A Generalized Markov Chain Model for IEEE 802.11 Distributed Coordination Function

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Abstract

To improve the accuracy and enhance the applicability of existing models, this paper proposes a generalized Markov chain model for IEEE 802.11 Distributed Coordination Function (DCF) under the widely adopted assumption of ideal transmission channel. The IEEE 802.11 DCF is modeled by a two dimensional Markov chain, which takes into account unsaturated traffic, backoff freezing, retry limits, the difference between maximum retransmission count and maximum backoff exponent, and limited buffer size based on the M/G/1/K queuing model. We show that existing models can be treated as special cases of the proposed generalized model. Furthermore, simulation results validate the accuracy of the proposed model.

Keywords: WLAN, IEEE 802.11, DCF, Markov chain model, M/G/1/K queuing model

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1. Introduction

IEEE 802.11 [1] is the leading global standard for Wireless Local Area Network (WLAN) and has been the research focus of the wireless community for many years. Distributed coordination function (DCF) is the fundamental technique used in IEEE 802.11 medium access control (MAC) layer. It employs Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) as the underlying random access scheme. The procedure of DCF is shown in **Fig. 1**.
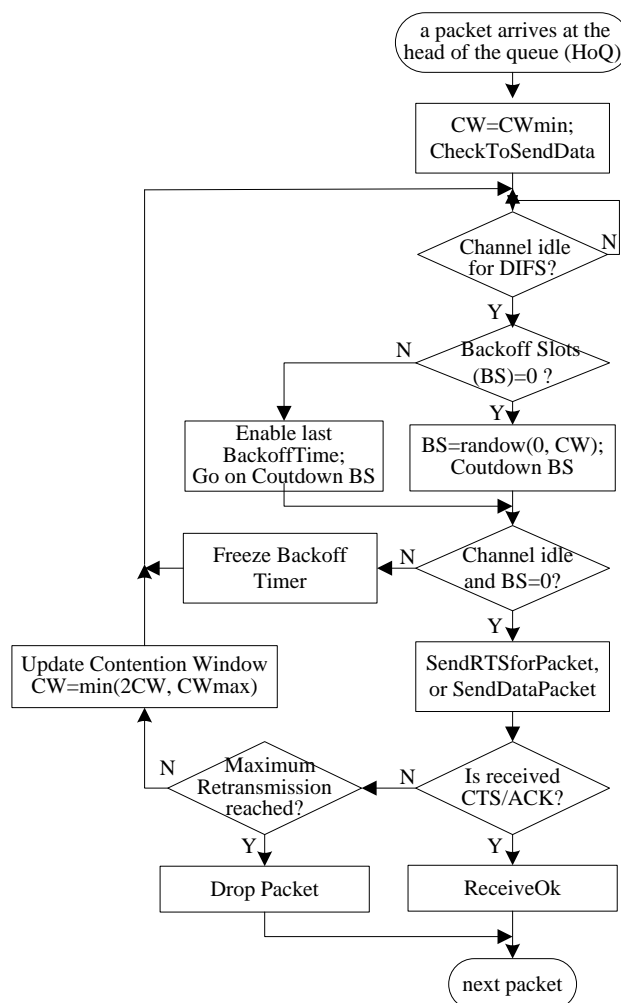

Fig. 1. Backoff algorithm in DCF of IEEE802.11 MAC

If a station has frames to transmit, it should wait for a free channel and start a backoff timer. The initial backoff time is a random value chosen from the contention window $[0, CW_{min}]$. If the channel is free during a system slot, the backoff counter subtracts 1; if the channel is busy, the timer freezes until the channel becomes idle. When the idle time exceeds a Distributed

Inter-Frame Space (DIFS), the timer is unfrozen and begins to decrement again. When the backoff timer is decremented to 0, a station starts to transmit frames. If the receiver successfully receives the transmitted frame, it will send an acknowledgement (ACK) to the sender after a Short Inter-Frame Space (SIFS). However, if the transmit is failed due to collisions or channel fading etc., the sender will not receive an ACK from the receiver. In this case, the contention window of the sender will be doubled according to the binary exponential backoff algorithm and the frame will be retransmitted. If the contention window reaches its maximum value, it will keep this window size; if the number of failed transmissions for a frame reaches the maximum retry count, this frame will be dropped and the station will start to send the next frame.

Many methods have been proposed to model and analyze the performance of IEEE 802.11 DCF. The most classic method proposed by G. Bianchi [2] used a two-dimensional Markov chain to model the binary exponential backoff mechanism in IEEE 802.11 DCF. By solving the steady state solution of the Markov chain, Bianchi's model can obtain the transmission probability in any given slot for each station. This probability can be used to calculate the main system performance indicators of saturation throughput. Nevertheless, Bianchi's paper adopted a rough framework for the backoff model of IEEE 802.11 DCF with some assumptions: 1) the wireless channel is an ideal channel so that there is no hidden/exposed terminal problem and channel error; 2) the traffics of each station are saturated, i.e. there are always data packets in the transmission queue; 3) every station has an infinite MAC layer transmit queue and queue overflow is ignored. However, other practical factors, such as the retry limits, backoff freezing, and the difference between maximum retransmissions count and maximum backoff exponent, are not taken into account in Bianchi's model.

To improve the accuracy and practicality of Bianchi's model in saturated traffic conditions, many researchers have presented several improved models to analyze the throughput, time delay, and energy consumption based on Bianchi's model. H. Wu et al [3] further developed the Markov model to consider limited retransmission count. H. Zhai et al [4] used the state transfer function to derive an approximate distribution function model of the system MAC layer service time under saturated conditions. They also proved that the M/G/1/K queuing model performs well in the simulation of Poisson Stream. Q. Ni et al [5] considered the saturation throughput for both congested and error-prone Gaussian channels, and extended Bianchi's model to consider finite number of packet retransmission. Their simulations revealed the connection between bit error rate and throughput. K. Szczypiorski et al proposed an extension model in [6] by considering backoff freezing. W.K. Kuo [7] built an energy consumption model based on the extended Bianchi's model with backoff freezing.

On the other hand, many literatures deal with the extension of Bianchi's model to unsaturated traffic conditions typically encountered in practical network operations. L.Y. Shyang et al [8] extended Bianchi's model to unsaturated traffic conditions and to take into account retry limits. The relationship between collision probability and hidden terminal was also investigated in [8]. G. Prakash et al [9] proposed a new analytical model taken into account failed transmissions due to channel errors in the unsaturated condition. In this model, if a channel error is detected, the backoff window is maintained at the same backoff stage. W. Yang et al [10] used a three-dimensional Markov chain and the M/G/1/K queuing model to establish a throughput model under the unsaturated condition. C. Xu et al [11] used the M/G/1/K queuing model to analyze the unsaturated traffic. Furthermore, Q. Zhao et al [12] investigated the impact of MAC buffer size on the throughput and time delay under the unsaturated condition. F. Daneshgaran et al [13] investigated the unsaturated throughput in

presence of fading channel and capture effect. They also validated the linear throughput model. Nevertheless, [13] didn't consider limited retry and backoff freezing.

Due to its simplicity and theoretical insight, the Markov chain analytical model has been broadly used as a reference model in wireless LAN protocol analysis. This paper is also based on the Markov chain analytic model presented by G. Bianchi. Latter, analytical models proposed in [3][4][5][6][7][8][9][10][11][12][13] extended Bianchi's model in different aspects, but none of these model can fully address the details of IEEE 802.11 DCF procedure as included in **Table 1**. In the table, "Y" is the short of Yes and means that the performance model has been considered in the corresponding model. There are clear limitations of existing models, for example, the model in [6] is not suitable for unsaturated conditions; the models in [8] and [11] do not consider time freezing during backoff progress. To our best knowledge, the model in [13] is the most comprehensive model in the current literature. Nevertheless, this model does not take into account limited retry and backoff freezing.

Table 1. Comparison of performance model for IEEE 802.11 DCF

Model Names	MAC Layer					Application Layer	Route Layer	Physical Layer	
	DCF				Queue size	Unsaturated	Hidden problem	Channel fading	Capture effects
	BEB	Retry limit	Maximum backoff exponents	Backoff freezing					
G. Bianchi [2]	Y								
H. Wu [3]	Y	Y	Y						
H. Zhai [4]	Y				Y				
Q. Ni [5]	Y	Y						Y	
K. Szczypiorski [6]	Y	Y	Y	Y					
W.K. Kuo [7]	Y	Y		Y	Y				
L.Y. Shyang [8]	Y	Y				Y	Y		
G. Prakash [9]	Y	Y	Y			Y		Y	
W. Yang [10]	Y		Y		Y				
C. Xu [11]	Y				Y	Y			
Q. Zhao [12]	Y				Y	Y			
F. Daneshgaran [13]	Y					Y		Y	Y
Our proposed	Y	Y	Y	Y	Y	Y			

Motivated by the lack of a comprehensive model, this paper proposes a generalized analytical model for IEEE 802.11 DCF performance evaluation under the assumption of ideal channel condition. Ideal channel condition means that interference-free physical layer communications are always reliable and packet losses can only be caused by collisions. The proposed model is an extension of the theoretical model in [2][3][4][5][6][7][8][9][10][11][12][13] and can be apply to both saturated and unsaturated traffic conditions. Our model in this paper considers a comprehensive MAC situation including the state transition during station freezing and the difference between the maximum number of retransmission and maximum backoff exponent. Furthermore, this paper also combines the M/G/1/K queuing model to analyze the MAC layer finite queue.

The remainder of this paper is organized as follows. A brief review of DCF is introduced in Section II. The proposed analytical model as well as a detailed analysis of the MAC queuing performance of IEEE 802.11 DCF is presented in Section III. Section IV presents numerical and simulation results of the throughput, using typical MAC layer parameters in IEEE 802.11 b. Finally, Section V concludes the paper.

2. 802.11 DCF Analytical Model

2.1 Assumptions and definitions

The main aim of this section is to propose an effective and accurate generalized analytical model of IEEE802.11 DCF protocol under idle channel conditions. We combine and extend existing models [2][5][6][9][10] according to IEEE802.11 DCF protocol specifications [1] and practical network considerations. For example, our model extends the model in [6] to address the unsaturated traffic condition, the model in [11] to include backoff timer freezing, and the model in [13] to take into account limited retry.

Bianchi's model [2] and related models in [3][4][5][6][7][8][9][10][11][12][13] rely on some limiting assumptions. To make our model as general as possible, here we try to adopt only a few fundamental assumptions as follows. (1) The number of stations is finite and the number of contending stations is denoted by n . (2) All stations have the same architecture and the same traffic mode, which is a Poisson progress with rate λ. (3) The transmission range is equal to the carrier sensing range and the interference range. There are no hidden terminals and no capture effect. (4) As in most analytical work [2][3][4][5][6][7][8][9][10][11][12], the transmission channel is assumed to be ideal, so that transmission failures are only caused by collisions. We note that in practical non-ideal channels, transmission failures can also be caused by channel fading [14][15][16] and/or burst errors of the digital channel [17]. However, modeling such details is commonly regarded as unnecessary for DCF performance analysis. (5) There is no post back-off, so that every packet experiences at least one back-off period. (6) The transmission buffer size of each station is finite, and the buffer size is measured in units of packets denoted as K. (7) Whenever a station transmit a packet, the probability of collision is fixed and independent of its past transmission history.

IEEE802.11 DCF has two access options [1]: the basic access and Request-To-Send/Clear-To-Send (RTS/CTS) access mechanisms. The main difference is that the latter uses the RTS/CTS to reserve the channel before sending data. However, the backoff procedure is the same in both mechanisms. Therefore, this paper only focuses on the basic access mechanism, where data frames and ACK frames are exchanged.

Let i be the backoff stage and W_i be the corresponding contention window size. According to the exponential backoff, the relation between the contention window size W_i at the ith stage and its initial size W_0 is:

$$\begin{cases} W_i = 2^i W_0 & for \quad 0 \le i \le m' \\ W_i = 2^{m'} W_0 & for \quad m' \le i \le m \end{cases} \tag{1}$$

where m is the maximum retransmissions count, m' is the maximum backoff exponent by which the contention window can be doubled from the minimum contention window CW_{min}

to the maximum contention window $CW_{\max}$. Here, $W_0 = CW_{\min} + 1$, $2^{m'} W_0 = CW_{\max} + 1$. For Direct Sequence Spread Spectrum (DSSS) physical layer (PHY) in 802.11, we have $m' = 5$. We note that the default values of m are different for data frame and RTS frame in 802.11 MAC.

In the remainder of the paper, we use τ to denote the probability of frame transmi- ssion in a randomly chosen time slot. Let p_{coll} be the probability that a station stays in its current state during backoff process, i.e., a station detects the busy channel and pauses its backoff counter during a backoff slot. Let p_f denote the probability of failed transmission, i.e., an ACK is not received.

2.2 Generalized Markov Chain Model

In [2], based on the assumption that collision probability is independent from the transmission history, a two-dimensional discrete-time Markov chain model represented as a stochastic process $(s(t), b(t))$ is defined, where $s(t)$ is the backoff stage i $(0,1,...,m)$ at time t and $b(t)$ is the backoff counter value k $(0,1,...,W_i - 1)$ at time t. We introduce an extra *idle* state for the unsaturated condition to simulate the free state of a station when there is no packet in the transmission buffer. **Fig. 2** shows the state diagram of the proposed two-dimensional Markov chain model, where each row represents a backoff stage and the states in a row represent different time slots within a stage. The solid lines present the saturated condition, while the extension of dashed lines represents unsaturated condition. In **Fig. 2**, η_0 is the probability that the queue is empty after transmission and q is the probability that packet arrived in a virtual slot. The virtual slot is the expected time per slot that is related the state of the Markov chain with the actual time spent in each state.

When a station has data to transmit and is on the backoff progress, if it detects the channel to be busy, it freezes its backoff timer. The probability that this occurs is p_{coll}. If the channel is idle, the station's backoff timer is decremented by 1. The probability that this occurs is $1 - p_{coll}$. When the backoff timer is decremented to 0, the station starts to transmit frames. The probability of successful transmission is $1 - p_f$. Therefore, a station has the probability of p_f to enter the next backoff stage. According to the exponential backoff algorithm within the limited retry, when the backoff stage reaches the maximum retransmission count, the station will enter an idle state with probability η_0 to wait for a new packet arrival or active a new round of exponential backoff process with probability $1 - \eta_0$.
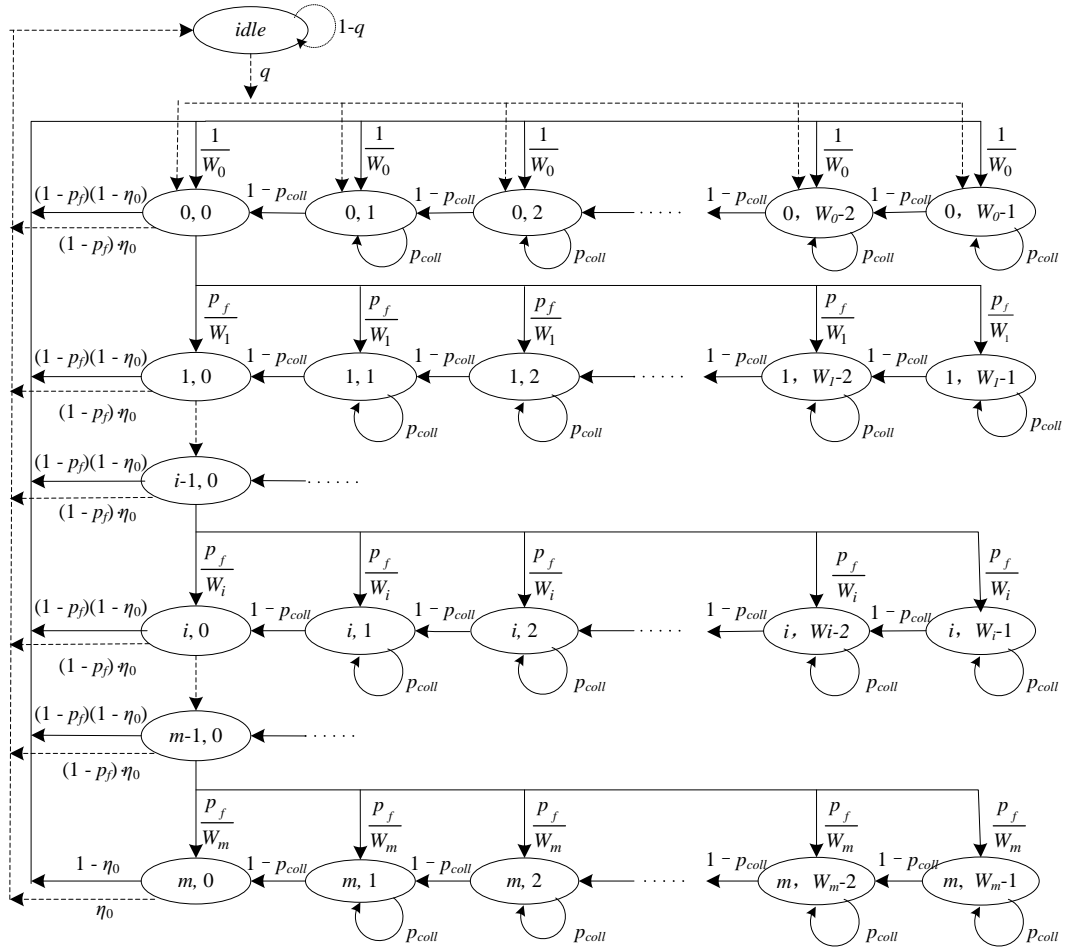
Fig. 2. State diagram of the proposed DCF model based on a two-dimentional Markov chain

Here, the one-step conditional state transition probability $P\{s(t+1)=i_1, b(t+1)=k_1 \mid s(t)=i_0, b(t)=k_0\}$ is denoted by the short notation $P\{i_1, k_1 \mid i_0, k_0\}$ in the following equations [2].

From **Fig. 2**, the non-null one-step transition probabilities are:

$$\begin{cases} P\{i,k \mid i,k+1\} = 1 - p_{coll} & i \in [0,m], \ k \in [0,W_i-2] & (a) \\ P\{i,k \mid i,k\} = p_{coll} & i \in [0,m], \ k \in [1,W_i-1] & (b) \\ P\{0,k \mid i,0\} = (1-p_f)(1-\eta_0)/W_0 & i \in (0,m], \ k \in [0,W_0-1] & (c) \\ P\{i,k \mid i-1,0\} = p_f / W_i & i \in [0,m), \ k \in [0,W_i-1] & (d) \\ P\{0,k \mid m,0\} = (1-\eta_0)/W_0 & k \in [0,W_0-1] & (e) \\ P\{0,k \mid Idle\} = q/W_0 & k \in [0,W_0-1] & (f) \\ P\{Idle \mid i,0\} = (1-p_f)\eta_0 & i \in [0,m) & (g) \\ P\{Idle \mid m,0\} = \eta_0 & & (h) \\ P\{Idle \mid Idle\} = 1 - q & & (i) \end{cases} \quad (2)$$

Let $b_{i,k} = \lim_{t\to\infty} P\{s(t)=i, b(t)=k\}$, $i \in [0,m], k \in [0,W_i-1]$ be the stationary distribution of any state $(s(t),b(t))$. As long as the retry stage does not exceed the retry limit, it increases after each failed transmission. It can be shown that

$$b_{i,0} = p_f \cdot b_{i-1,0} = p_f^i \cdot b_{0,0} \qquad 0 < i \leq m \qquad (3)$$

For each state, the sum of all outgoing/output transitions must be 1, i.e. a station must transfer into the next state. Of course, the station can stay at the same state. This is the situation of backoff freezing and it occurs with probability p_{coll}.

Let p_{idle} denote the steady state probability of the system being in the *idle* state, where the queue is empty as shown in **Fig. 2**. The probability can be calculated as follows:

$$p_{idle} = \frac{\eta_0}{q} \left(\sum_{j=0}^{m-1} (1-p_f)b_{j,0} + b_{m,0} \right) \qquad (4)$$

Due to chain regularity, $b_{i,k}$ can be written in a more general case according to **Fig. 2** as:

$$b_{i,k} = \frac{(W_i - k) \cdot p_f^i \cdot b_{0,0}}{W_i(1 - p_{coll})} \qquad i \in [0,m], k \in (0,W_i-1] \qquad (5)$$

$$p_{idle} = \frac{\eta_0}{q} b_{0,0} \qquad (6)$$

Sum up all states probabilities, we obtain:

$$\sum_{i=0}^{m} \sum_{k=0}^{W_i-1} b_{i,k} + P_{idle} = 1 \qquad (7)$$

All the steady state probabilities of the Markov chain can be expressed as the function of $b_{0,0}$ which can be obtained from Eqn. (7) after some mathematical manipulations. Since the

maximum retransmissions count m will be different from the maximum backoff exponent m', we get two cases:

$$b_{0,0} = \begin{cases} \left[\dfrac{W_0\left(1-p_f\right)[1-(2p_f)^{m'+1}]-\left(1-2p_f\right)(1-p_f^{m'+1})}{2\left(1-2p_f\right)\left(1-p_f\right)\left(1-p_{coll}\right)} + \dfrac{1-p_f^{m'+1}}{1-p_f} + \dfrac{\eta_0}{q} \right]^{-1}, & m \le m' \\[4mm] \left[\dfrac{\psi}{2\left(1-2p_f\right)\left(1-p_f\right)\left(1-p_{coll}\right)} + \dfrac{1-p_f^{m+1}}{1-p_f} + \dfrac{\eta_0}{q} \right]^{-1}, & m > m' \end{cases} \tag{8}$$

where $\psi = (1-p_f)W_0(1-(2p_f)^{m'+1})-(1-2p_f)(1-p_f^{m+1})+W_0 2^{m'}p_f^{m'+1}\left(1-p_f\right)\left(1-p_f^{m-m'}\right)$.

We note that the proposed model is a general model that includes many existing models as special cases. When $\eta_0 = 0$ and $q = 1$, the station is approaching saturated traffic conditions, and the proposed model becomes the same as that in [6]. If $\eta_0 = 0$, $q = 1$, $p_{coll} = 0$, and time freezing is not considered, Eqn. (8) becomes the same as that presented in [8] under saturated traffic conditions.

Furthermore, if $\eta_0 = 0$, $q = 1$, $p_{coll} = 0$, and setting unlimited retry $m \to \infty$ so that $p_f^{m+1} = 0$, and $m > m'$, Eqn. (8) can be rewritten as:

$$\lim_{m \to \infty} b_{0,0} \to \frac{2(1-2p_f)(1-p_f)}{(1-2p_f)(W_0+1)+p_f W_0(1-(2p_f)^{m'})} \tag{9}$$

Eqn. (9) is the same as the station state probability $b_{0,0}$ given by Bianchi [2] under saturated conditions.

Similarly, if $\eta_0 \ne 0$, $p_{coll} = 0$, and $p_f^{m+1} = 0$, Eqn. (8) becomes the same as $b_{0,0}$ in [11] under unsaturated conditions. In addition, if we set $\eta_0 = 1-q$, $p_{coll} = 0$, and $p_f^{m+1} = 0$, the proposed model reduces to that of F. Daneshgaran proposed in [13] under unsaturated conditions.

Therefore, the Markov chain model given in Eqn. (8) can be regarded as a generalized model. The relationship between the proposed model and other existing models are illustrated in **Fig. 3,** which clearly shows a key advantage of our model.
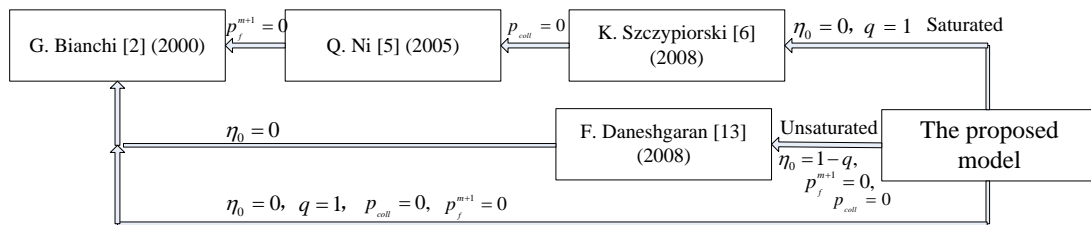

Fig. 3. Relationship between the proposed model and other exisiting models

2.3 Probability of Transmission and Probability of Failure Transmission

The probability of transmission τ is the sum of the steady state probabilities of transmission. Hence, the probability that a station transmits a packet in a randomly chosen time slot is:

$$\tau = \sum_{i=0}^{m} b_{i,0} = \frac{1 - p_f^{m+1}}{1 - p_f} \cdot b_{0,0} \tag{10}$$

In practical applications, m', m, W_0 are parameters that are typically fixed, while p_f, p_{coll}, q, η_0 are parameters that can be adjusted. The next step is to determine optimized values for p_f, p_{coll}, q, and η_0 given m', m, W_0.

Transmission failures can happen due to many reasons such as frame errors, channel noise, collision without capture etc. Here, we consider only transmission failure caused by collision. The failure transmission probability p_f is expressed as follows:

$$p_f = 1 - (1 - \tau)^{n-1} \tag{11}$$

When a station is in the backoff process, backoff counter will be suspended if a collision occurs (i.e., there is at least one other station sending a packet so that the channel is busy) during a backoff process. In this case, the station enters backoff freezing. The probability of collision is equal to one minus the probability that the channel is idle in a time slot. It is given by

$$p_{coll} = 1 - (1 - \tau)^n \tag{12}$$

2.4 The Probability q of Packet Arrival in a Virtual Slot

We have assumed that packet arrival follows a Poisson process. Therefore, q is dependent on the size of the virtual slot and the packet arrival rate λ. The probability that k packets arrive during a time interval T in a Poisson process with rate λ is given by

$$P\{a(T) = k\} = e^{-\lambda T} \frac{(\lambda T)^k}{k!} \tag{13}$$

The probability that at least one packet in a queue arrives during the virtual slot E_{slot} can be calculated by

$$q = 1 - P\{a(E_{slot}) = 0\} = 1 - e^{-\lambda E_{slot}} \tag{14}$$

A station has three states when it is on the decrease process of backoff: idle, collision and success transmit. The probabilities of these states are denoted as p_I, p_c and p_s. The duration of these states are denoted as T_I, T_c and T_s. Obviously, these state probabilities depend on the transmit probabilities of other n-1 stations. In the basic access case, the procedure of frame transmission is data-ACK two-way handshaking. Due to channel sharing policies, we get

$$\begin{cases} p_I = (1-\tau)^{n-1} \\ p_S = (n-1)\cdot\tau\cdot(1-\tau)^{n-2} \\ p_C = 1-p_I-p_S = 1-(1-\tau)^{n-1}-(n-1)\cdot\tau\cdot(1-\tau)^{n-2} \end{cases} \tag{15}$$

and

$$\begin{cases} T_I = \sigma \\ T_S = t_{data}+\delta+t_{SIFS}+t_{ack}+\delta+t_{DIFS} \\ T_C = t_{data}+ACK_{timeout}+t_{DIFS} \end{cases} \tag{16}$$

where σ is the duration of an empty system slot time, δ is the propagation delay, t_{data} and t_{ack} are the durations to transmit a data frame and a ACK frame, respectively, including the time in PHY and MAC, t_{SIFS} is the Short Inter-Frame Space (SIFS), t_{DIFS} is the Distributed Inter-Frame Space (DIFS), $ACK_{timeout}$ is the time setting for ACK responding time.

It follows that the average size of the virtual slot can be calculated by

$$E_{slot} = p_S T_S + p_I T_I + p_c T_C \tag{17}$$

2.5 The probability η0 that the queue is empty after transmission

Since packet processing at each station can be considered as a single serve system[4][10], we can use the M/G/1/K queuing model to analyze the queuing performance under the assumption of Poisson arrivals of packets. Let η_k be the probability that k packets remain in the queuing system after a packet's transmission at the steady state. Let X_k be the number of packets remain in the queue after the kth departure. The stochastic process X_k forms a discrete time Markov chain. Let a_k be the probability that there are k arrivals during the MAC service time.

The average channel access delay denoted as D is the average service time, which starts from a packet arrival at the head of the queue (HoQ) and ends when the packet is successfully transmitted or dropped. Let T_{max} be the maximum allowing time for a packet to stay in the buffer queue and T_{wait} be the waiting time in the queue. If T_{wait} is greater than T_{max}, the packet will be dropped. We assume that T_{max} is large enough, so D is less than $(T_{max}-T_{wait})$ and the packet will not be dropped during the access process.

The K-dimensional matrix of state transition probability $\mathbf{P}=\{p_{ij}\}=\Pr\{X_{k+1}=j\,|\,X_k=i\}$ is written as [4][10]:

$$\mathbf{P} = \begin{bmatrix} a_0 & a_1 & a_2 & \dots & a_{k-2} & 1-\sum_{k=0}^{K-2} a_k \\ a_0 & a_1 & a_2 & \dots & a_{k-2} & 1-\sum_{k=0}^{K-2} a_k \\ 0 & a_0 & a_1 & \dots & a_{k-3} & 1-\sum_{k=0}^{K-3} a_k \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_0 & 1-a_0 \end{bmatrix} \tag{18}$$

where $a_k = \int_0^\infty \Pr(a_k = k \mid D = t)dG(t)$, $G(t)$ is the probability distribution of access service time. Due to the Poisson inputs, we get

$$a_k = \sum_{D=0}^{\infty} \frac{e^{-\lambda D}(\lambda D)^k}{k!} \Pr\{time = D\} \approx \sum_{D=0}^{T_{max}-T_{wait}} \frac{e^{-\lambda D}(\lambda D)^k}{k!} \Pr\{time = D\} \tag{19}$$

Given $\eta = [\eta_0, \eta_1, \dots, \eta_{K-1}]$, according to the balance equation and normalization equation, we have

$$\eta \cdot \mathbf{P} = \eta \tag{20}$$

$$\sum_{k=0}^{K-1} \eta_k = 1 \tag{21}$$

From the above equations, we can compute the vector η, including the value of η_0. The solution to calculate a_k will be introduced in the next section.

2.6 The probability ak that there are k arrivals during the MAC service time

Average service time is also called average access delay, it can be divided into three parts: the successful transmission time t_s, failure transmission time t_f, and backoff time $t_{backoff}$. We have

$$D = t_s + t_f + t_{backoff} \tag{22}$$

where, $t_s = T_S$, $t_f = iT_C = iT_S$, $i \in [0, m]$.

For each backoff stage, the average selected backoff slot has half-length of the current contention window due to random uniform choosing [3]. The average backoff time $t_{backoff}$ is the sum of all backoff time in every backoff stage, and is equal to product of the average backoff slots and the average slot size. We have

$$t_{backoff} = E_{slot} \sum_{j=0}^{i} \frac{(W_j - 1)}{2} \tag{23}$$

Since we assume that the maximum number of retry is limited and $T_{\max}$ is sufficiently large, there is no packet dropped during the channel access. According to the M/G/1/K queuing model, a_k can be expressed as

$$
\begin{aligned}
a_k &= \sum_{D=0}^{T_{\max}-T_{wait}} \frac{e^{-\lambda D}(\lambda D)^k}{k!} \Pr\{time = D\} \\
&= \sum_{i=0}^{m} \frac{e^{-\lambda(T_s + iT_c + t_{backoff}(i))}[\lambda(T_s + iT_c + t_{backoff}(i))]^k}{k!} \Pr(t = T_s + iT_c + t_{backoff}(i)) \\
&= \sum_{i=0}^{m-1} \frac{e^{-\lambda(T_s + iT_c + t_{backoff}(i))}\left[\lambda(T_s + iT_c + t_{backoff}(i))\right]^k}{k!} p_f^i (1 - p_f) \\
&\quad + \frac{e^{-\lambda(T_s + mT_c + t_{backoff}(m))}\left[\lambda(T_s + mT_c + t_{backoff}(m))\right]^k}{k!} p_f^m
\end{aligned}
\tag{24}
$$

Substitute (24) into (18) and (20), η_0 can be expressed as a function of p_f as follows:

$$
\eta_0 = f(p_f, \lambda, T_s, T_c, m, W_0, \sigma, \tau, n)
\tag{25}
$$

For an arbitrary buffer size K, and an exponential distribution of service time, η_0 can be approximately calculated using the M/M/1/K queuing model. The approximation is given by [11]

$$
\eta_0 = \frac{1}{1 + \rho + \rho^2 + \ldots + \rho^K}
\tag{26}
$$

where ρ is the traffic intensity or queue utilization, and $\rho = \lambda D$.

We can calculate the five parameters. τ, p_f, p_{coll}, q and η_0 from equations (10), (11), (12), (14) and (25) by numerical methods.

2.7 Throughput

Let S be the normalized system throughput defined as successfully transmitted payload bits during a randomly selected time slot [10]:

$$
S = \frac{E[data]}{E[slot]} = \frac{8 L_{data} p_S'}{p_S' T_S + p_I' T_I + p_C' T_C}
\tag{27}
$$

where p_S' is the probability that a channel is busy for a successfully transmission over the network and its corresponding time denoted as $T_S{}'$, p_I' is the probability that the channel is idle and its corresponding time denoted as $T_I{}'$, and p_C' is the probability of collision when more than one stations are sending data simultaneously and its corresponding time denoted

as $T_C{}'$. These probabilities can be expressed respectively as:

$$\begin{cases} p'_s = \mathrm{n}\tau\left(1-\tau\right)^{n-1} \\ p'_I = \left(1-\tau\right)^n \\ p'_C = 1-\left(1-\tau\right)^n - \mathrm{n}\tau\left(1-\tau\right)^{n-1} \end{cases} \tag{28}$$

3. Model Validations and Analysis

The proposed analytical model is verified by the NS-3.11[18] simulator by comparing the simulation results with the analytical results. The simulated time is set to be 100 seconds and the results are averaged over 10 simulation runs.

We assume that all stations can communicate with each other and DSSS is used as the PHY modulation technique. The default parameters are shown in **Table 2**. The channel bit rate is 1Mbps. The PHY header (including PLCP header and preamble) and MAC header (including frame header and trailer) are 24 bytes and 28 bytes respectively. Hence, the duration of data frame and ACK frame can be calculated as: $t_{data} = 192bits@1Mbps + (L_{data}+28)bytes@1Mbps = 8608\mu s$, $t_{ack} = 192bits@1Mbps + 14bytes@1Mbps = 304\mu s$.

Table 2. Parameter values for simulation and model

Parameter	Value
Data rate	1 Mbps
Data packet length (L_{data})	1024 bytes
Contention window minimum (CW_{min})	31
Contention window maximum (CW_{max})	1023
SIFS	10 μs
DIFS	50 μs
Slot time (σ)	20 μs
Maximum retries (m)	5
δ	1 μs
T_s	8974 μs
T_c	8974 μs
t_{data}	8608 μs
t_{ack}	304μs

Fig. 4 show the throughput versus traffic load with different station numbers *n*=5 and *n*=10 respectively under large enough buffer size in the network. In each figure, the throughput increases as function of the traffic load until it reaches the saturated condition. It is observed that given the same traffic load, saturation occurs earlier in **Fig. 4-(b)**, i.e., it is easier to reach the saturation condition in denser networks. In both figures, only slight differences are observed between the numerical and simulation results. This validates the accuracy of our analytical model.
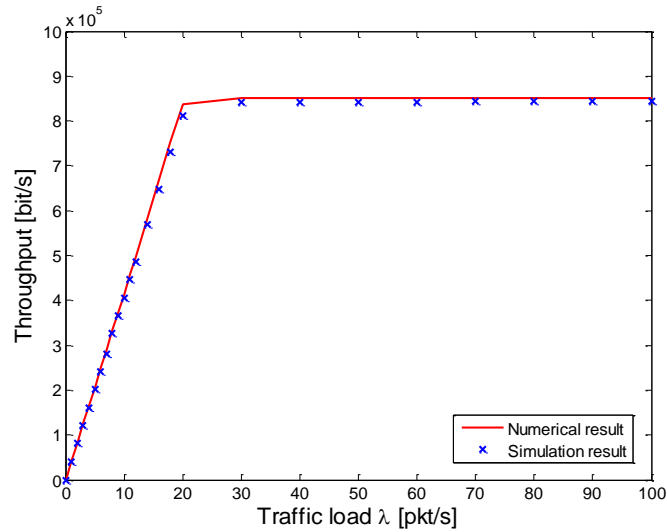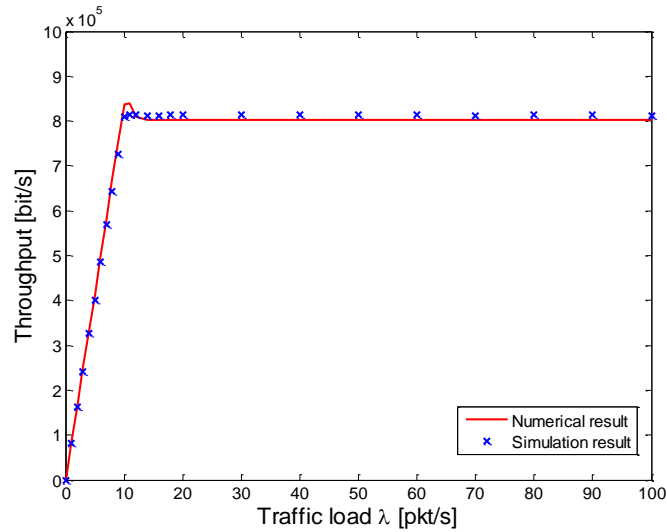
(a) *n*=5

(b) *n*=10

Fig. 4. Throughput versus traffic load

Fig. 5 plots the throughput versus the traffic load for two buffer sizes *K*=10 and *K*=100. The number of stations is fixed to *n*=10. We can see that increasing the buffer size has little impact on the throughput. When the traffic load is greater than the saturated point of traffic, there are always some packets in the queue required to transmit regardless of the buffer size, therefore the throughput reaches at saturated state. Again, a close match between the numerical and simulation results is observed.
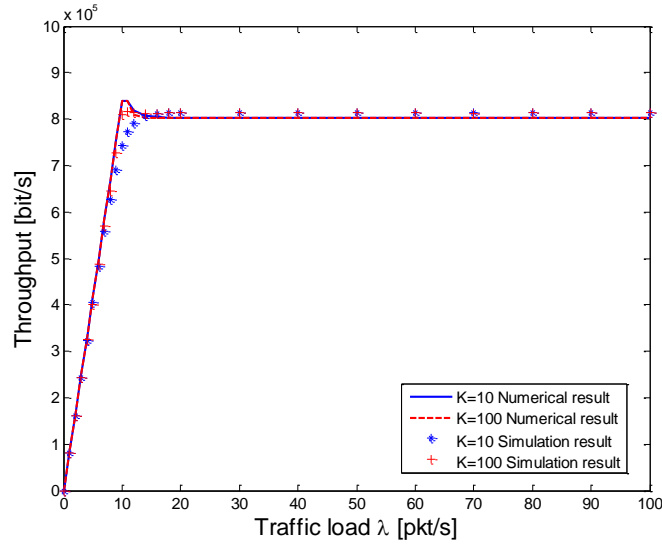
Fig. 5. Throughput versus traffic load under different buffer sizes (K=10, 100)

When $K=1$, η_0 is always equal to 1. Under the saturated conditions, η_0 is always equal to 0. These two situations are unrelated to the queuing model. Thus, we consider the situation of $K>1$, and compare the M/G/1/K queuing model with other models. Among all existing models, F. Daneshgaran's model in [13] is the more comprehensive model and also the most related one to the proposed model. We therefore choose this model to be compared with the proposed model.

Fig.6 shows the numerical results obtained from the proposed model with the M/G/1/K queuing model and the M/M/1/K model. Numerical results obtained from the model presented by F. Daneshgaran in [13] as well as NS3-based simulation results are also presented for comparison purpose. We observe that our proposed model with the M/G/1/K queuing model provides the best approximation to the simulation result. We note that the model in [13] does not consider the limited retransmission count so that the numerical value of throughput in [13] is higher than that in our proposed model and the simulated result.

Fig. 7 presents throughput as a function of stations number n when the buffer size is sufficiently large. We can see that along with the increasing stations number, the throughput increases until it reaches the saturated condition.

In **Fig. 4**, **Fig. 5**, **Fig. 6** and **Fig. 7**, the proposed model closely matches the corresponding simulated results. The minor discrepancy between the simulation and numerical results may be due to the random influence of the PHY layer such as channel fading.
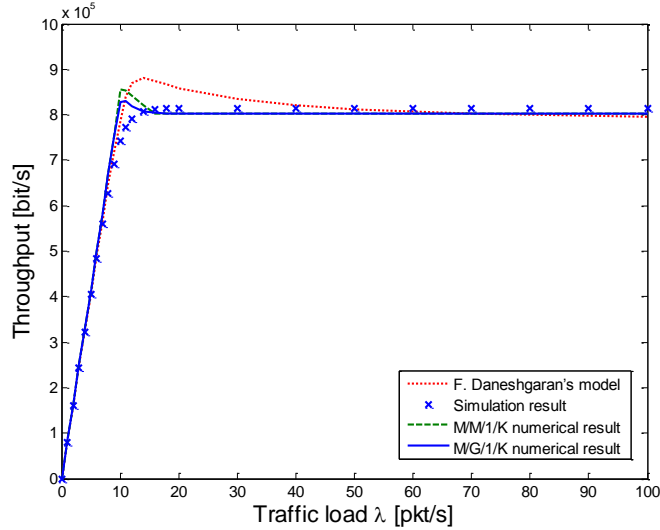
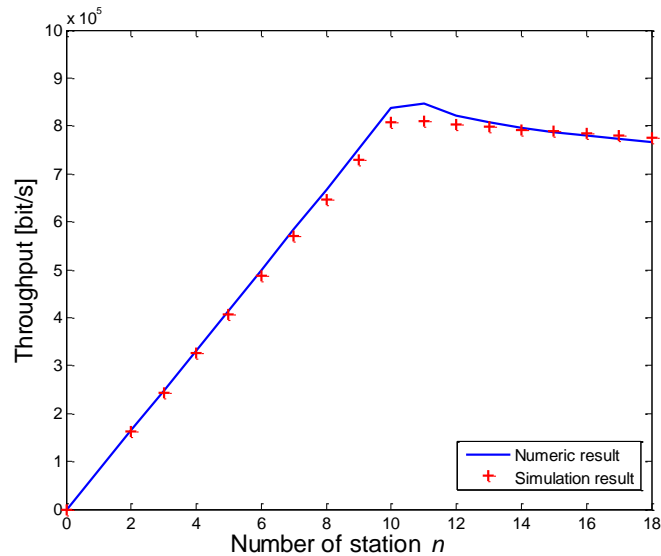**Fig. 6.** Throughput versus traffic load for different queuing model

Fig. 7. Throughput versus numbers of station

4. Conclusion

The analytic model proposed in this paper considers the fully backoff progress of IEEE 802.11 DCF. The proposed model is able to give theoretical insights into the complex relationships between various system parameters (e.g., traffic type, buffer size, and station number) and the resulting system performance. In addition, the proposed model uses the M/G/1/K queuing model to characterize the performance of 802.11 under the saturated and non-saturated conditions. We have demonstrated that the proposed model can be used to numerically predict system performance that highly matches corresponding simulation results. Furthermore, we

have shown that our model outperforms existing models in terms of accuracy and generality. In conclusion, the proposed model is shown to be a powerful analytical tool for DCF performance analysis. It can be used for various purposes such as parameter optimization and protocol design. Our future work will aim to develop analytical models for non-Poisson traffic types.

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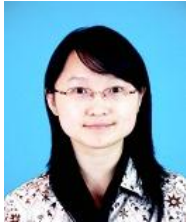
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