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LIST EDGE AND LIST TOTAL COLORINGS OF PLANAR GRAPHS WITHOUT 6-CYCLES WITH CHORD

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Abstract. Giving a planar graph G, let $\chi'_l(G)$ and $\chi''_l(G)$ denote the list edge chromatic number and list total chromatic number of G respectively. It is proved that if a planar graph G without 6-cycles with chord, then $\chi'_l(G) \le \Delta(G) + 1$ and $\chi''_l(G) \le \Delta(G) + 2$ where $\Delta(G) \ge 6$.

1. Introduction

The terminology and notation used but undefined in this paper can be found in [1]. Let $G = (V, E)$ be a graph. We use $V(G)$, $E(G)$, $F(G)$, $\Delta(G)$ and $\delta(G)$ to denote the vertex set, edge set, face set, maximum degree, and minimum degree of G, respectively. Let $d_G(x)$ or simply $d(x)$, denote the degree of a vertex (face) x in G. A vertex (face) x is called a *k-vertex* (*k-face*), *k^+-vertex* (*k^+-face*), *k^--vertex*, if $d(x) = k$, $d(x) \ge k$, $d(x) \le k$. We use $(d_1, d_2, \ldots, d_n)$ to denote a face f if $(d_1, d_2, \ldots, d_n)$ are the degree of vertices incident to the face f. If u_1, u_2, …, u_n are the vertices on the boundary walk of a face f, then we write $f = u_1u_2 \cdots u_n$. Let $\delta(f)$ denote the minimal degree of vertices incident to f. We use $f_i(v)$ denote the number of i-faces incident to v for each $v \in V(G)$. Let $n_i(f)$ denote the number of i-vertices incident to f for each $f \in F(G)$. A cycle C of length k is called k-cycle, if $xy \in E(G)\backslash E(C)$ and x, $y \in V(C)$, the cycle C is called *k-cycle with chord*.

The mapping L is said to be a *total assignment* for a graph G if it assigns a list $L(x)$ of possible colors to each element $x \in V \cup E$. If G has a proper total coloring $\phi(x) \in L(x)$ for all $x \in V \cup E$, then we say that G is *total-L-colorable*. Let $f : V \cup E \to N$ be a function into the positive integers. We say that G is *total-f-choosable* if it is total-L-colorable for every total assignment L satisfying $|L(x)| = f(x)$ for all $x \in V \cup E$. The *list total coloring number* $\chi''_l(G)$ of G is the smallest integer k such that G is total-f-choosable when $f(x) = k$ for each $x \in V \cup E$. The *list edge coloring number* $\chi'_l(G)$ of G is

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defined similarly in terms of coloring edges alone; and so are the concept of *edge-f-choosable*. If G is a graph that can not be edge-$(\Delta(G)+1)$-choosable or total-$(\Delta(G)+2)$-choosable graph with the fewest vertices and edges, then we call it a critical graph. On the list coloring number of a graph G, there is a famous conjecture known as the List Coloring Conjecture.

Conjecture 1. *For a multigraph G,*

$$\text{(a) } \chi'_l(G) = \chi'(G); \text{ (b) } \chi''_l(G) = \chi''(G).$$

Part (a) of the above conjecture was formulated independently by Vizing, by Gupta, by Alberson and Collins, and by Bollobás and Harris [5, 11]. It is well known as the *List Coloring Conjecture*. Part (b) was formulated by Borodin, Kostochka and Woodall [2]. Part (a) and Part (b) has been proved for outerplanar graphs [13], and graphs with $\Delta \geq 12$ which can be embedded in a surface of nonnegative characteristic [2]. List Coloring Conjecture has been proved for a few other special graphs, such as bipartite multigraphs [4], complete graphs of odd order [6]. There are several related results for planar graphs without some short cycles or by adding girth restrictions [7, 8, 9, 3, 16, 14, 15, 10].

In this paper, we shall show that if G is a planar graph without 6-cycles with chord, then $\chi'_l(G) \leq \Delta(G)+1$ and $\chi''_l(G) \leq \Delta(G)+2$ where $\Delta(G) \geq 6$.

2. Planar graphs without 6-cycles with chord

First let us introduce an important lemma.

Lemma 2.1. *Let G be a critical planar graph without 6-cycles with chord. If $\Delta(G) \geq 6$, then there is an edge $uv \in E(G)$ such that $\min\{d(u), d(v)\} \leq \lfloor \frac{\Delta(G)+1}{2} \rfloor$ and $d(u)+d(v) \leq \max\{8, \Delta(G)+2\}$.*

Proof. For G is a critical planar graph with $\Delta(G) \geq 6$, then G contains no $(4,4,5^-)$-face $f = uvw$. By contradiction, let L' and L'' be any list assignments such that $|L'(e)| = \Delta(G)+1$ for each $e \in E(G)$ and $|L''(x)| = \Delta(G)+2$ for each $x \in V(G) \cup E(G)$.

Let $G' = G - \{uv, vw, wu\}$. By G is a critical graph, G' is edge-L'-colorable. Now there are at least three colors available for uv, and at least two colors for vw and wu. We can easily color vw, uw, and uv successively. So G is edge-L'-colorable, a contradiction.

For the same reason, G' is total-L''-colorable. Erase the colors of the vertices u, v, w. For each element x incident with f, we define a reduced total list $\bar{L''}(x)$ such that $\bar{L''}(x) = L''(x) \backslash \{\phi(x') \mid x'$ is incident with or adjacent to x, and, x' is not incident with $f\}$ where $\phi(x')$ denotes the color of the element x'. Then $|\bar{L''}(u)| \geq 4$, $|\bar{L''}(v)| \geq 4$, $|\bar{L''}(w)| \geq 2$, $|\bar{L''}(uv)| \geq 4$, $|\bar{L''}(uw)| \geq 3$, $|\bar{L''}(vw)| \geq 3$. If there is a color $\alpha \in \bar{L''}(uw) \backslash \bar{L''}(u)$, then we can color uw with the color α, and color w, wv, v, uv, and u successively. So $\bar{L''}(uw) \subseteq \bar{L''}(u)$. Similarly, $\bar{L''}(vw) \subseteq \bar{L''}(v)$. If there is a color $\beta \in \bar{L''}(u) \backslash \bar{L''}(v)$, then we can color u with β, and color w, uw, vw, uv and v successively. So $\bar{L''}(u) = \bar{L''}(v)$. Thus there is a color $\gamma \in \bar{L''}(uw) \bigcap \bar{L''}(v)$. We color uw and v with γ, and color w, vw, uv and u successively. From the above discussion, in any case, f is total-$\bar{L''}$-colorable. So G is total-L''-colorable, a contradiction.

In the following, we show that for the critical planar graph without 6-cycles with chord, if $\Delta(G) \geq 6$, then there is an edge $uv \in E(G)$ such that $\min\{d(u), d(v)\} \leq \lfloor \frac{\Delta(G)+1}{2} \rfloor$ and $d(u) + d(v) \leq \max\{8, \Delta(G) + 2\}$. By contradiction, we have $d(u)+d(v) \geq \max\{9, \Delta(G)+3\}$ for each edge $uv \in E(G)$ such that $\min\{d(u), d(v)\} \leq \lfloor \frac{\Delta(G)+1}{2} \rfloor$. It is clear that $\delta(G) \geq 3$.

By Euler's formula $|V|-|E|+|F| = 2$ and $\sum_{v\in V(G)} d(v) = \sum_{f\in F(G)} d(f) = 2|E|$, we have

$$\sum_{v\in V(G)} (2d(v) - 6) + \sum_{f\in F(G)} (d(f) - 6) = -6(|V| - |E| + |F|) = -12.$$

Define an initial charge function w on $V(G)\cup F(G)$ by setting $w(v) = 2d(v)-6$ if $v \in V(G)$ and $w(f) = d(f)-6$ if $f \in F(G)$, so that $\sum_{x\in V(G)\cup F(G)} W(x) = -12$. Now redistribute the charge according to the following discharging rules.

For convenience, let $\bar{w(v)}$ denote the total charge transferred from a vertex v to all its incident 4- and 5-faces where $d(v) = 4$.

$D1$. If f is a 3-face incident with a vertex v, then v gives f charge $\frac{2-\bar{w(v)}}{f_3(v)}$ if $d(v) = 4$, $\frac{4}{3}$ if $d(v) = 5$, $\frac{3}{2}$ if $d(v) \geq 6$.

$D2$. If f is a 4-face incident with a vertex v, then v gives f charge $\frac{1}{2}$ if $d(v) = 4$ or 5, 1 if $d(v) \geq 6$.

$D3$. If f is a 5-face incident with a vertex v, then v gives f charge $\frac{1}{5}$ if $d(v) = 4$ or 5, $\frac{1}{3}$ if $d(v) \geq 6$.

Let the new charge of each element x be $w'(x)$ for each $x \in V(G) \cup F(G)$.

In the following, let us check the new charge of each element $x \in V(G) \cup F(G)$.

Suppose $d(v) = 3$. Then $w'(v) = w(v) = 0$.

Suppose $d(v) = 4$. Then $w(v) = 2$, $f_3(v) \leq 4$. If $f_3(v) \geq 1$, then $w'(v) \geq 2 - \frac{2-\bar{w(v)}}{f_3(v)} f_3(v) - \bar{w(v)} = 0$ by $D1$. Otherwise, i.e., $f_3(v) = 0$, then $f_4(v) \leq 2$, $f_5(v) \leq 4$ for G contains no 6-cycles with chord. We have $w'(v) > 2 - \frac{1}{2} \times 2 - \frac{1}{5} \times 4 = \frac{1}{5} > 0$ by $D2$ and $D3$.

Suppose $d(v) = 5$. Then $w(v) = 4$, $f_3(v) \leq 3$ for G contains no 6-cycles with chord. If $f_3(v) = 3$, then $f_4(v) = 0$ and $f_5(v) = 0$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 4 - \frac{4}{3} \times 3 = 0$ by $D1$. If $f_3(v) = 2$, then $f_4(v) \leq 1$ and $f_5(v) \leq 1$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 4 - \frac{4}{3} \times 2 - \frac{1}{2} - \frac{1}{5} = \frac{19}{30} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 1$, then $f_4(v) \leq 2$ and $f_5(v) \leq 2$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 4 - \frac{4}{3} - \frac{1}{2} \times 2 - \frac{1}{5} \times 2 = \frac{19}{15} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 0$, then $f_4(v) \leq 2$ or $f_5(v) \leq 5$ for G contains no 6-cycles with chord, we have $w'(v) > 4 - \frac{1}{2} \times 2 - \frac{1}{5} \times 5 = 2 > 0$ by $D2$ and $D3$.

Suppose $d(v) = 6$. Then $w(v) = 6$, $f_3(v) \leq 4$ for G contains no 6-cycles with chord. If $f_3(v) = 4$, then $f_4(v) = 0$ and $f_5(v) = 0$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 6 - \frac{3}{2} \times 4 = 0$ by $D1$. If $f_3(v) = 3$, then $f_4(v) \leq 1$ and $f_5(v) \leq 1$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 6 - \frac{3}{2} \times 3 - 1 - \frac{1}{3} = \frac{1}{6} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 2$, then $f_4(v) \leq 2$ and $f_5(v) \leq 2$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 6 - \frac{3}{2} \times 2 - 1 \times 2 - \frac{1}{3} \times 2 = \frac{1}{3} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 1$, then $f_4(v) \leq 3$ and $f_5(v) \leq 3$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 6 - \frac{3}{2} - 1 \times 3 - \frac{1}{3} \times 3 = \frac{1}{2} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 0$, then $f_4(v) \leq 3$ or $f_5(v) \leq 6$ for G contains no 6-cycles with chord, we have $w'(v) > 6 - 1 \times 3 - \frac{1}{3} \times 6 = 1 > 0$ by $D2$ and $D3$.

Suppose $d(v) = 7$. Then $w(v) = 8$, $f_3(v) \leq 5$ for G contains no 6-cycles with chord. If $f_3(v) = 5$, then $f_4(v) = 0$ and $f_5(v) = 0$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 8 - \frac{3}{2} \times 5 = \frac{1}{2} > 0$ by $D1$. If $f_3(v) = 4$, then $f_4(v) \leq 1$ and $f_5(v) = 0$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 8 - \frac{3}{2} \times 4 - 1 = 1 > 0$ by $D1$ and $D2$. If $f_3(v) = 3$, then $f_4(v) \leq 2$ and $f_5(v) \leq 2$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 8 - \frac{3}{2} \times 3 - 1 \times 2 - \frac{1}{3} \times 2 = \frac{5}{6} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 2$, then $f_4(v) \leq 3$ and $f_5(v) \leq 3$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 8 - \frac{3}{2} \times 2 - 1 \times 3 - \frac{1}{3} \times 3 = 1 > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) \leq 1$, then $f_4(v) \leq 3$ or $f_5(v) \leq 7$ for G contains no 6-cycles with chord, we have $w'(v) > 8 - \frac{3}{2} - 1 \times 3 - \frac{1}{3} \times 7 = \frac{7}{6} > 0$ by $D1$, $D2$ and $D3$.

Suppose $d(v) = 8$. Then $w(v) = 10$, $f_3(v) \leq 6$ for G contains no 6-cycles with chord. If $f_3(v) = 6$ or 5, then $f_4(v) = 0$ and $f_5(v) = 0$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 10 - \frac{3}{2} \times 6 = 1 > 0$ by $D1$. If $f_3(v) = 4$, then $f_4(v) \leq 1$ and $f_5(v) \leq 1$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 10 - \frac{3}{2} \times 4 - 1 - \frac{1}{3} = \frac{8}{3} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 3$, then $f_4(v) \leq 2$ and $f_5(v) \leq 3$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 10 - \frac{3}{2} \times 3 - 1 \times 2 - \frac{1}{3} \times 3 = \frac{5}{2} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) = 2$, then $f_4(v) \leq 4$ and $f_5(v) \leq 4$ for G contains no 6-cycles with chord. We can get $w'(v) \geq 10 - \frac{3}{2} \times 2 - 1 \times 4 - \frac{1}{3} \times 4 = \frac{5}{3} > 0$ by $D1$, $D2$ and $D3$. If $f_3(v) \leq 1$, then $f_4(v) \leq 4$ or $f_5(v) \leq 8$ for G contains no 6-cycles with chord, we have $w'(v) > 10 - \frac{3}{2} - 1 \times 4 - \frac{1}{3} \times 8 = \frac{11}{6} > 0$ by $D1$, $D2$ and $D3$.

Suppose $d(v) \geq 9$. Then $w(v) = 2d(v) - 6$, $f_4(v) \leq d(v) - \frac{4}{3} \times f_3(v)$, $f_5(v) \leq d(v) - \frac{4}{3}f_3(v)$ for G contains no 6-cycle with chord. So $w'(v) \geq 2d(v) - 6 - \frac{3}{2}f_3(v) - f_4(v) - \frac{1}{3}f_5(v) \geq 2d(v) - 6 - \frac{3}{2}f_3(v) - d(v) + \frac{4}{3}f_3(v) - \frac{1}{3}d(v) + \frac{4}{9}f_3(v) \geq \frac{2}{3}d(v) - 6 + \frac{5}{18}f_3(v)$ by $D1$, $D2$ and $D3$. For $0 \leq f_3(v)$, we have $w'(v) \geq \frac{2}{3}d(v) - 6 \geq 0$.

Suppose $d(f) = 3$. Then $w(f) = -3$.

Case 1. $\delta(f) = 3$, then f is a $(3, 6^+, 6^+)$-face by assumption. We have $w'(f) = -3 + \frac{3}{2} \times 2 = 0$ by $D1$.

Case 2. $\delta(f) = 4$, then f is a $(4, 4, 6^+)$- or $(4, 5^+, 5^+)$-face for G contains no $(4, 4, 5^-)$-face.

Case 2.1. f is a $(4, 4, 6^+)$-face. For convenience, let $f = uvw$ where $d(u) = d(v) = 4$. If one of the 4-vertex is incident to at least three 3-faces, without loss of generality, let $f_3(u) \geq 3$, then $f_4(u) = f_5(u) = 0$ and $f_3(v)+f_4(v)+f_5(v) \leq 2$ for G contains no 6-cycles with chord. We have $w'(f) \geq -3+\frac{1}{2}+\frac{3}{2}+\frac{2-\bar{w(v)}}{f_3(v)} \geq -3+\frac{1}{2}+\frac{3}{2}+1 = 0$ by $D1$, $D2$ and $D3$. Otherwise, i.e., $f_3(u) \leq 2$, $f_3(v) \leq 2$, then we have $\frac{2-\bar{w(v)}}{f_3(v)} \geq \frac{4}{5}$ for G contains no 6-cycles with chord and by $D1$, $D2$ and $D3$. So $w'(f) \geq -3+\frac{4}{5}\times 2+\frac{3}{2} = \frac{1}{10} > 0$ by $D1$.

Case 2.2. f is a $(4, 5^+, 5^+)$-face. By $D1$, $D2$ and $D3$, we have $\frac{2-\bar{w(v)}}{f_3(v)} \geq \frac{1}{2}$. So $w'(f) \geq -3+\frac{1}{2}+\frac{4}{3}\times 2 = \frac{1}{6} > 0$ by $D1$.

Case 3. $\delta(f) \geq 5$, then we have $w'(f) = -3+\frac{4}{3}\times 3 = 1 > 0$ by $D1$.

Suppose $d(f) = 4$. Then $w(f) = -2$. If $\delta(f) = 3$, then f is a $(3, 3^+, 6^+, 6^+)$-face by assumption. We have $w'(f) \geq -2+1\times 2 = 0$ by $D2$. If $\delta(f) \leq 4$, then f is a $(4^+, 4^+, 4^+, 4^+)$-face. We have $w'(f) \geq -2+\frac{1}{2}\times 4 = 0$ by $D2$.

Suppose $d(f) = 5$. Then $w(f) = -1$.

Case 4. $\delta(f) = 3$, then $n_3(f) \leq 2$ by assumption. If $n_3(f) = 2$, then f is a $(3, 3, 6^+, 6^+, 6^+)$-face by assumption. We have $w'(f) \geq -1+\frac{1}{3}\times 3 = 0$ by $D3$. If $n_3(f) = 1$, then f is a $(3, 4^+, 4^+, 6^+, 6^+)$-face by assumption. We have $w'(f) \geq -1+\frac{1}{3}\times 2+\frac{1}{5}\times 2 = \frac{1}{15} > 0$ by $D3$.

Case 5. $\delta(f) = 4$, then we have $w'(f) \geq -1+\frac{1}{5}\times 5 = 0$ by $D3$.

Suppose $d(f) \geq 6$. Then $w'(f) = w(f) \geq 0$.

From the above discussion, we can obtain $\sum_{x\in V(G)\cup F(G)} w'(x) \geq 0 > -12$, a contradiction. □

In the following, let us give the proof of the main theorem.

Theorem 2.2. *If G is a planar graph without 6-cycles with chord, then $\chi'_l(G) \leq \Delta(G)+1$ and $\chi''_l(G) \leq \Delta(G)+2$ where $\Delta(G) \geq 6$.*

Proof. By contradiction, let G' and G'' be minimal counterexamples (i.e., critical graphs) to the conclusions for χ'_l and χ''_l respectively, and let L' and L'' be list assignments such that $|L'(e)| = \Delta+1$ for each $e \in E(G)$ and G' is not edge-L'-colorable, and $|L''(x)| = \Delta+2$ for each $x \in V(G)\cup E(G)$ and G'' is not total-L''-colorable. By Lemma 2.1, G' and G'' contains an edge $uv \in E(G)$ such that $\min\{d(u), d(v)\} \leq \lfloor\frac{\Delta(G)+1}{2}\rfloor$ and $d(u)+d(v) \leq \max\{8, \Delta(G)+2\}$.

Let $\bar{G'} = G' - uv$. Then $\bar{G'}$ is edge-L'-colorable by assumption. For $d(u)+d(v) \leq \max\{8, \Delta(G)+2\}$, there is at most $\Delta(G)$ edges which are adjacent with uv in $\bar{G'}$. So there is at least one color in $L'(uv)$ which we can use to color uv. Then G' is edge-L'-colorable, a contradiction.

Let $\bar{G''} = G'' - uv$. Then $\bar{G''}$ is total-L''-colorable by assumption. Without loss of generality, let $d(u) = \min\{d(u), d(v)\}$. Erase the color of u, then there is at least one color in $L''(uv)$ which we can use to color uv for $d(u)+d(v) \leq \max\{8, \Delta(G){+}2\}$. For $d(u) \leq \lfloor \frac{\Delta(G)+1}{2} \rfloor$, then u is adjacent to at most $\lfloor \frac{\Delta(G)+1}{2} \rfloor$ vertices or is incident to $\lfloor \frac{\Delta(G)+1}{2} \rfloor$ edges. So there is at least one color in $L'(u)$ which we can use to color u. Then G'' is total-L''-colorable, a contradiction. So $\chi'_l(G) \leq \Delta(G)+1$ and $\chi''_l(G) \leq \Delta(G)+2$ □

References

[1] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, North-Holland, New York, 1976.

[2] O. Borodin, A. Kostochka, and D. Woodall, *List edge and list total colourings of multigraphs*, J. Combin. Theory Ser. B **71** (1997), no. 2, 184–204.

[3] J. Cai, J. Hou, X. Zhang, and G. Liu, *Edge-choosability of planar graphs without non-induced 5-cycles*, Inform. Process. Lett. **109** (2009), no. 7, 343–346.

[4] F. Galvin, *The list chromatic index of a bipartite multigraph*, J. Combin. Theory Ser. B **63** (1995), no. 1, 153–158.

[5] R. Häggkvist and A. Chetwynd, *Some upper bounds on the total and list chromatic numbers of multigraphs*, J. Graph Theory **16** (1992), no. 5, 503–516.

[6] R. Häggkvist and J. Janssen, *New bounds on the list-chromatic index of the complete graph and other simple graphs*, Combin. Probab. Comput. **6** (1997), no. 3, 295–313.

[7] J. Hou, G. Liu, and J. Cai, *Edge-choosability of planar graphs without adjacent triangles or without 7-cycle*, Discrete Math. **309** (2009), no. 1, 77–84.

[8] ———, *List edge and list total colorings of planar graphs without 4-cycles*, Theoret. Comput. Sci. **369** (2006), no. 1-3, 250–255.

[9] ———, *List edge and list total colorings of planar graphs without short cycles*, Inform. Process. Lett. **108** (2008), no. 6, 347–351.

[10] J. Hou, G. Liu, and J. Wu, *Some results on list total colorings of planar graphs*, Lecture Note in Computer Science **4489** (2007), 320–328.

[11] T. Jensen, B. Toft, *Graph Coloring Problem*, Wiley-Interscience, New York, 1995.

[12] A. Kostochka, *List edge chromatic number of graphs with large girth*, Discrete Math. **101** (1992), no. 1-3, 189–201.

[13] W. Wang and K. Lih, *Choosability, edge-choosability and total choosability of outerplane graphs*, European J. Combin. **22** (2001), no. 1, 71–78.

[14] ———, *Choosability and edge choosability of planar graphs without five cycles*, Appl. Math. Lett. **15** (2002), no. 5, 561–565.

[15] ———, *Structural properties and edge choosability of planar graphs without 6-cycles*, Combin. Probab. Comput. **10** (2001), no. 3, 267–276.

[16] L. Zhang and B. Wu, *Edge choosability of planar graphs without small cycles*, Discrete Math. **283** (2004), no. 1-3, 289–293.

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