해, 확률 분포 사이의 대수 차를 측정하는 Kullback-Leiber divergence기반의 새로운 스펙트럼 센싱 기술을 제안한다. 제안된 센싱 기법은 특정 센싱 구간에서의 국부 센싱 측정 값들이 잡음 분포에서 발생하였는지, 기사용자 신호에서 발생하였는지를 Kullback-Leiber divergenc를 이용하여 판단한다. 시뮬레이션을 통해, 제안된 Kullback-Leiber divergence기반의 스펙트럼 센싱 기법이 동일 조건에서 에너지 검출 기반의 스펙트럼 센싱 기법보다 더 좋은 성능을 제공할 수 있음을 보였다. 특히, 페이딩 환경 및 기사용자 신호의 SNR값이 낮은 경우에 에너지 검출 기반의 스펙트럼 센싱 기법과 비교할 때 제안된 기법의 성능이 크게 향상됨을 보였다.

ABSTRACT

In the paper, an information divergence called ***Kullback-Leiber divergence***, which measures the average of the logarithmic difference between two probability density functions, is utilized to derive a novel method for spectrum sensing in cognitive radio systems. In the proposed sensing method, we test whether the observed samples are drawn from the noise distribution by using Kullback-Leiber divergence. It is shown by numerical results that under the same conditions, the proposed Kullback-Leiber divergence-based spectrum sensing always outperforms the energy detection based spectrum sensing significantly, especially in low SNR regime and in fading circumstance.

☞ keyword : Cognitive radio, spectrum sensing, Kullback-Leiber divergence.

1. Introduction

Recently, cognitive radio (CR) has been proposed as a feasible solution to improve spectrum utilization by introducing the opportunistic usage of the frequency bands that are not heavily occupied by licensed user (LU) [1]. To avoid causing interference to the LUs, CR users (CUs) are allowed to use the licensed bands opportunistically when such bands are not occupied, and must abandon its contemporary band to seek a new idle spectrum again when the frequency band is suddenly accessed by the LUs. This causes spectrum sensing to play a key role in CR.

Based on the observations from the band of interest, many spectrum sensing techniques have been proposed, such as match filtering approach [2][3], feature detection approach [4][5] and energy detection (ED) approach [6]-[9]. The match filtering method and the feature detection method take advantage of known patterns contained in LU's signal such as pilot patterns, frame structure, cyclostationary features, etc, to detect the presence of the LU's signal. However, if the assumption about the parameters related to the known patterns is invalid or imprecise, sensing performance of these methods will be degraded. On the other hand, the ED approach does not require any information about the LU's signal. The principle of the ED is based on the difference between the energy of the signal and that of the noise. When the time-varying natures of wireless channel (e.g.,

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shadowing, fading) is obvious or the signal-to-noise ratio (SNR) is low, this difference will be small for distinguishing between the signal and the noise. Subsequently, the detection performance of the ED can be very poor.

In this work, to detect the presence of the LU's signal, we test whether the observed samples are drawn from the noise distribution. Hence, the spectrum sensing can be implemented only based on the noise distribution, and the prior knowledge of the LU's signal becomes unnecessary. Firstly, an information divergence called Kullback-Leiber divergence (KLD) is utilized to derive a test statistic for spectrum sensing when the noise is known Gaussian. Furthermore, an improved spectrum sensing algorithm is also proposed for the case that the noise is unknown. Numerical results reveal that under the same sensing conditions and channel environments, the KLD based spectrum sensing always outperforms the ED based spectrum sensing significantly, especially in low SNR regime and in fading environment.

2. Overview of Kullback- Leibler divergence

In probability theory and information theory, the Kullback-Leibler divergence [10]-[12] (also called information divergence, or information gain) is a measure of the divergence from probability distribution P to probability distribution Q and it is defined as:

$$KL[P\|Q]=\int_{-\infty}^{+\infty}p(x)\log_2\left[\frac{p(x)}{q(x)}\right]dx \tag{1}$$

where $p(x)$ and $q(x)$ are probability density functions (pdf) of P and Q, respectively.

The KLD is always non-negative, $KL[P\|Q]\geq 0$, and equals zero if and only if $P=Q$. For two Gaussian distributions $P=N(m_1,v_1)$ and $Q=N(m_2,v_2)$, the KLD has a closed expression [12]:

$$KL[N(m_1,v_1)\|N(m_2,v_2)]= \frac{1}{2}\left[\log_2\frac{v_2}{v_1}+\frac{v_1}{v_2}+\frac{(m_1-m_2)^2}{v_2}-1\right] \tag{2}$$

3. KLD based spectrum sensing under known Gaussian noise

Spectrum sensing can be formulated as a binary hypothesis testing problem as follows:

$$\begin{cases} H_0: LU's\, signal\, is\ absent, \\ H_1: LU's\, signal\, is\, present. \end{cases} \tag{3}$$

Depending on the status of the LU's signal, the received signal at the CU is given as follows

$$y(t)=\begin{cases} n(t) & H_0 \\ h(t)s(t)+n(t) & H_1 \end{cases} \tag{4}$$

where $y(t)$ represents the received signal at the CU, $h(t)$ denotes the amplitude gain of the channel between the CU and the LU, $s(t)$ represents the signal transmitted by the LU, and $n(t)$ is the additive noise. Without loss of generality, we assume that the noise is Gaussian distribution with mean zero and variance unity. Hence, the pdf of noise is:

$$p_0(y)=\frac{1}{\sqrt{2\pi}}e^{-\frac{y^2}{2}} \tag{5}$$

Let $Y=\{Y_i\}_{i=1}^{N}$ be N local observations at the CU. When the LU's signal is absent, Y_i's are samples drawn from the noise and then can be regarded as independent and identically distributed (i.i.d.) sequence drawn from common distribution $p_0(y)$. On the other hand, when there is transmission signal from the LU, the observations Y_i's do not come from the common distribution $p_0(y)$. Therefore, detecting the presence of LU's signal is now equivalent to testing the null hypothesis:

$$H_0:\ Y\ is\ a\ sequence\ drawn \tag{6}$$

against the general alternative that Y is not a sequence drawn from common distribution $p_0(y)$.

Assume that Y_i's follow a Gaussian distribution of which mean and variance are given by maximum likelihood as:

$$m = \frac{1}{N}\sum_{i=1}^{N} Y_i;\ v = \frac{1}{N}\sum_{i=1}^{N}(Y_i - m)^2 \tag{7}$$

Let $\hat{p}_{Y(y)}$ denote the empirical pdf of the observations set Y, we have

$$\widehat{p_Y}(y) = \frac{1}{\sqrt{2\pi v}} e^{-\frac{(y-m)^2}{2v}} \tag{8}$$

Under the null hypothesis, it is clear that $\widehat{p_{Y(y)}}$ will be fairly close to the noise pdf $p_0(y)$ when N is large enough. If $\widehat{p_{Y(y)}}$ deviates significantly from $p_0(y)$, then it is reasonable to reject the null hypothesis $H_0$and further to declare the hypothesis H_1. It means that the distance between $\hat{p}_{Y(y)}$ and $p_0(y)$ can be utilized to make the decision on the presence of LU's signal.

In the paper, we use KLD to measure the distance from the empirical pdf $\hat{p}_{Y(y)}$ to the noise pdf $p_0(y)$ to obtain the test statistic as follows:

$$\begin{aligned} T &= KL[N(m,v)\|N(0,1)] \\ &= \frac{1}{2}\left[-\log_2 v + v + m^2 - 1\right] \end{aligned} \tag{9}$$

The spectrum sensing decision is then made by comparing T with a decision threshold λ as follows:

$$\begin{cases} T \geqq \lambda, & decide\, H_1 \\ T < \lambda, & decide\, H_0 \end{cases} \tag{10}$$

In sum, the proposed KLD based spectrum sensing algorith under known Gaussian noise can be performed as follows:

Step 1: Take N observations from the frequency band of interest, and calculate mean m and variance v of the observations.

Step 2: Calculate the value T according to the formula Eqn. (9).

Step 3: Reject the null hypothesis H_0 in favor of the presence of LU's signal if $T > \lambda$; otherwise, declare that the frequency band of interest is not in use.

Step 4: Go to Step 1 for the next sensing cycle.

4. KLD based spectrum sensing under unknown noise

In Section 3, the proposed KLD based spectrum sensing algorithm is derived under the assumption that the noise distribution $p_0(y)$ is perfectly known. However, it is very difficult to extract the noise distribution in practice since the noise at the CU consists of thermal noise, receiver noise, and environmental noise, which can vary over time and location. To deal with this issue, instead of using the exact noise pdf $p_0(y)$, we now use empirical noise pdf $\hat{p}_0(y)$ to develop a KLD based spectrum sensing for the unknown noise case. The empirical noise pdf $\hat{p}_0(y)$ is obtained by listening to the band of interest if we know that it is free in this stage, or by listening to a special channel which is rarely used. For example, channel 37 in United States is reserved for radio astronomy and is used in very few occasions [9]. Let $X = \{X_j\}_{j=1}^{M}$ be M local noise observations at the CU. The noise distribution is approximated as a Gaussian distribution with mean and variance given by maximum likelihood as follows:

$$m_0 = \frac{1}{M}\sum_{j=1}^{M} X_j;\quad v_0 = \frac{1}{M}\sum_{j=1}^{M}(X_j - m_0)^2 \tag{11}$$

Therefore, the empirical pdf of the noise is expressed as follows:

$$\hat{p}_0(y) = \frac{1}{\sqrt{2\pi v_0}} e^{-\frac{(y-m_0)^2}{2v_0}} \tag{12}$$

After obtaining the empirical pdf $\hat{p}_Y(y)$by N observations from the frequency band of interest, the KLD from $\hat{p}_Y(y)$ to $\hat{p}_0(y)$ is used to determine the test statistic:

$$
\begin{aligned}
T &= KL\left[\hat{p}_Y(y) \| \hat{p}_0(y)\right] \\
&= KL\left[N(m,v) \| N(m_0,v_0)\right] \\
&= \frac{1}{2}\left[\log_2 \frac{v_0}{v} + \frac{v}{v_0} + \frac{(m-m_0)^2}{v_0} - 1\right]
\end{aligned} \tag{13}
$$

As a result, the proposed KLD based spectrum sensing algorithm in unknown noise can be implemented as follows:

Step 1: Take M observations $\{X_j\}_{j=1}^{M}$ from the special channel which is known to be rarely used, and calculate mean m_0 and variance v_0 of the noise samples.

Step 2: Take N observations from the frequency band of interest, and calculate mean m and variance v of the observations.

Step 3: Calculate the test statistic T using Eqn. (13).

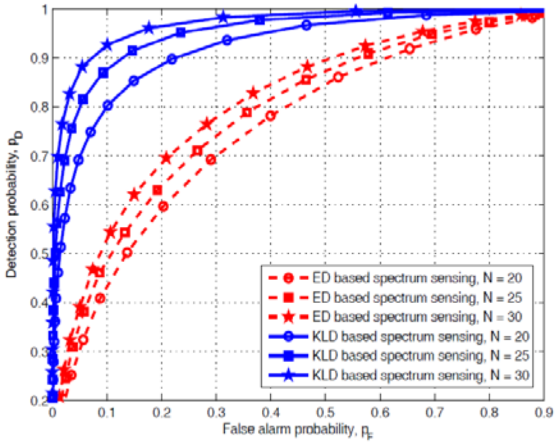
Step 4: Reject the null hypothesis H_0 in favor of the presence of LU's signal if $T > \lambda$; otherwise, declare that the channel of interest is idle.

Step 5: Go to Step 2 for the next sensing cycle.

5. Simulation results

To evaluate the performance of the proposed KLD based spectrum sensing algorithms, Monte-Carlo simulations are carried out. The sensing performance of the proposed method is compared to the sensing performance of the ED based spectrum sensing method under same conditions.

Firstly, simulation is performed under the condition that the SNR of LU's signal at the CU is - 4 dB; and noise is white Gaussian noise with zero mean and unit variance, and is known to the CUs. The receiver operating characteristic (ROC) curves of the proposed method and the comparison method are plotted in the Fig. 1. It can be seen that when the number of samples N increases, the sensing performances of both the proposed spectrum sensing method and the ED based one are improved. For a fixed N, the sensing performance of the proposed sensing method is superior to the one of the ED based sensing method. For example, with the number of samples $N=25$ and the false alarm probability $p_F=0.10$, the detection probability p_D of the proposed sensing method and ED based sensing method are 0.48 and 0.87, respectively. This performance improvement

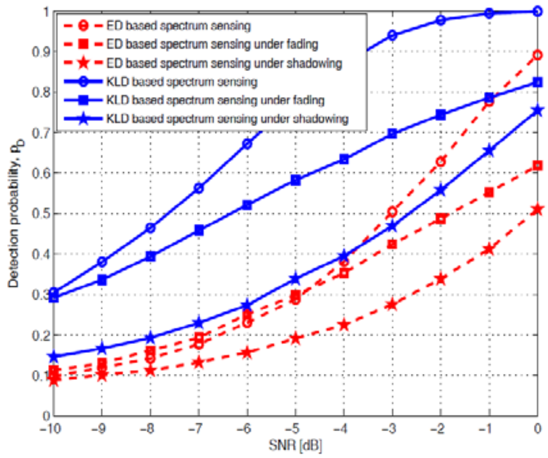


(Fig.1) The ROC curves of the proposed KLD based spectrum sensing and ED based spectrum sensing under condition that $SNR=-4dB$, and noise is Gaussian and is known to the CUs.

of the proposed scheme is mainly due to the fact that the ED is only based on the difference between the energy of the signal and that of the noise while the proposed scheme utilizes the difference between the distribution of noise and the distribution of measured energies.

Secondly, the detection probability is evaluated with $N=30$, $p_F=0.05$, and the SNR varies from - 10 dB to 0 dB. Both Rayleigh fading channel and log-normal shadowing channel with 6 dB of standard deviation are also considered in this simulation. As plotted in Fig. 2, the detection probability of the proposed method is always higher than that of the ED based method. This result clearly proves that the proposed KLD based spectrum sensing outperforms the ED based one significantly, especially in low SNR regime (SNR < -10dB) and in shadowing/fading channel.

Finally, simulation is performed under the condition that the SNR of LU's signal at the CU is - 4 dB; and noise is Laplacian noise with mean zero and variance unity, and is unknown to the CUs. The empirical noise distribution $\hat{p}_0(y)$ is extracted by the CU with $M=1000$ noise samples. The ROC curves of the proposed method and the comparison method are illustrated in the Fig. 3. Similar to the previous results, the sensing performance of the proposed sensing method outperforms the ED based sensing method



(Fig.2) The detection probability of the proposed KLD based spectrum sensing and ED based spectrum sensing under condition that $N=30$, $p_F=0.05$, and noise is Gaussian and known to the CUs

significantly. For example, with $p_F=0.1$, the detection probability p_D of the proposed sensing method is 0.84, while the one for the ED based sensing method can achieve 0.33 only.

6. Conclusion

Spectrum sensing is a fundamental problem in CR networks. In this paper, we have proposed a spectrum sensing method based on KLD in both known and unknown noise conditions. In the proposed sensing method, we have tested whether the observed samples are drawn from the noise distribution by using Kullback-Leiber divergence. Numerical results have shown that under same conditions, the proposed spectrum sensing method always outperforms the ED based spectrum sensing method especially in low SNR regime (SNR < -10dB) and in shadowing/fading channel.

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