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SHADOWABLE CHAIN TRANSITIVE SETS OF C^1-GENERIC DIFFEOMORPHISMS

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Abstract. We prove that a locally maximal chain transitive set of a C^1-generic diffeomorphism is hyperbolic if and only if it is shadowable.

1. Introduction

Transitive sets, homoclinic classes and chain components of a diffeomorphism f on a closed C^∞ manifold M are natural candidates to replace the Smale's hyperbolic basic sets in non-hyperbolic theory of differentiable dynamical systems. Many recent papers have explored their hyperbolic-like properties such as dominated splitting, partial hyperbolicity and hyperbolicity.

For instance, Sakai *et al.* proved in [7, 10] that if the chain component $C_f(p)$ of a diffeomorphism f containing a hyperbolic periodic point p is robustly shadowable (i.e., there is a C^1 neighborhood $\mathcal{U}(f)$ of f such that the chain component $C_g(p_g)$ of $g \in \mathcal{U}(f)$ containing the continuation p_g is shadowable for g), then $C_f(p)$ is hyperbolic. Moreover Lee *et al.* in [6, 8, 9, 11] obtained sufficient conditions for the homoclinic classes to be hyperbolic. It is known by Bonatti and Crovisier in [3] that, in the C^1-generic context, every chain component with a periodic point is a homoclinic class.

In this paper, we study the hyperbolicity of shadowable chain transitive sets of C^1-generic diffeomorphisms f on a closed C^∞ manifold M. Note that every transitive set, homoclinic class and chain component of f are examples of chain transitive sets of f.

Let $\mathrm{Diff}(M)$ be the space of diffeomorphisms of M endowed with the C^1-topology. Denote by d the distance on M induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle TM. Let $f \in \mathrm{Diff}(M)$. For $\delta > 0$, a sequence of points $\{x_i\}_{i=a}^b$ in M ($-\infty \le a < b \le \infty$) is called a δ-*pseudo-orbit* (or δ-*chain*) of f if $d(f(x_i), x_{i+1}) < \delta$ for all $a \le i \le b-1$. For a closed f-invariant set $\Lambda \subset M$, we say that f has the *shadowing property* (or Λ is *shadowable* for f) if for every $\epsilon > 0$, there is $\delta > 0$ such that for any δ-pseudo-orbit $\{x_i\}_{i=a}^b \subset \Lambda$

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of f ($-\infty \le a < b \le \infty$), there is $y \in M$ satisfying $d(f^i(y), x_i) < \epsilon$ for all $a \le i \le b-1$. In this case, $\{x_i\}_{i=a}^{b}$ is said to be *ϵ-shadowed* by the point y. Notice that only δ-pseudo-orbits of f contained in Λ are allowed to be ϵ-shadowed, but the shadowing point $y \in M$ is not necessarily contained in Λ.

Given$f \in \mathrm{Diff}(M)$, a closed f-invariant set $\Lambda \subset M$ is said to be *chain transitive* if for any points $x, y \in \Lambda$ and $\delta > 0$, there exists a δ-pseudo orbit $\{x_i\}_{i=a_\delta}^{b_\delta} \subset \Lambda$ ($a_\delta < b_\delta$) of f such that $x_{a_\delta} = x$ and $x_{b_\delta} = y$. For given points $x, y \in M$, we write $x \rightsquigarrow y$ if for any $\delta > 0$, there is a δ-pseudo-orbit $\{x_i\}_{i=a_\delta}^{b_\delta}(a_\delta < b_\delta)$ of f such that $x_{a_\delta} = x$ and $x_{b_\delta} = y$. The set $\{x \in M : x \rightsquigarrow x\}$ is called the *chain recurrent set* of f and is denoted by $\mathcal{CR}(f)$. Define a relation $\sim$ on $\mathcal{CR}(f)$ by $x \sim y$ if $x \rightsquigarrow y$ and $y \rightsquigarrow x$. It is clear that $\sim$ is an equivalent relation on $\mathcal{CR}(f)$. The equivalence classes are called the *chain components* (or *chain recurrent classes*) of f. Clearly every chain component is a maximal chain transitive set; that is, a set which are maximal in the family of all chain transitive sets of f ordered by inclusion.

A closed f-invariant set $\Lambda \subset M$ is said to be *transitive* if there is a point $x \in \Lambda$ such that the ω-limit set $\omega(x)$ of x coincides with Λ; and Λ is said to be *locally maximal* if there is an open neighborhood V of Λ such that $\Lambda = \bigcap_{n\in\mathbb{Z}} f^n(V)$.

Recall that a closed f-invariant set $\Lambda \subset M$ is called *hyperbolic* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E^s \oplus E^u$ and there exist constants $C > 0, 0 < \lambda < 1$ such that

$$\|Df^n|_{E^s(x)}\| \le C\lambda^n$$

and

$$\|Df^{-n}|_{E^u(x)}\| \le C\lambda^n$$

for all $x \in \Lambda$ and $n \ge 0$. Moreover, we say that Λ admits a *dominated splitting* if the tangent bundle $T_\Lambda M$ has a Df-invariant splitting $E \oplus F$ and there exist constants $C > 0, 0 < \lambda < 1$ such that

$$\|Df^n|_{E(x)}\| \cdot \|Df^{-n}|_{F(f^n(x))}\| \le C\lambda^n$$

for all $x \in \Lambda$ and $n \ge 0$.

We say that a subset $\mathcal{R} \subset \mathrm{Diff}(M)$ is *residual* if $\mathcal{R}$ contains the intersection of a countable family of open and dense subsets of Diff(M); in this case $\mathcal{R}$ is dense in $\mathrm{Diff}(M)$. A property (P) is said to be (C^1)-*generic* if (P) holds for all diffeomorphisms which belong to some residual subset of $\mathrm{Diff}(M)$.

Recently Abdenur and Díaz [2] obtained a necessary and sufficient condition for a locally maximal transitive set Λ of a C^1-generic diffeomorphism f to be hyperbolic as follow: either Λ is hyperbolic, or there are a C^1-neighborhood $\mathcal{U}(f)$ of f and a neighborhood V of Λ such that every $g \in \mathcal{U}(f)$ does not have the shadowing property on the neighborhood V.

The main result of this paper is the following.

Theorem A. *A locally maximal chain transitive set of a C^1-generic diffeomorphism is hyperbolic if and only if it is shadowable.*

It is explained in [1] that every C^1-generic diffeomorphism comes in one of two types: *tame* diffeomorphisms, which have a finite number of homoclinic classes and whose nonwandering sets admit partitions into a finite number of disjoint transitive sets; and *wild* diffeomorphisms, which have an infinite number of (disjoint and different) homoclinic classes and whose nonwandering sets admit no such partitions. It is easy to show that if a diffeomorphism has a finite number of chain components, then every chain component is locally maximal, and so every chain component of a tame diffeomorphism is locally maximal. Hence we can get the following result by Theorem A.

Theorem B. *There is a residual set $\mathcal{R} \subset \mathrm{Diff}(M)$ such that if $f \in \mathcal{R}$ is tame, then the following two conditions are equivalent*:

(1) $\mathcal{CR}(f)$ *is hyperbolic.*
(2) $\mathcal{CR}(f)$ *is shadowable.*

2. Proof of Theorem A

In dynamical systems the periodic orbits play an important role. Some dynamical invariants are associated to them; in general, they also can be followed after perturbation of the dynamics. Pugh's closing lemma implies that any transitive set Λ of a C^1-generic diffeomorphism f is the Hausdorff limit of a sequence of periodic orbits P_n of f: i.e., $\lim_{n\to\infty} P_n = \Lambda$.

Recently Crovisier [4] provides us with a remarkable result for the following question, in terms of chain transitivity: what is the class of compact sets that may be approximated by a sequence of periodic orbits? He proved that the chain transitive sets of C^1-generic diffeomorphisms are approximated in the Hausdorff topology by periodic orbits.

First we state some results which will be used in the proof of Theorem A.

Lemma 2.1. *There is a residual set $\mathcal{R}_1 \subset \mathrm{Diff}(M)$ such that every $f \in \mathcal{R}_1$ satisfies the following properties*:

(1) *Every periodic point of f is hyperbolic and all their invariant manifolds are transverse* (*Kupka-Smale*).
(2) *A compact f-invariant set Λ is chain transitive if and only if Λ is the Hausdorff limit of a sequence of periodic orbits of f* ([4]).

Lemma 2.2. *There is a residual set $\mathcal{R}_2 \subset \mathrm{Diff}(M)$ such that every $f \in \mathcal{R}_2$ satisfies the following property*: *For any closed f-invariant set $\Lambda \subset M$, if there are a sequence of diffeomorphisms f_n converging to f and a sequence of hyperbolic periodic orbits P_n of f_n with index k verifying* $\lim_{n\to\infty} P_n = \Lambda$, *then there is a sequence of hyperbolic periodic orbits Q_n of f with index k such that Λ is the Hausdorff limit of Q_n, where the index of a hyperbolic periodic orbit P is the dimension of the stable manifold of P.*

Proof. Let $\mathcal{K}(M)$ be the space of all nonempty compact subsets of M with the Hausdorff metric, and take a countable basis $\beta = \{\mathcal{V}_n\}_{n=1}^{\infty}$ of $\mathcal{K}(M)$. For each pair (n,k) with $n \geq 1$ and $k \geq 0$, we denote by $\mathcal{H}_{n,k}$ the set of diffeomorphisms f such that f has a C^1-neighborhood $\mathcal{U}$ in Diff(M) with the following property: for every $g \in \mathcal{U}$, there is a hyperbolic periodic orbit $Q \in \mathcal{V}_n$ of g with index k. Let $\mathcal{N}_{n,k}$ be the set of diffeomorphisms f such that f has a C^1-neighborhood $\mathcal{U}$ in Diff(M) with the following property: for every $g \in \mathcal{U}$, there is no hyperbolic periodic orbit $Q \in \mathcal{V}_n$ of g with index k. It is clear that $\mathcal{H}_{n,k} \cup \mathcal{N}_{n,k}$ is open in Diff(M). To show that $\mathcal{H}_{n,k} \cup \mathcal{N}_{n,k}$ is a dense in Diff(M), we take $f \in \text{Diff}(M) - \mathcal{N}_{n,k}$. Then for any C^1-neighborhood $\mathcal{U}$ of f, there is $g \in \mathcal{U}$ such that g has a hyperbolic periodic orbit $Q \in \mathcal{V}_n$ with index k. The hyperbolicity of Q for g implies that $g \in \mathcal{H}_{n,k}$. This means that $f \in \overline{\mathcal{H}_{n,k}}$, and so $\overline{\mathcal{H}_{n,k} \cup \mathcal{N}_{n,k}} = \text{Diff}(M)$.

Let

$$\mathcal{R}_2 = \bigcap_{n \in \mathbb{Z}^+, k=0,\ldots,\dim(M)} \mathcal{H}_{n,k} \cup \mathcal{N}_{n,k}.$$

Then $\mathcal{R}_2$ is a residual subset of Diff(M). Let $f \in \mathcal{R}_2$, and let Λ be a closed f-invariant subset of M. Assume that there is a sequence of diffeomorphisms f_n converging to f and a sequence of periodic orbits P_n of f_n with index k such that Λ is the Hausdorff limit of P_n. For any neighborhood $\mathcal{V}$ of Λ in $\mathcal{K}(M)$, take $\mathcal{V}_m \in \beta$ such that $\Lambda \in \mathcal{V}_m \subset \mathcal{V}$. Then we have $f \notin \mathcal{N}_{m,k}$, and so $f \in \mathcal{H}_{m,k}$. Hence f has a periodic orbit, say Q_m, in $\mathcal{V}_m$ with index k. This completes the proof. □

We say that a point x in M is *well closable* for $f \in \text{Diff}(M)$ if for any $\varepsilon > 0$, there are $g \in \text{Diff}(M)$ with $d_{C^1}(g,f) < \varepsilon$ and a periodic point p of g such that $d(f^n(x), g^n(p)) < \varepsilon$ for all $0 \leq n \leq \pi(p)$, where $\pi(p)$ is the period of p. Let $\sum(f)$ denote the set of well closable points of f. Mane's ergodic closing lemma [6] says that $\mu(\sum(f)) = 1$ for any f-invariant Borel probability measure μ on M.

Let $\mathcal{M}$ be the space of all Borel measures μ on M endowed with the weak* topology. It is easy to check that, for any ergodic measure $\mu \in \mathcal{M}$ of f, μ is supported on a periodic orbit $P = \{p, f(p), \ldots, f^{\pi(p)-1}(p)\}$ of f if and only if

$$\mu = \frac{1}{\pi(p)} \sum_{i=0}^{\pi(p)-1} \delta_{f^i(p)},$$

where δ_x is the atomic measure respecting x.

The following lemma comes from the Mane's ergodic closing lemma in [6] which gives the measure theoretical viewpoint on the approximation by periodic orbits.

Lemma 2.3. *There is a residual set $\mathcal{R}_3 \subset \text{Diff}(M)$ such that every $f \in \mathcal{R}_3$ satisfies the following property: Any ergodic invariant measure μ of f is the limit of sequence of ergodic invariant measures supported by periodic orbits P_n*

of f in the weak topology. Moreover, the orbits P_n converges to the support of μ in the Hausdorff topology.*

Proof. Let $\beta = \{\mathcal{V}_n\}_{n=1}^{\infty}$ be a countable basis of $\mathcal{M}$. For each positive integer n, we denote by $\mathcal{H}_n$ the set of diffeomorphisms f such that f has a C^1-neighborhood $\mathcal{U}$ in $\mathrm{Diff}(M)$ with the following property: for any $g \in \mathcal{U}$, there is a periodic point p of g such that

$$\frac{1}{\pi(p)} \sum_{i=0}^{\pi(p)-1} \delta_{g^i(p)} \in \mathcal{V}_n.$$

Let $\mathcal{N}_n$ be the set of diffeomorphisms f such that f has a C^1-neighborhood $\mathcal{U}$ in $\mathrm{Diff}(M)$ with the following property: for any $g \in \mathcal{U}$, there is no periodic point p of g such that $\frac{1}{\pi(p)} \sum_{i=0}^{\pi(p)-1} \delta_{g^i(p)} \in \mathcal{V}_n$. It is obvious that $\mathcal{H}_n \cup \mathcal{N}_n$ is open in $\mathrm{Diff}(M)$. To show that $\mathcal{H}_n \cup \mathcal{N}_n$ is a dense in $\mathrm{Diff}(M)$, we take $f \in \mathrm{Diff}(M) - \mathcal{N}_n$. Then for any C^1-neighborhood $\mathcal{U}$ of f, there is $g \in \mathcal{U}$ such that g has a periodic point p such that $\frac{1}{\pi(p)} \sum_{i=0}^{\pi(p)-1} \delta_{g^i(p)} \in \mathcal{V}_n$. With a small perturbation, we may assume that the periodic orbit is hyperbolic. The hyperbolicity of p implies that $g \in \mathcal{H}_n$. This means that $f \in \overline{\mathcal{H}_n}$, and so $\mathcal{H}_n \cup \mathcal{N}_n$ is dense in $\mathrm{Diff}(M)$.

Let

$$\mathcal{R}_3 = \bigcap_{n \in \mathbb{Z}^+} \mathcal{H}_n \cup \mathcal{N}_n.$$

Then $\mathcal{R}_3$ is a residual subset of $\mathrm{Diff}(M)$. Let $f \in \mathcal{R}_3$, and let μ be an ergodic invariant measure of f. For any neighborhood $\mathcal{V}$ of μ in $\mathcal{M}$, there is $\mathcal{V}_n \in \beta$ such that $\mu \in \mathcal{V}_n \subset \mathcal{V}$. By the Mane's ergodic closing lemma and Birkhoff ergodic theorem, there is a well closable point x in the support of μ such that μ is the limit point of $\frac{1}{n} \sum_{i=0}^{n-1} \delta_{f^i(x)}$ under the weak* topology and the support of μ equal the closure of the positive orbit of x. Since x is well closable, one can see that $f \notin \mathcal{N}_n$, and so $f \in \mathcal{H}_n$. Hence there is a periodic point, say p_n, of f such that $\frac{1}{\pi(p_n)} \sum_{i=0}^{\pi(p_n)-1} \delta_{f^i(p_n)} \in \mathcal{V}_n \subset \mathcal{V}$. This means that there is an ergodic invariant measure of f in $\mathcal{V}$ whose support is a periodic orbit $P_n = \{f^i(p_n)\}_{i \in \mathbb{Z}}$ of f. By our construction, we can see that the support of μ is the Hausdorff limit of P_n, and so completes the proof. □

In the following lemma, we can see that every periodic point of a shadowable chain transitive set Λ of $f \in \mathcal{R}_1$ has the same index; that is, the dimensions of stable manifolds of all periodic points in Λ are the same.

Lemma 2.4. *Let $f \in \mathcal{R}_1$, and Λ be a shadowable chain transitive set of f. Then all periodic points in Λ have the same index.*

Proof. Let p and q be two periodic points of f in Λ, and let $\varepsilon > 0$ be a small constant such that the local stable manifold $W^s_\varepsilon(p)$ and the local unstable manifold $W^u_\varepsilon(q)$ are well defined. Take a constant $\delta > 0$ such that every δ-pseudo orbit in Λ is ε-shadowed by a point in M. Since Λ is chain transitive, there is a δ-pseudo orbit $\{x_0, x_1, \ldots, x_n\}$ in Λ such that $x_0 = q$ and $x_n = p$. Construct a δ-pseudo orbit ξ in Λ as follows:

$$\xi = \{\ldots, f^{-2}(q), f^{-1}(q), q, x_1, \ldots, p, f(p), f^2(p), \ldots\}.$$

Then there is an orbit $Orb(y)$ which ε shows ξ, where $Orb(y) = \{f^n(y) : n \in \mathbb{Z}\}$. Since $Orb(y) \cap W^s_\varepsilon(p) \neq \emptyset$ and $Orb(y) \cap W^u_\varepsilon(q) \neq \emptyset$, we have $y \in W^s(p) \cap W^u(q)$. This implies that the index of p and index of q should be same. Otherwise it will contradicts the fact that the stable manifold $W^s(p)$ and the unstable manifold $W^u(q)$ are transverse, and so completes the proof. $\square$

Now we define the residual subset $\mathcal{R}$ of $\mathrm{Diff}(M)$ required in the statement of Theorem A as follow: $\mathcal{R} = \mathcal{R}_1 \cap \mathcal{R}_2 \cap \mathcal{R}_3$. Then we have the following proposition which is crucial to prove Theorem A.

Proposition 2.1. *Let $f \in \mathcal{R}$, and let Λ be a shadowable chain transitive set of f which is locally maximal. Then there exist constants $m > 0$ and $0 < \lambda < 1$ such that for any periodic point $p \in \Lambda$,*

$$\prod_{i=0}^{\pi(p)-1} \|Df^m|_{E^s(f^{im}(p))}\| < \lambda^{\pi(p)},$$

$$\prod_{i=0}^{\pi(p)-1} \|Df^{-m}|_{E^u(f^{-im}(p))}\| < \lambda^{\pi(p)}$$

and

$$\|Df^m|_{E^s(p)}\| \cdot \|Df^{-m}|_{E^u(f^m(p))}\| < \lambda^2,$$

where $\pi(p)$ denote the period of p.

Proof. Since $f \in \mathcal{R}_2$, and all periodic points in Λ have the same index and Λ is locally maximal, we can choose a C^1-neighborhood $\mathcal{U}(f)$ of f and a neighborhood U of Λ such that every $g \in \mathcal{U}(f)$ has no nonhyperbolic periodic orbit which is contained in U. Suppose not. Then, for any C^1-neighborhood $\mathcal{V}(f)$ of f and a neighborhood V of Λ, we can take $g_1, g_2 \in \mathcal{V}(f)$ and hyperbolic periodic orbits Q_1 and Q_2 (in V) of g_1 and g_2, respectively, such that $\mathrm{index} Q_1 \neq \mathrm{index} Q_2$. Consequently we can select two sequences of diffeomorphisms g_n and $g_n^{'}$ which converge to f, and two sequences of hyperbolic periodic orbits $Q_n, Q_n^{'}$ of g_n and $g_n^{'}$, respectively, such that $\lim_{n\to\infty} Q_n = \Lambda = \lim_{n\to\infty} Q_n^{'}$ and $\mathrm{index} Q_n \neq \mathrm{index} Q_n^{'}$ for each $n \in \mathcal{N}$. Without loss of generality, we may assume that $\mathrm{index} Q_n = \mathrm{index} Q_m$ and $\mathrm{index} Q_n^{'} = \mathrm{index} Q_m^{'}$ for all $m, n \in \mathcal{N}$ by taking a subsequence if necessary. From Lemma 2.2, we can choose two sequences of periodic orbits P_n and $P_n^{'}$ of f such that $\mathrm{index} P_n =$index Q_n, $\mathrm{index} P_n^{'} = \mathrm{index} Q_n^{'}$ and Λ is the Hausdorff limit of $\{P_n\}$ and $\{P_n^{'}\}$, respectively. Since Λ is locally maximal, we may assume that $P_n, P_n^{'} \subset \Lambda$ for sufficiently large n. Since index $P_n \neq$ index $P_n^{'}$, we arrive at the contradiction by Lemma 2.4. Moreover we may assume that all of the indices of periodic orbits of $g \in \mathcal{U}(f)$ are the same. Hence we can apply Lemma II.3 in [6], and so we get the constants $K > 0, m_0 \in \mathbb{Z}^+$ and $0 < \lambda < 1$ such that for any periodic point $p \in \Lambda$ with $\pi(p) \geq K$,

$$\prod_{i=0}^{\pi(p)-1} \|Df^{m_0}|_{E^s(f^{im_0}(p))}\| < \lambda^{\pi(p)},$$

$$\prod_{i=0}^{\pi(p)-1} \|Df^{-m_0}|_{E^u(f^{-im_0}(p))}\| < \lambda^{\pi(p)}$$

and

$$\|Df^{m_0}|_{E^s(p)}\| \cdot \|Df^{-m_0}|_{E^u(f^{m_0}(p))}\| < \lambda^2.$$

Let Λ_0 be the set of all periodic points in Λ whose periods are less than K. Since every periodic point of f is hyperbolic, there are only a finite number of periodic points in Λ_0, and so Λ_0 is hyperbolic for f. Let k be a positive integer such that $\|Df^{km_0}|_{E^s(x)}\| < \lambda$ and $\|Df^{-km_0}|_{E^u(x)}\| < \lambda$ for all $x \in \Lambda_0$. If we let $m = km_0$, then we know that m and λ are the required constants. □

End of the proof of Theorem A. By Lemma 2.1 and the third property of Proposition 2.1, we can see that Λ admits a dominated splitting $T_\Lambda M = E \oplus F$ which satisfies $E(p) = E^s(p)$ and $F(p) = E^u(p)$ for every periodic point $p \in \Lambda$. To complete the proof of Theorem A, it is enough to show that Df is contracting on E and Df is expanding on F if Λ is shadowable for f. Suppose Df is not contracting on E. Then, by a simple calculation, we can find a "bad" point $b \in \Lambda$ such that

$$\|Df^k|_{E(b)}\| \geq 1$$

for any $k > 0$. Denote by δ_x the atomic measure respecting x. Let us consider a sequence $\{\frac{1}{n}\sum_{i=0}^{n-1} \delta_{f^{im}(b)} : n \in \mathbb{Z}^+\}$ in $\mathcal{M}$, and take an accumulation point $\mu \in \mathcal{M}$ of the sequence. Then we can see that μ is a f^m-invariant probability measure on M with $\mathrm{supp}(\mu) \subset \Lambda$ which satisfies $\int \log(\|Df^m|_{E(x)}\|) df^l_*\mu \geq 0$ for any $l \in \mathbb{Z}$. Take

$$\nu = \frac{1}{m}\sum_{l=0}^{m-1} f^l_*\mu.$$

We can easily see that ν is a f-invariant measure supported on Λ which satisfies $\int \log(\|Df^m|_{E(x)}\|) d\mu \geq 0$. Note here that we can extend E continuously to the whole manifold M. By the ergodic decomposition theorem, there is an ergodic measure μ_0 with $\mathrm{supp}(\mu_0) \subset \Lambda$ such that

$$\int \log(\|Df^m|_{E(x)}\|) d\mu_0 \geq 0.$$

Then, by Lemma 2.3, we can take a sequence of ergodic f-invariant measures μ_n such that the support of each μ_n is a periodic orbit P_n of f, $\{\mu_n\}$ converges to μ_0 and $\{P_n\}$ converges to the support of μ_0. Since Λ is locally maximal, we may assume that every P_n is contained in Λ for sufficiently large n.

If we apply Proposition 2.1, then we have

$$\int \log(\|Df^m|_{E(x)}\|)d\mu_n < \log \lambda$$

for sufficiently large n. Since μ_n converges to μ_0 in the weak* topology, we have

$$\int \log(\|Df^m|_{E(x)}\|)d\mu_n \to \int \log(\|Df^m|_{E(x)}\|)d\mu_0$$

as $n \to \infty$. Hence we get $\int \log(\|Df^m|_{E(x)}\|)d\mu_0 < 0$. The contradiction proves that Df is contracting on E. Similarly we can show that Df is expanding on F. □

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