

TM Mode Analysis of a Periodic Thick Mushroom Structure

Dae Woong Woo[†], Wee Sang Park[†]

[†]Department of Electrical Engineering, Graduate School of Engineering Mastership,
Pohang University of Science and Technology, Korea

Abstract

We analyzed a periodic thick mushroom structure for use as an artificial magnetic conductor using mode-matching method. The fields in each region were represented by either Floquet modes or waveguide modes. By applying tangential electric and magnetic field continuity conditions and using matrix equations, unknown coefficients and dispersion diagram were calculated. The proposed model can account for the effects of oblique incidence. Simulation time using the method was much faster than the commercial tools. We found that the current method produces accurate results of reflection phase and dispersion diagram.

Key words: Artificial magnetic conductor (AMC), Mushroom, Mode matching

1. INTRODUCTION

Periodic mushroom structures are widely used in the analysis and design of antennas and waveguides, because of their unique features as Artificial Magnetic Conductor (AMC) or Electromagnetic Bandgap (EBG) [1-3]. In conventional mushroom structures, conductors are assumed to be very thin, and there are few research about thick conductor. In this paper, plane wave scattering by the periodic mushroom structure with a conductor of finite thickness is analyzed utilizing a rigorous and accurate technique based on [4]. As an example, TM polarized plane wave incidence is studied. However, our analysis could be easily generalized to various configurations of periodic mushroom-like structure with different type and location of excitation sources.

2. FORMULATION

Firstly, we divide the mushroom structure (Fig.1) into three regions (*a*, above the mushroom cap; *b*, the cap; and *c*, between the base of the cap and the substrate) and represent

the electromagnetic fields in each region. The incident magnetic and electric waves are represented by

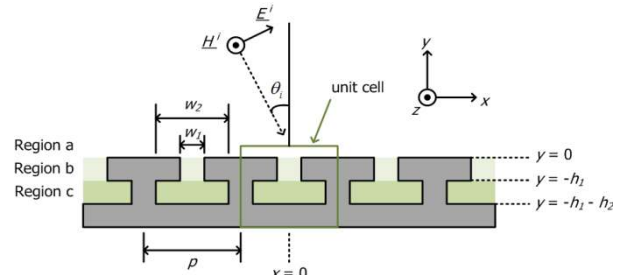


Fig. 1. Geometry of the periodic thick mushroom.

$$H_z^i(x, y) = e^{-j(k_{x0}x - k_{y0}y)} + \sum_{m=-\infty}^{\infty} A_m e^{-j(k_{xm}x + k_{ym}y)} \quad (1a)$$

$$E_x^i(x, y) = \frac{k_{y0}}{\omega \epsilon_a} e^{-j(k_{x0}x - k_{y0}y)} - \frac{A_m k_{ym}}{\omega \epsilon_a} e^{-j(k_{xm}x + k_{ym}y)} \quad (1b)$$

where

$$k_{xm} = k_{x0} + \frac{2\pi m}{p} \quad (2a)$$

$$k_{ym} = \begin{cases} \sqrt{k_a^2 - k_{xm}^2}, & |k_a| \geq |k_{xm}| \\ -j\sqrt{k_{xm}^2 - k_a^2}, & |k_a| < |k_{xm}| \end{cases} \quad (2b)$$

Manuscript received : Sept. 28, 2012 / revised : Nov. 1, 2012

[†]Corresponding Author: woodw@postech.ac.kr

Tel: +82-54-279-2930, Fax: +82-54-279-2930,

Det. of Electrical Engineering, Graduate School of Engineering Mastership, Pohang University of Science and Technology, Korea

where m is the Floquet mode number, and the $-j$ term in (2b)

means that fields decay with distance from the periodic structure (radiation condition). When the EM frequency is less than the grating frequency, only the zeroth mode ($m = 0$) propagates from the periodic mushroom structure; the other modes are evanescent.

The scattered waves in region b are represented using an infinite number of the forward and the backward waveguide modes by

$$H_z^b(x, y) = \sum_{u=0}^{\infty} \cos \xi_u (x + w_1 / 2) (C_u^+ e^{-j\beta_u(y+h_1)} + C_u^- e^{j\beta_u y}) \quad (3a)$$

$$E_x^b(x, y) = \sum_{u=0}^{\infty} \frac{\beta_u \cos \xi_u (x + w_1 / 2)}{\omega \epsilon_b} (-C_u^+ e^{-j\beta_u(y+h_1)} + C_u^- e^{j\beta_u y}) \quad (3b)$$

where

$$\xi_u = \frac{u\pi}{w_1} \quad (4a)$$

$$\beta_u = \begin{cases} \sqrt{k_b^2 - \xi_u^2}, & |\xi_u| \geq |k_b| \\ -j\sqrt{\xi_u^2 - k_b^2}, & |\xi_u| < |k_b| \end{cases} \quad (4b)$$

Here, u is the waveguide mode number in region b . When the frequency is below first cut-off frequency, only zeroth mode ($u = 0$) propagates and the other modes are evanescent throughout that region.

In the same manner, the scattered waves in region c can be represented by

$$H_z^c(x, y) = \sum_{v=0}^{\infty} D_v \cos \xi_v (x + w_2 / 2) \cos \beta_v (y + h_1 + h_2) \quad (5a)$$

$$E_x^c(x, y) = -\frac{1}{j\omega \epsilon_c} \sum_{v=0}^{\infty} D_v \beta_v \cos \xi_v (x + w_2 / 2) \sin \beta_v (y + h_1 + h_2) \quad (5b)$$

where

$$\xi_v = \frac{v\pi}{w_2} \quad (6a)$$

$$\beta_v = \begin{cases} \sqrt{k_c^2 - \xi_v^2}, & |\xi_v| \geq |k_c| \\ -j\sqrt{\xi_v^2 - k_c^2}, & |\xi_v| < |k_c| \end{cases} \quad (6b)$$

In (5), v is the waveguide mode number in region c . The boundary condition ($E_z = 0$ at $y = -h_1 - h_2$) was applied in each waveguide modes. Since we assumed the infinite periodic structure, the analysis of a unit cell is sufficient. The required boundary conditions are

$$E_{x,y=0+} = E_{x,y=0-}, |x| \leq p/2 \quad (7a)$$

$$H_{z,y=0+} = H_{z,y=0-}, |x| \leq w_1/2 \quad (7b)$$

$$E_{x,y=-h_1+} = E_{x,y=-h_1-}, |x| \leq w_2/2 \quad (7c)$$

$$H_{z,y=-h_1+} = H_{z,y=-h_1-}, |x| \leq w_1/2 \quad (7d)$$

Table I. Field notation

Region	Field function	Orthogonal function
a	m	n
b	u	r
c	v	s

To obtain the unknown coefficients (A_n , C_u^+ , C_u^- , D_v), we should multiply some orthogonal functions defined for each region (Table I). After applying the tangential continuity, the unknown coefficients can be calculated using the following equations.

$$pk_{y0}\delta_{n0} - pk_{yn}A_n = \frac{w_1\epsilon_a}{2\epsilon_b} \sum_{u=0}^{\infty} (-C_u^+ e^{-j\beta_u h_1} + C_u^-) G_{1un}^+ \quad (8a)$$

$$\sum_{u=0}^{\infty} \beta_u (C_u^+ - C_u^- e^{-j\beta_u h_1}) \cdot G_{2us} = \frac{(1 + \delta_{s0})w_2}{2} \frac{\epsilon_b}{j\epsilon_c} \beta_s D_s \sin \beta_s h_2 \quad (8b)$$

$$G_{1r0}^- + \sum_{m=-\infty}^{\infty} A_m G_{1rm}^- = (1 + \delta_{r0}) (C_r^+ e^{-j\beta_r h_1} + C_r^-) \quad (8c)$$

$$\frac{(1 + \delta_{r0})w_1}{2} (C_r^+ + C_r^- e^{-j\beta_r h_1}) = \sum_{v=0}^{\infty} G_{2rv} D_v \cos \beta_v h_2 \quad (8d)$$

To solve the equation, C_u^+ and C_u^- must be transformed in terms of A_m and D_v . From (8c) and (8d), we can obtain

$$C_u^+ = \frac{1}{-j2(1 + \delta_{u0}) \sin \beta_u h_1} \left(G_{1u0}^- + \sum_{m=-\infty}^{\infty} A_m G_{1um}^- - \frac{2}{w_1} \sum_{v=0}^{\infty} G_{2uv} D_v \cos \beta_v h_2 \cdot e^{j\beta_u h_1} \right) \quad (8a)$$

$$C_u^- = \frac{1}{j2(1 + \delta_{u0}) \sin \beta_u h_1} \left(\left(G_{1u0}^- + \sum_{m=-\infty}^{\infty} A_m G_{1um}^- \right) e^{j\beta_u h_1} - \frac{2}{w_1} \sum_{v=0}^{\infty} G_{2uv} D_v \cos \beta_v h_2 \right) \quad (8b)$$

Applying (9a) and (9b) to (8a) and (8b) yields

$$\left[A_n + \sum_{m=-\infty}^{\infty} A_m \Gamma_{1mn} \right] + \left[\sum_{v=0}^{\infty} D_v \Gamma_{2vn} \right] = [\delta_{n0} - \Gamma_{10n}] \quad (9a)$$

$$\left[\sum_{m=-\infty}^{\infty} A_m \Gamma_{3ms} \right] + \left[-\beta_s D_s + \sum_{v=0}^{\infty} D_v \Gamma_{4vs} \right] = [-\Gamma_{30s}] \quad (9b)$$

where

$$\Gamma_{1mn} = \frac{-jw_1}{2pk_{yn}} \frac{\epsilon_a}{\epsilon_b} \sum_{u=0}^{\infty} \frac{\beta_u}{(1+\delta_{u0})} \cot \beta_u h_1 \cdot G_{1um}^- \cdot G_{1un}^+ \quad (10a)$$

$$\Gamma_{2vn} = \frac{j \cos \beta_v h_2}{pk_{yn}} \frac{\epsilon_a}{\epsilon_b} \sum_{u=0}^{\infty} \frac{\beta_u}{(1+\delta_{u0})} \csc \beta_u h_1 \cdot G_{2uv}^- \cdot G_{1un}^+ \quad (10b)$$

$$\Gamma_{3ms} = \frac{-2}{(1+\delta_{s0})w_2 \sin \beta_s h_2} \frac{\epsilon_c}{\epsilon_b} \sum_{u=0}^{\infty} \frac{\beta_u}{(1+\delta_{u0})} \csc \beta_u h_1 \cdot G_{1um}^- \cdot G_{2us} \quad (10c)$$

$$\Gamma_{4vs} = \frac{4 \cos \beta_v h_2}{(1+\delta_{s0})w_1 w_2 \sin \beta_s h_2} \frac{\epsilon_c}{\epsilon_b} \sum_{u=0}^{\infty} \frac{\beta_u}{(1+\delta_{u0})} \cot \beta_u h_1 \cdot G_{2uv}^- \cdot G_{2us} \quad (10d)$$

Truncation is required to solve the two matrix equations (9). If the maximum Floquet mode number is M and the maximum waveguide number is V , the matrix size of (9) is $(2M + V + 2) \times (2M + V + 2)$. In general, the reflection phase $\angle A_0$ of the dominant mode and dispersion diagram is of prime interest in AMC. Dispersion diagram can be obtained using the determinant of (9). If $\det(\text{matrix}) = 0$, then $R \rightarrow \infty$ and only surface waves exist.

3. RESULTS

We plotted some reflection phase and dispersion diagrams. Simulation using the formulated matrix equation was done within a few seconds, which is much faster than commercial simulators. The reflection amplitude of zeroth and first mode, and the reflection phase of zeroth mode are converged when the truncated mode number ($M \geq 12$) (Fig. 2). In dispersion diagram, cutoff frequencies of the TM mode are 12 GHz, 16.5 GHz, and 8.7 GHz, respectively. We found that the results of the reflection phase (Fig. 3) and dispersion diagram (Fig. 4) agreed well with the output of commercial simulators (Ansoft HFSS and CST MWS).

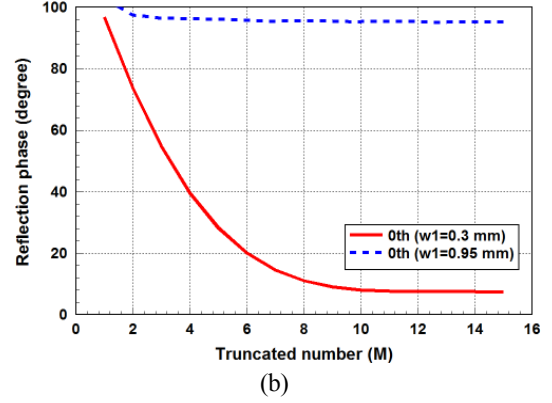
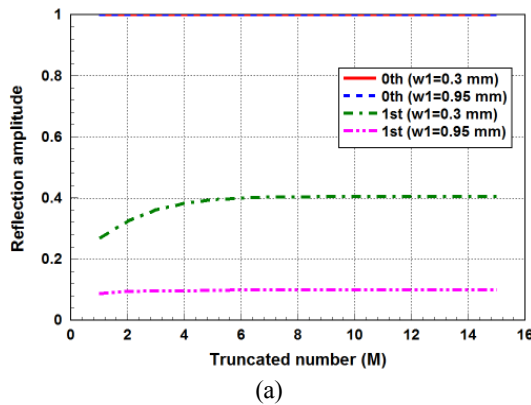


Fig. 2. Reflection (a) amplitude and (b) phase with the truncation number (M). $p = 5$ mm, $h_1 = 0.5$ mm, $h_2 = 1$ mm, $w_2 = 0.5$ mm, $\epsilon_{ra} = \epsilon_{rb} = \epsilon_{rc} = 1$, $\mu_{ra} = \mu_{rb} = \mu_{rc} = 1$.

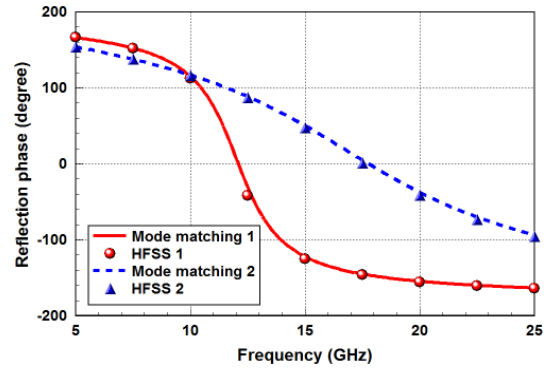


Fig. 3. Reflection phase of 0th mode. $p = 5$ mm, $h_1 = 0.5$ mm, $h_2 = 1$ mm, $w_2 = 0.5$ mm, $\epsilon_{ra} = \epsilon_{rb} = \epsilon_{rc} = 1$, $\mu_{ra} = \mu_{rb} = \mu_{rc} = 1$, (a) $w_1 = 0.3$ mm, $\theta_i = 0^\circ$. (b) $w_1 = 0.95$ mm, $\theta_i = 60^\circ$.

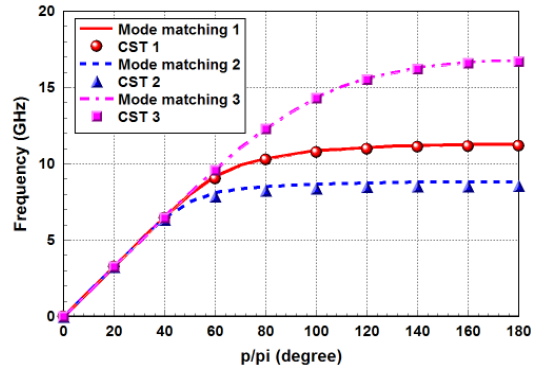


Fig. 4. Dispersion diagram. $h_1 = 0.5$ mm, $h_2 = 1$ mm, $w_2 = 4.7$ mm, (a) $w_2 = 0.3$ mm, $\epsilon_{rc} = 1.0$, (b) $w_2 = 0.3$ mm, $\epsilon_{rc} = 4.0$, (c) $w_2 = 2.5$ mm, $\epsilon_{rc} = 4.0$.

4. CONCLUSION

We used the mode-matching technique to analyze a periodic thick mushroom AMC. The fields in each region were represented by Floquet or waveguide modes. By applying tangential electric and magnetic field continuity

conditions and using matrix equations, unknown coefficients and dispersion diagrams were calculated. The proposed method was much faster than the conventional simulation tool (CST, HFSS) and had good accuracy.

propagation in power cable, frequency selective surface, and RFID for conducting structure. Professor Park is a member of the IEEE Microwave Theory and Techniques Society and the Antennas and Propagation Society.

ACKNOWLEDGMENT

This work was supported by the Brain Korea 21 Project in 2012.

References

- [1] F. Yang, and Y. Rahmat-Samii, "Reflection phase characterizations of the EBG ground plane for low profile wire antenna applications," *IEEE Trans. Antennas Propag.*, Vol. 51, No. 10, pp. 2691-2703, Oct. 2003.
- [2] D. W. Woo, D. R. Shin, J. P. Kim, J. K. Ji, W. M. Seong, and W. S. Park, "Bandwidth Analysis of a Jerusalem AMC Based on Equivalent Circuit Model," *Journal of the Institute of Webcasting, Internet and Telecommunication(IWIT)*, Vol. 10, No. 5, pp. 128-135, Oct. 2010. Vol. 10, No. 5, pp. 7-12, Oct. 2010.
- [3] S. J. Muhn, and W. S. Park, "Regression Progress to Evaluate Metal Scale Thickness using Microwave," *Journal of the Institute of Webcasting, Internet and Telecommunication(IWIT)*, Vol. 10, No. 5, pp. 128-135, Oct. 2010. Vol. 10, No. 5, pp. 1-5, Oct. 2010.
- [4] *Electromagnetic Theory and Applications for Photonic Crystals*, K.Yasumoto, Ed. Boca Raton, FL: CRC Press, 2005.



Dae Woong Woo received a B.S. degree in electronic and electrical engineering from Kyungpook National University, Daegu, Korea, in 2007. He is currently working toward a Ph. D. degree at the Pohang University of Science and technology, Pohang, Korea. His

research interests include mobile antenna, periodic structure, frequency selective surface and artificial magnetic conductor.



Wee Sang Park received a B.S. degree in electronic engineering from Seoul National University, Korea, in 1974, and an M.S. and a Ph.D. degrees in electrical engineering from the University of Wisconsin-Madison in 1982 and 1986, respectively. Since 1988, he has been with the Department of Electrical Engineering,

Pohang University of Science and Technology. His research interests are wireless power transfer, RF