

# A Parallel Combinatory OFDM System with Weighted Phase Subcarriers

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## Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is usually regarded as a spectral efficient multicarrier modulation technique, yet it suffers from a high peak-to-average power ratio (PAPR) problem. Among all the existing PAPR reduction techniques in OFDM systems, side information based PAPR reduction techniques such as partial transmit sequence (PTS) and selective mapping (SLM) schemes, have attracted the most attention. However, the transmission of side information results in somewhat spectral loss and this does not significantly improve the bit error rate (BER) performance. Parallel combinatory (PC) OFDM yields higher spectral efficiency (SE) and better BER performance on Gaussian channels, while is a little but not obvious PAPR improvement over the ordinary OFDM system. This investigation aimed to design a 'perfect' OFDM system. We introduce the side information to rotate the subcarrier phases of our novel PC-OFDM system structure, and call this new system the SIPC(Side information based Parallel Combinatory)-OFDM system. The proposed system achieves better PAPR and SE performance. In addition, considering the tradeoff of system parameters, the proposed system also has the properties of a higher BER.

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**Keywords:** Parallel combinatory, peak-to-average power ratio, side information, spectral efficiency, BER

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## 1. Introduction

Recently, as diverse communication networks provide new options for the connectivity of products, this connectivity of various communication networks creates a diverse data transmission rate, which includes the connectivity to Internet or to a computer using a home network. Applications such as digital audio broadcasting (DAB), digital video broadcasting (DVB) and wireless multimedia require a high transmission rate, in other words, higher spectral efficiency (SE). As wireless communication becomes an ever-more important and pervasive part of our daily life, radio frequency bandwidth usage increases greatly. Improving the SE while guaranteeing the quality of service is critical. In order to solve this problem, it is necessary that we pay closer attention to spectral and power efficiency issues.

Orthogonal Frequency Division Multiplexing (OFDM), as the most well-known multicarrier modulation technique, has been widely used or recommended for the wireless communication, because of its inherent advantage of high tolerance to multipath fading in wireless channels with a high transmission data rate. However, OFDM signals suffer from a high peak-to-average power ratio (PAPR) which leads to power inefficiency in the radio frequency portion of the transmitter. Many PAPR reduction techniques have been proposed in the literature, among them, partial transmit sequence (PTS) and selective mapping (SLM) are the most attractive approaches since they are distortionless. These techniques use side information to reduce the peak power, which results in lower spectral efficiency (SE) to some extent.

Parallel combinatory (PC) signaling was first proposed for spectrum system in [1] as a method for increasing the SE. Then it was introduced to the OFDM system as a PC-OFDM system using M-PSK modulation, which selects a subset of  $N$  available subcarriers and transmits  $(M+1)^N$  different kinds of information for the selected subset at each symbol interval, rather than  $M^N$  different waveforms because PAPR grows linearly with the number of subcarriers in the conventional OFDM system. As for the PC-OFDM system, the selected subcarriers are modulated by points from an  $(M+1)$ -PSK signal constellation, which is an M-PSK signal constellation diagram and the remaining subcarriers are modulated with zero amplitude point. Because the number of selected subcarriers is less than that of an ordinary OFDM system, the PC-OFDM system can be carefully designed to have a lower PAPR and higher SE. In addition, this new system was also demonstrated to have a lower bit error rate (BER) over AWGN (Additive White Gaussian Noise) and Rayleigh fading channel in [2]. And it was also applied to a UWB system in [3][4] and optical wireless communication systems in [5].

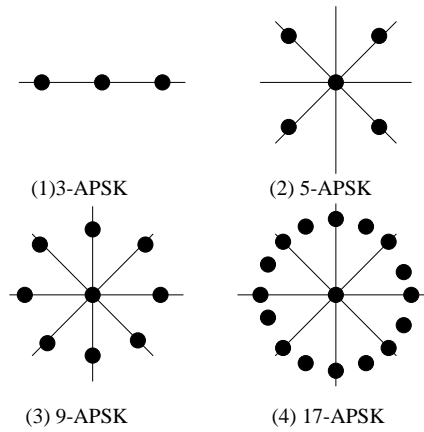
Moreover, since PTS and SLM require the side information at the receiver to recover the original transmitted sequence, the SE will be less. The proposed side information generator used in our proposed side information based PC-OFDM wireless communication system enables the side information detection with only one state index value to be transmitted at the transmitter. This novel system can enhance PAPR performance and BER performance, in addition to achieving higher SE.

The subsequent sections of this paper are organized as follows. The conventional PC-OFDM system is described in section II. The proposed OFDM wireless system and its algorithm are described in section III. In section IV, numerical analysis and simulation results of SE, PAPR, and BER are evaluated and compared to the conventional OFDM systems. Finally, the conclusion of this paper is given in Section V.

## 2. Conventional System Model

PTS and SLM are usually introduced into an OFDM system to reduce the high peak power. In brief, PTS divides the incoming long data stream into disjointed sub-blocks and optimizes the phases for each sub-block signal. Similarly, the incoming data stream of SLM is duplicated and optimized by the side information so that the combined signal streams has a lower PAPR.

While PTS and SLM schemes require a bank of IFFTs to generate a set of candidate signals, the phase rotation sequence (side information), which is inserted into candidate signals, requires extra subcarriers to transmit them, thus SE is reduced due to the need to share the side information with the receiver. Additionally, it was demonstrated in [6] that PTS and SLM have no improvement on the BER performance over the conventional OFDM system.



**Fig.1.** A constellation diagram of (M+1)-ary APSK

In this section, we mainly focus on the conventional PC-OFDM system which was proposed for increasing SE.

We consider OFDM as the underlying multicarrier modulation communication technique since it has emerged as a major candidate for the future broadband wireless communication systems, in which the serial data is first converted to parallel and then the parallel data streams are transmitted to the orthogonal channels synchronously. This parallel transmission scheme benefits the PC code to be available to a parallel data transmission scheme.

We apply the PC code to the new OFDM modulation system. First, we introduce a (M+1)-APSK (amplitude and phase shift keying) modulation method, as we mentioned above, i.e., M-ary constellation signals are extended with a zero-amplitude point at the origin of the signal space as shown in Fig.1 with different levels of modulation. Introducing this new modulation scheme to the conventional OFDM system, we can obtain the whole PC-OFDM system as shown in Fig. 2. At the transmitter side, all the  $N$  subcarriers are comprised of two parts,  $N_{PSK}$  nonzero amplitude subcarriers and  $N_{Null}$  zero amplitude subcarriers.  $N_{PSK}$  subcarriers are modulated by the points from the ordinary PSK constellation diagram and

$N_{Null}$  are mapped by  $\left\lceil \log_2 \binom{N}{N_{PSK}} \right\rceil$  bits to decide which subset of subcarriers will be applied.

Here, we map  $m_{PSK}$  bits to modulate the phases of the selected  $N_{PSK}$  carriers and map  $m_{PC}$  bits to determine the subset of selected  $N_{PSK}$  subcarriers out of a total of  $N$  subcarriers,

which can be realized in  $\binom{N}{N_{PSK}}$  ways.

Thus the total bits of the subcarrier consist of the PSK bits and PC bits  $m_{tot} = [m_{PSK}, m_{PC}]$ , which can be transmitted by one PC-OFDM symbol as

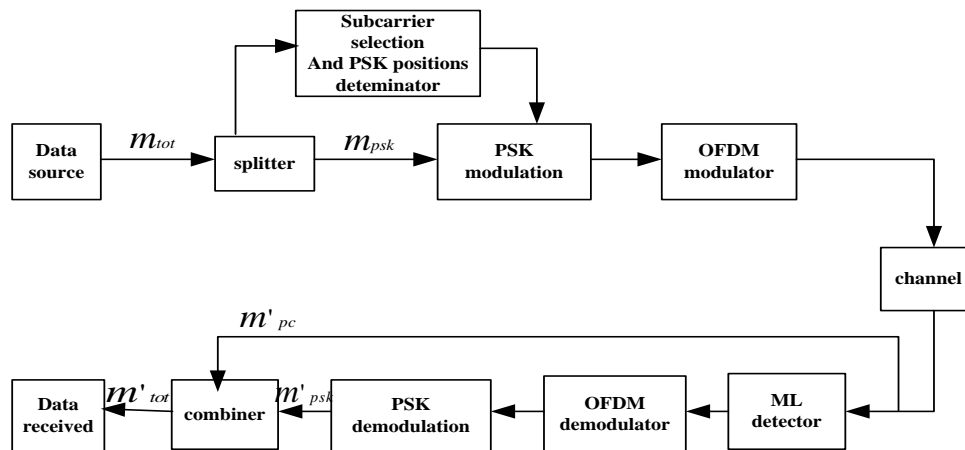
$$m_{tot} = m_{psk} + m_{pc} = N_{psk} \cdot \log_2 M + \left\lfloor \log_2 \binom{N}{N_{psk}} \right\rfloor (N_{Null} = N - N_{psk}) \quad (1)$$

where  $\lfloor x \rfloor$  represents the largest integer smaller than or equal to  $x$ .

The following steps in the transmitter are the same as the ordinary OFDM system, which include serial to parallel conversion, IFFT (Inverse Fast Fourier Transform) operation and transmission to the channel.

At the receiver side, as suggested in [2], we use an ML (Maximum Likelihood) detector to find the point in the  $(M+1)$ -APSK constellation diagram that is closest to the received data in each subcarrier after FFT (Fast Fourier Transform) operation.

The problem of the above PC-OFDM system is the mapping procedure at the transmitter. First we need to determine those subcarriers that are used for phase modulation and for zero amplitude. The easiest way is to generate a lookup table of  $2^{m_{pc}}$  columns and  $N$  rows thus it can be considered as  $2^{m_{pc}}$  vectors with each vector length of  $N$ . Then we can fill the zero amplitude subcarriers' position with value "0" and M-ary modulated subcarriers' positions with value " $N_{psk}$ " respectively. However, we can see that this table fails because of the exponential increase in storage when  $m_{pc}$  increases. It will occupy a large storage of memory and take longer time for searching for the subcarrier subset. Thus, we choose an easier solution in the next section.



**Fig.2.** A block diagram of the parallel combinatory OFDM (PC-OFDM) system

### 3. Proposed System Description

The PAPR is the most serious handicap of OFDM systems. Many PAPR reduction techniques

have been proposed, such as clipping [7], tone reservation (TR) [8] and tone injection (TI) [9], constellation extension [10][11][12], and so on. Among them selective mapping (SLM) [13] and partial transmit sequence (PTS) [14][15][16] are well-known due to their substantial PAPR reducing capability without sacrificing the signal fidelity. Their key idea of them is both to generate  $U > 1$  statistically independent OFDM symbol sequences and to transmit the  $u_{ih}$  sequence with the lowest PAPR. The side information is introduced to rotate the original transmitting data sequences from one to another to minimize the peak  $|W_n|$  of the signal. Since PTS and SLM have similar performance as pointed out in [17], for convenience, we will only mention about the PTS scheme in an OFDM system in the following parts of this paper.

In the conventional PC-OFDM system, since not all of the subcarriers are occupied, a certain amount of data information will be lost. However, the selected subcarriers are associated with the combination type, so that the lost data information can be compensated for. This system has been proved to outperform the conventional OFDM system in many aspects [1-4]. Usually we know, it would be better if an OFDM system is capable of high power efficiency and high SE. However it is hard to be realized in OFDM since there is always a tradeoff between them unless other complicated extra techniques are utilized. Most of the techniques have their own advantages superiority and inferiority. Fortunately, we observed that applying the side information to rotate the subcarriers of the PC-OFDM will assure a better performance and higher efficiency.

We exploit the inherent characteristics of a PC-OFDM system and do some modifications to get a novel system, called as SIPC-OFDM system. The block diagram of its transmitter is given in Fig. 3.

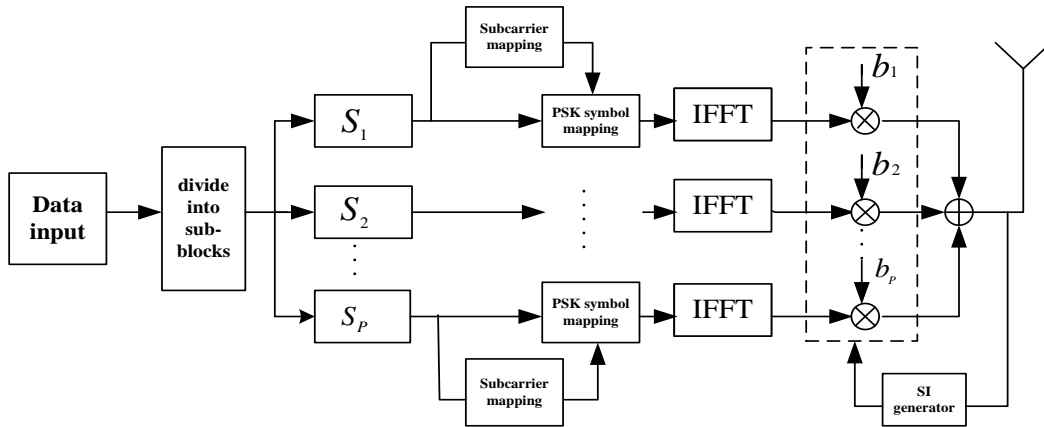


Fig.3. A transmitter block diagram of the SIPC-OFDM system

Suppose rectangular pulse shaping is employed, then the input data transmitted on the  $K_{th}$  subcarrier can be expressed as follows:

$$S[n] = \begin{cases} \frac{1}{\sqrt{N}} \sum_{k=1}^N D_k e^{j2\pi kn/N} & n \in [1, N] \\ 0 & otherwise \end{cases} \quad (2)$$

where  $N$  is the total number of subcarriers, and  $D_k$  represents the data symbol transmitted on the  $K_{th}$  subcarrier.

$P$  statistically independent sequences are first generated from the message-bearing bits sequence  $S[n](n = 0, 1, \dots, N - 1)$ , and these  $P$  sequences, which consist of a contiguous set of equal size subcarriers  $Q(QP = N)$ . Let the parallel data sequence  $S_i$  be expressed as an  $M$ -dimensional vector  $S_i = [S_0^i, S_1^i, \dots, S_{Q-1}^i](i = 1, 2, \dots, P)$ , the total number of PSK modulated subcarriers is determined as  $N_{PSK}$ , ( $N_{PSK} \leq N$ ) according to (1), we can select the first  $m_{PC}^i$  bits and the remaining bits  $m_{PSK}^i$  of the vector  $S_i$  to denote the PC bits and PSK bits respectively. The parallel combinatory operation, which is located before the sub-block division in the conventional PC-OFDM system, is shifted behind the sub-block division in our novel system. The mapping procedure of PSK bits  $m_{PSK}^i$  follows the Gray code according to  $\{11 \rightarrow 1, 10 \rightarrow -j, 01 \rightarrow j, 00 \rightarrow -1\}$  under QPSK modulation, and the pairs of PSK bits are mapped to PSK symbols.

Assuming that  $m_{tot}^i = [1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1]$ , for the  $i_{th}$  sub-block,

we can produce the vector  $C = [j, -j, 1, -1, -j, j, 1]$  under the assumption that

$M = 4$  ( $M$ : modulation level), ( $N = 8$ ),  $N_{PSK} = 7$ , the first 3  $\left( \left\lfloor \log_2 \left( \frac{N}{N_{PSK}} \right) \right\rfloor = 3 \right)$  bits (1,0,1) are transmitted in the choice of which subcarriers are zero amplitude and the following 14bits in the received phase of the selected subcarriers.

Further in this case, we will establish a look-up table of  $2^3 = 8$  vectors for the  $i_{th}$  sub-block as in the **Table 1**, where PC bits (1,0,1) denotes PC symbols positions vectors  $[1, 2, 0, 3, 4, 5, 6, 7]$ . Then the digital modulated symbols will be placed on the corresponding subcarriers as the order of the positions vector and the '0' will be inserted accordingly as  $C = [j, -j, 0, 1, -1, -j, j, 1]$ .

By applying the  $N$ -point IFFTs operation of the frequency-domain sequence  $D_i$ , the same operation in each sub-block, we can obtain the total  $P$  time-domain sequence  $x_i$  as

$$x_i = IFFT \{D_i\} \quad (3)$$

The objective of PTS or SLM is to form a weighted combination of the  $P$  disjoint sets. Here the weighting factors  $\{b_i\}$ , which is also called as side information, they are further assumed to be pure rotations (*i.e.*  $b_i = e^{j\phi_i}$ ,  $\phi_i = (0, 2\pi)$ ), and are inserted to each sub-block. To reduce the minimization complexity, we use a suboptimal choice by limiting the phase rotation factors  $\phi_i$  of the  $i_{th}$  sub-block to be uniformly uniform distributed between 0 and  $2\pi$ , a vector of  $(P - 1)$  weighting phase factors are equally distributed in the unit circle as the

values of  $V = \left[ e^{j\left(\frac{2\pi}{P}\right)}, e^{j\left(\frac{2\pi}{P-1}\right)}, \dots, e^{j(2\pi)} \right]$ ,  $b = [1, V]$ . The vector  $V$  is permuted  $P!$  times to construct the codebook  $B$ . Then we can get the vector of the weighting phase factors as  $\left[ e^{i\phi_1}, e^{i\phi_2}, \dots, e^{i\phi_p} \right]^T$ . Thus, the whole new symbol sequence  $\tilde{x}_i$  which will be transmitted to the channel can be expressed as

$$\tilde{x} = \sum_{i=1}^P x_i b_i = \sum_{i=1}^P x_i b_i \quad (4)$$

Due to the side information  $\sum_{i=1}^P \{b_i\}$  the phase angles of the subcarriers in each sub-block are changed, and a number of *opt* iterations are performed before obtaining the optimally combined sequences. Finally, the optimal PAPR can be found using the equality:

$$\varepsilon_{\max} = \min_{b_1, b_2, \dots, b_p} \left( \max_{0 \leq n \leq N} \left| \sum_{i=1}^P x_i b_{i\_opt} \right|^2 \right) \quad (5)$$

where  $\sum_{i=1}^P \{b_{i\_opt}\}$  labels the optimal weighting phase factors as the selected side information at the transmitter side.

In order to recover the original transmitted signal perfectly, a portion of the spectrum must be allocated for the transmission of side information. Many novel algorithms have been suggested to embed the side information at the transmitter or decode the original transmitted signals without side information at the receiver.

In the following, we introduce our proposed suboptimal strategy to show the side information decoding process by only one state index value  $m$  instead of transmitting the whole weighted phase sequence at the transmitter.

At the transmitter, suppose the optimal weighted phase sequence  $\sum_{i=1}^P \{b_{i\_opt}\}$  has been found and *opt* has been memorized after a number of iterations. Now we explain the encoding process which can be described as

Initialization

$$\text{Set state index } m = [1 : P!], (i = 1, 2, \dots, P)$$

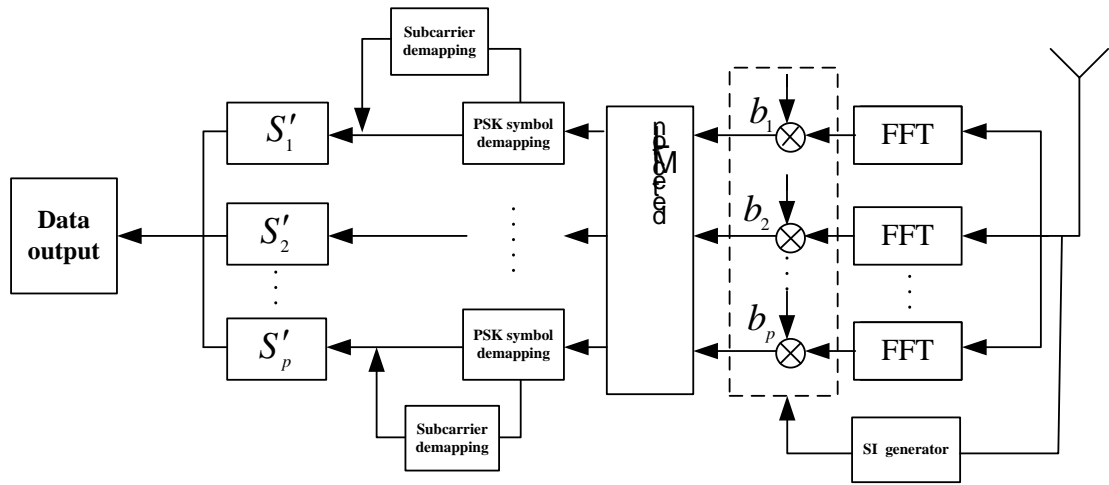
Let  $m$  denote the iteration time of the optimal side information sequence  $\sum_{i=1}^P \{b_{i\_opt}\}$  i.e.,

$$m = \text{opt}$$

Generate an empty  $P \times 1$  matrix  $G$  to wait for the input elements  $\sum_{i=1}^P \{G_{m,i}\}$

Pair the map  $\sum_{i=1}^P \{b_{i\_opt}\}$  to the elements of the matrix  $G = \sum_{i=1}^P \{G_{m,i}\}$  and save the new matrix  $G$

The above process is viewed as a ‘side information (SI) generator’ in this paper. We just need to know which state index  $m$  has been put into the SI generator and we can get the optimal side information  $\sum_{i=1}^P \{b_{i\_opt}\}$  at the output.



**Fig. 4.** A receiver block diagram of the SIPC-OFDM system

At the receiver, the time domain optimized signal  $\tilde{x} = \sum_{i=1}^P \tilde{x}_i$  will be transformed into the frequency-domain by a FFT operation as follows

$$r = H\tilde{x} + noise \tag{6}$$

where  $n$  represents the additive Gaussian white noise vector with zero mean and variance  $\sigma_n^2$  in the channel.  $H = [H_1, H_2, \dots, H_P]^T$ , vector  $H$  is the frequency channel response of all the  $N$  subcarriers with  $P$  sub-blocks.

The signals are converted to the frequency domain by FFT operation, which can be computed as

$$R_i = FFT\{r_i\} = FFT\{H\tilde{x}_i + noise\} \tag{7}$$

where  $H_i = [H_{i,1}, H_{i,2}, \dots, H_{i,Q}]^T$  ( $0 \leq i \leq P$ ) denotes the channel coefficients of the  $i_{th}$  sub-block of size  $Q$ .

Generally, few papers have dealt with the decoding process in the receiver in a PTS scheme until now. One exception was a novel PTS proposed in [14], in which the computation complexity was reduced but no attention has been paid to how to decode the optimized signal



sequence at the receiver side. We apply the state  $m$  to the SI generator to retrieve side information  $\sum_{i=1}^P \{b_{i\_opt}\}$  of the  $i_{th}$  sub-block. Since the state  $m$  only needs a few bits to transmit, compared to a long information bits sequence, the loss of these bits can be ignored. To remove the side information, we only need to multiply the inverse rotation phase  $\{b_{i\_opt}^*\}$  of the side information to the received signals. Thus, (7) can be rewritten as

$$R'_i = R_i \cdot b_{i\_opt}^* = FFT \left\{ \left( H \sum_{i=1}^P x_i b_{i\_opt} + n \right) \cdot b_{i\_opt}^* \right\} = FFT \left\{ \sum_{i=1}^P H_i x_i \right\} + noise' \quad (8)$$

where  $*$  denotes the inverse rotation of the side information phase and  $noise'$  indicates the affected frequency domain noise.

Further, the ML detector is utilized to find the point  $\hat{D}_n$  as in the above process, which is the most approximate to the received value  $R_n$  in the  $(M+1)$ -PSK constellation diagram for each subcarrier. Suppose  $N_{PSK}'$  non-zero symbols are detected by the ML detector at the receiver's side, which are closest to the constellation points. These detected symbols are conserved in the float memory and conveniently convenient to be accessed. There are three cases that we need to consider respectively.

$N_{PSK}' = N_{PSK}$ , we detect the same number of non-zero amplitude subcarriers as that of the transmitter. It is the simplest case that we do not need to explain it explicitly by figure.

$N_{PSK}' > N_{PSK}$ , there are much more non-zero subcarriers than that of in the transmitter, supposing  $N_{PSK}' = 8$  in the above example, for the  $i_{th}$  sub-block, one zero amplitude subcarrier is necessary for finding the corresponding subcarrier mapping type. The non-zero  $(N_{PSK} - N_{PSK}')$  subcarriers that have the smallest amplitude are detected to be zero. As Fig. 5-(a) shows that only six signals are ML detected to be zero amplitude so that the signal just outside the zero decision border is detected to be zero amplitude signal. (this signal is indicated as the error)

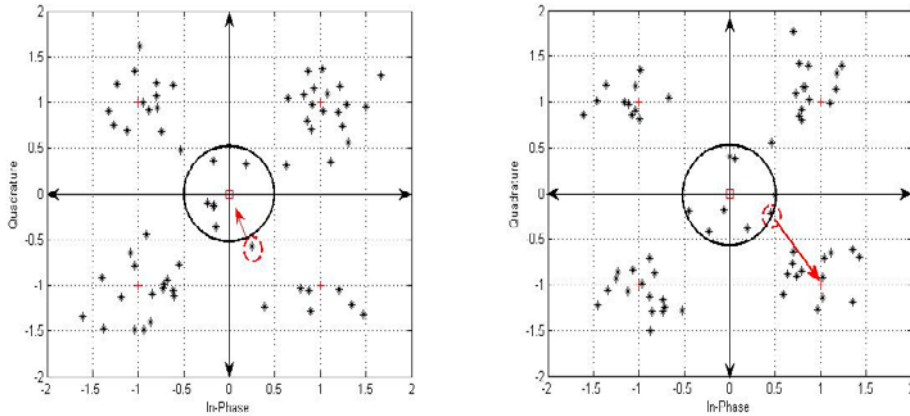
$N_{PSK}' < N_{PSK}$ , fewer non-zero amplitude signals are detected. Suppose  $N_{PSK}' = 6$ , we only need to focus on the detected zero amplitude signals. Comparing these signal amplitudes, the ones with the largest amplitudes will be viewed as the non-zero amplitude signals. Fig. 5-(b) shows that eight signals are detected as zero amplitude signals after ML detection, while only seven of them are required, so the signal which is nearest to the zero decision border is detected to be non-zero amplitude signal. (this signal is indicated as the error)

**Table.1.** The look up table of mapping the PC bits and PSK symbols positions

(1-7: the order of the PSK symbols' subcarrier 0: the zero amplitude subcarrier position)

PC bits	PSK symbols positions
[0,0,0]	[1,2,3,4,5,6,7,0]
[0,0,1]	[1,2,3,4,5,6,0,7]

[0,1,0]	[1,2,3,4,5,0,6,7]
[0,1,1]	[1,2,3,4,0,5,6,7]
[1,0,0]	[1,2,3,0,4,5,6,7]
[1,0,1]	[1,2,0,3,4,5,6,7]
[1,1,0]	[1,0,2,3,4,5,6,7]
[1,1,1]	[0,1,2,3,4,5,6,7]



(a) case 2

(b) case 3.

**Fig.4.** A constellation diagram of the ML detected signals with  $N = 8, N_{PSK} = 7, P = 8$

(square: the zero amplitude signals plus: the transmitted signal asterisk: the detected signal)

Through the above process, the zero amplitude positions can be determined for the  $i_{th}$  sub-block. With the benefit of these positions, the corresponding PC bits sequence is determined according to the PC bits mapping table as **Table 1**. The positions of the PC bits and PSK symbols are one-to-one pair-wise mapped. Suppose we receive the same vector as before,  $C' = [0, j, -j, 1, -1, -j, j, 1]$ , the '0' position indicates that the vector of its PSK symbols positions is  $[0, 1, 2, 3, 4, 5, 6, 7]$ , then we will decode it to the vector  $m_{tot}' = [1, 1, 1, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 0, 1, 1, 1]$  for the  $i_{th}$  sub-block. The entire bits sequence  $S' = \sum_{i=1}^P S'_i$  is obtained for all the sub-blocks.

## 4. Numerical Analysis

### 1.1 Spectral Efficiency(SE) of the SIPC-OFDM System

We can explain SE by the information data rate, the frequency spacing is identical to that of the OFDM system, but more bits can be transmitted in the same symbol interval, a higher data transmission rate can be achieved than that of OFDM system thus the SE is increased. Next, we explain it by numerical analysis.

Firstly, for SE, which depends on the number of subcarriers  $N$  and the pulse shaping technique employed, rectangular pulse shaping is assumed in (2). Less frequently but unambiguously, the SE is measured in (bits/s)/Hz [18]. Second, since in the same symbol

duration  $T$ , we may define it as  $\eta_{SE} = \frac{m_{tot}}{BW_{sub}}$ , where  $m_{tot}$  is the number of total bits that can

be transmitted in one symbol duration and  $BW_{sub}$  is the system subcarrier bandwidth under the assumption that each subcarrier occupies the same bandwidth. Thus we can get the conventional PC-OFDM system and novel PC-OFDM system as follows:

$$\eta_{SE_1} = \frac{p \cdot N_{psk} \cdot \log_2 M + \left\lfloor \log_2 \binom{N}{N_{Null}} \right\rfloor}{BW_{sub}} \quad (9)$$

$$\eta_{SE_2} = \frac{p \cdot \left( N_{psk} \cdot \log_2 M + \left\lfloor \log_2 \binom{N}{N_{Null}} \right\rfloor \right)}{BW_{sub}} \quad (10)$$

Suppose we use modulation level  $M = 4$ , the number of subcarriers in one sub-block  $N = 10$ , the number of PSK modulated subcarriers in one sub-block  $N_{psk} = 8$  and the number of sub-blocks  $P = 8$ , in the novel PC-OFDM system, one OFDM symbol can transmit

$$m_{tot}' = p \cdot \left( N_{psk} \cdot \log_2 M + \left\lfloor \log_2 \binom{N}{N_{Null}} \right\rfloor \right) = 8 * \left( 8 * \log_2 4 + \left\lfloor \log_2 \binom{10}{8} \right\rfloor \right) = 168bits$$

where 5bits  $\left( \left\lfloor \log_2 \binom{N}{N_{Null}} \right\rfloor = 5 \right)$  will be used to determine which subcarriers are nonzero

amplitude and 16 bits  $(8 * \log_2 4 = 16)$  will be mapped to the PSK constellation points of those nonzero amplitude subcarriers for each sub-block.

However, in the conventional PC-OFDM system, the mapping table of PSK symbols positions is established immediately the long incoming bits streams comes, whose length can be calculated as

$$m_{tot} = p \cdot N_{psk} \cdot \log_2 M + \left\lfloor \log_2 \binom{N}{N_{Null}} \right\rfloor = 8 * 8 * \log_2 4 + \left\lfloor \log_2 \binom{10}{8} \right\rfloor = 133bits$$

The look up table is established to map 128bits  $(8 * 8 * \log_2 4 = 128)$  to their constellation points, which also use 5bits to determine which subcarriers have nonzero amplitude. The

remaining 128 bits will be mapped to these specified nonzero amplitude subcarriers. In this case, the size of the lookup table is up to

$$size = \left[ \binom{PN-1}{PN_{Null}-1} + \binom{PN-1}{PN_{Null}} \right] \times PN = (54 + 52) \times 80$$

While in the former, the lookup table size is reduced to

$$size' = \left[ \binom{N-1}{N_{Null}-1} + \binom{N-1}{N_{Null}} \right] \times N = (5 + 3) \times 10$$

Instead of a lookup table for PSK symbols position designation of size  $(108 \times 80)$ , we establish a much shorter PSK symbols position's look up table of size  $(8 \times 10)$ . Hence, we can observe that the novel PC-OFDM greatly saves the storage space and complexity. Furthermore, we can observe that almost

$\Delta\eta = (\eta_{SE_2} - \eta_{SE_1}) / \eta_{SE_2} = (168 - 133) / 168 \times 100\% = 20.83\%$  improvement can be obtained by this modification in the proposed system.

## 1.2 The PAPR Performance of the Proposed SIPC-OFDM System

We assume the transmitter adopts two cases of subset when  $(N, N_{PSK}) = (20, 15)$  and  $(20, 12)$  respectively to construct the time domain OFDM signals with BPSK, QPSK modulation of the nonzero amplitude subcarriers, respectively. Consider the OFDM system using  $N$  subcarriers, the PAPR of the transmitted signal of multicarrier systems can be defined as

$$PAPR = 10 \log_{10} \left[ \frac{\max |S_n|^2}{P_{av}} \right] \quad (1 \leq n \leq N) \quad (11)$$

where  $P_{av}$  presents average signal power.

In the case of SIPC-OFDM with  $N_{PSK}$  nonzero amplitude subcarrier, the PAPR, which is lower than the conventional OFDM can be given by

$$PAPR = 10 \log_{10} \left[ \frac{N_{psk}}{N} \cdot \frac{\max |S_n|^2}{P_{av}} \right] \quad (1 \leq n \leq N) \quad (12)$$

## 1.3 The BER Performance of the Proposed SIPC-OFDM System Over an AWGN Channel

We analyze the bit error performance of the SIPC-OFDM system over AWGN channels. The bit error probability of the conventional PC-OFDM system was given by Frenger and Svensson [2]. Assume the side information is perfectly extracted at the receiver side, the upper

bound of BER can approach to

$$p_{b,PC} = P \left[ \frac{1}{2} \frac{m_{pc}}{m_{tot}} + \frac{m_{psk}}{m_{tot}} \left( \frac{N_{PSK} - 2}{N_{PSK}} p_{b,PSK} + \frac{1}{2} \frac{2}{N_{PSK}} \right) \right] \quad (1)$$

where,

$$p_{b,PSK} = \frac{1}{\log_2 M} \operatorname{erfc} \left( \sqrt{\frac{m_{tot}}{N_{PSK}} \frac{E_b}{N_0} \sin \left( \frac{\pi}{M} \right)} \right) \quad (2)$$

$N_{PSK} \geq 2$  and  $E_b = A^2 N_{PC} T / m_{tot}$ . With the probability  $p_b$ , we may choose the wrong subset of subcarriers.

In PTS scheme, the side information recovery has an important effect on the overall system performance. We are now ready to derive the BER of the PC-PTS OFDM system on an AWGN channel, which may serve as a lower bound approximation of the slow fading channel.  $P_{b,SI}$  is the error probability of the side information recovery at the receiver side. The overall bit error probability can be computed as

$$P_{b,SIPC} = 1 - (1 - P_{b,SI})(1 - P_{b,PC}) = P_{b,SI} + P_{b,PC} - P_{b,SI}P_{b,PC} \quad (3)$$

Note that by the side information generator, the side information can be perfect in our system model, so that the BER decreased by the side information recovery is not taken into consideration in this paper.

## 5. Simulation Results

In this section, we present the simulation results to show the PAPR, SE and BER performance of the proposed SIPC-OFDM system. We also show the tradeoff between SE and BER.

**Table.2.** Specifications of the simulation parameters

modulation	BPSK, QPSK
the number of sub-blocks	8
Channel	AWGN
Synchronization (frequency, time)	perfect
Bit mapping error of PC	no
Amplifier	SSPA

The specifications of the common simulation parameters are listed in **Table 2**. The IBO (Input Backoff) of the SSPA (Solid State Power Amplifier) is defined as 3dB. **Fig. 6** shows the SEs of the conventional PC-OFDM system and the proposed PC-OFDM system under different phase modulation levels ( $M = 2, 4, 8, 16$ ). The solid curves represent conventional PC-OFDM with MPSK modulation. The dashed curves represent the maximum SE of the novel PC-OFDM systems using  $(M+1)$ -APSK modulation. The star and the triangle in the figure represent the maximum SE can be obtained by the proposed PC-OFDM system and the ordinary PC-OFDM system for a given order of modulation  $M$  respectively.

We can observe that the proposed novel PC-OFDM systems are always capable of achieving higher SE than the corresponding conventional systems, which shows the curves of the former is always above the latter. For obtaining the maximum SE, the conventional PC

systems need to utilize all the subcarriers as  $N_{PSK} / N = 1$ . While in our proposed system, the maximum SE can be achieved when not all of the subcarriers are occupied, as the modulation level increases, more subcarriers need to be utilized to achieve the optimal SE. For BPSK,  $N_{PSK} / N = 0.65$ , 65% non-zero amplitude subcarriers can promise the maximum SE, whereas for using 16PSK,  $N_{PSK} / N = 0.95$ , 95% of total subcarriers need to guarantee the maximum SE. Additionally, we can find that for the same SE, the higher level ( $M = 16$ ) modulation system requires fewer subcarriers than the lower level ( $M = 4$ ) modulation system in both the conventional and novel systems.

We use the complementary cumulative density function (CCDF) of PAPR to represent the probability of OFDM symbols with a PAPR exceeding some threshold  $\lambda$ . From (12), we know that the PAPR is increased with the number of non-zero amplitude subcarriers  $N_{PSK}$ . This conclusion can be proved by comparing the PAPR performance with  $N_{PSK} = 15$  and  $N_{PSK} = 12$ . **Fig. 7** and **Fig. 8** show the CCDF of the proposed SIPC-OFDM system for PAPR reduction with  $P = 8$ . For comparison, the results of conventional OFDM, PTS based OFDM, and SIPC based OFDM with different subcarrier mappings are also included. It can be seen that the proposed SIPC based OFDM system achieves the best PAPR performance compared to the other techniques using BPSK modulation at the threshold level of  $10^{-2}$ , the conventional OFDM makes the maximum PAPR value 10[dB] while can be reached 5.7[dB] and 6.0[dB] can be reached in the proposed structure with different non-zero amplitude subcarriers  $N_{PSK}$ . Still there is at least 1[dB] improvement over the PTS based OFDM system and almost 3[dB] over the conventional PC-OFDM system. When all these systems use the QPSK modulation, we can draw a similar conclusion with respect to PAPR performance as shown in **Fig. 8**. We simulate the BER performance of the proposed SIPC-OFDM system when the subcarriers adopts  $(N, N_{PSK}) = (15, 14), (15, 12)$  and  $(15, 10)$  using BPSK and QPSK modulation in the AWGN channel. The SSPA is introduced to the simulation, to show that the distortion is caused by the high power amplifier. **Fig. 9** and **Fig. 10** show the BER performance of the system in AWGN channels when  $P = 8$ . In **Fig. 9**, first, the results can be separated into two groups, one with SSPA and one without SSPA. It can be seen that the bit error probability decreases when the number of non-zero amplitude subcarriers  $N_{PSK}$  decreases in each group, the BER performance of BER when  $N_{PSK} = 10$  is much better than its performance when  $N_{PSK} = 12$  and  $N_{PSK} = 14$ . Then comparing the two groups, it is not difficult to find that in each value of  $N_{PSK}$ , the system using SSPA always has the worst BER performance.

Simultaneously, we need to note that the difference of group  $N_{PSK} = 12$  and  $N_{PSK} = 10$  is much larger than the group of  $N_{PSK} = 14$  and  $N_{PSK} = 12$ . The lowest possible BER is obtained when  $N_{PSK} = 1$ . However, as shown in **Fig. 6**, such a system has a very low SE. Instead choosing the value of  $N_{PSK}$  appropriately, there is a tradeoff between BER and SE. **Fig. 9** shows that the SIPC-OFDM system requires equal or lower  $E_b/N_0$  to obtain a BER of  $10^{-4}$  for  $N_{PSK} \leq 10$  using 3-APSK.

It is also verified that when SSPA is not applied, the conventional PTS has no improvement on the final BER compared to the ordinary OFDM system since the side information is assumed to be perfectly detected at the receiver side, which in [13], it was concluded that the

performance of PTS using SSPA has some degradation.

Similar trends can be seen in Fig. 10. We can see that the BER of the SIPC-OFDM system degraded greatly when  $E_b/N_0$  was greater than 3 compared to that in conventional OFDM as in Fig.10.

Therefore, the relationships of SE,PAPR and BER performance can be summarized. Better BER and PAPR performance are promised when many more non-zero amplitude subcarriers  $N_{PSK}$  are acquired, while the respective SE performance is degraded.

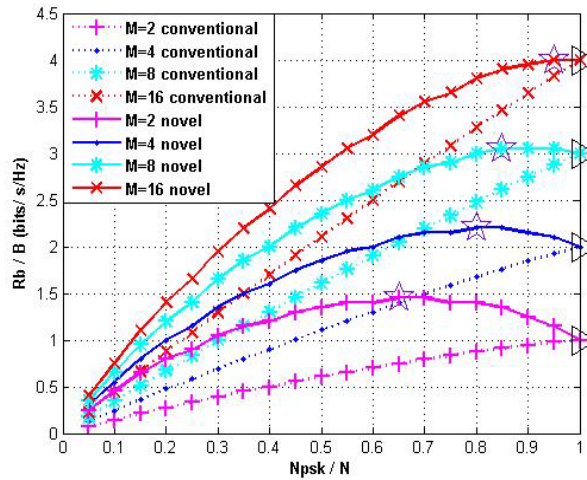


Fig.5. Spectral efficiency of novel PC-OFDM system with (M+1)-APSK modulation and conventional PC-OFDM system with M-PSK modulation

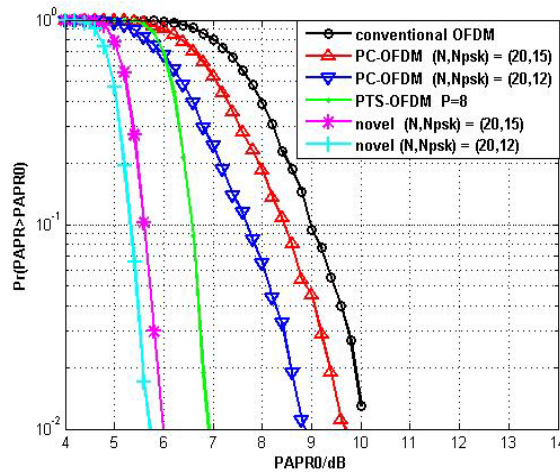


Fig.6. CCDF of the conventional and proposed schemes under BPSK modulation with sub-block number equals to 8.

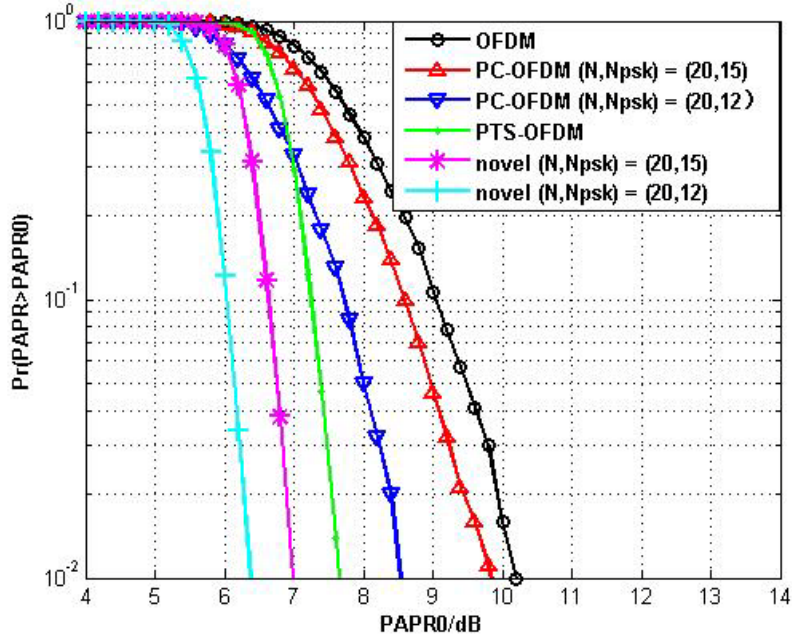


Fig.7. CCDF of the conventional and proposed schemes under QPSK modulation with sub-block number equals to 8.

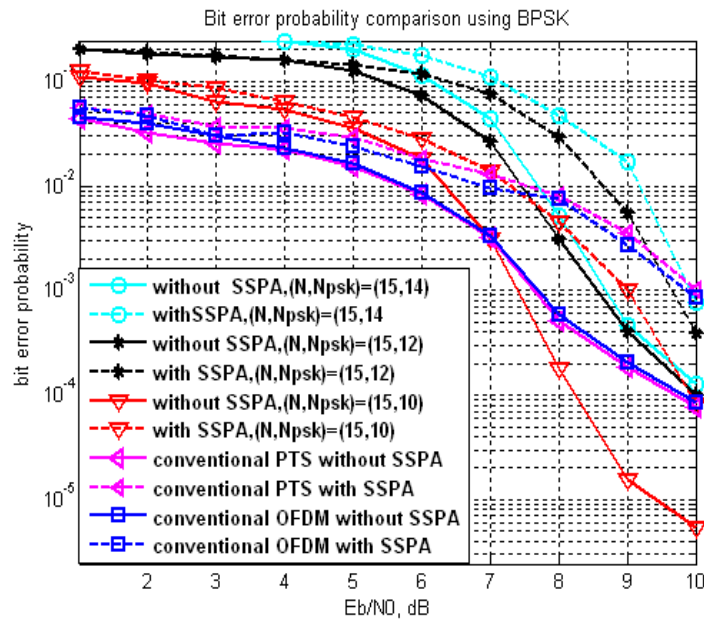
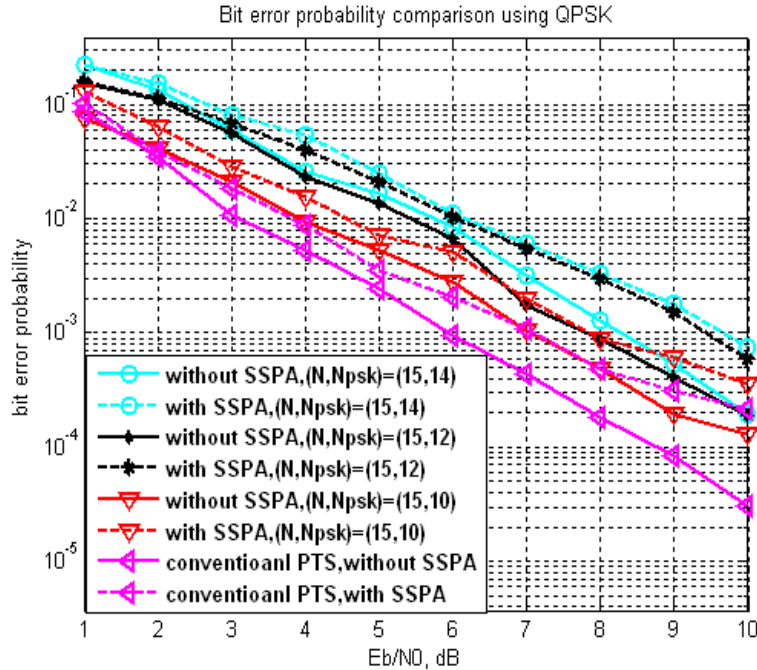


Fig.8. Comparison of BER performance of the conventional PTS-OFDM and proposed SIPC-OFDM system with BPSK modulation over an AWGN channel





**Fig.9.** Comparison of BER performance of the conventional PTS-OFDM and proposed SIPC-OFDM system with QPSK modulation over an AWGN channel

## 6. Summary

This paper proposed a novel SIPC-OFDM framework for the wireless communication systems. First, we improved the conventional PC-OFDM system by considering the complexity of PSK symbols determination. The new PC-OFDM system improved the SE and reduced the storage space greatly. Second, by introducing side information to this novel PC-OFDM scheme, we obtained the proposed SIPC-OFDM system structure. This proposed system utilizes the SI generator to let the receiver decode the weighting factors sequence with only one value and the ML detector at the receiver to detect the signal mapping type. It is capable of achieving a better PAPR characteristic and SE with an improvement in the BER performance for a given  $E_b/N_0$  by proper selection of the system parameters  $(M, P, N, N_{pc})$  over the conventional OFDM system or other OFDM based techniques .

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