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HYDROMAGNETIC ROTATING DISK FLOW OF A NON-NEWTONIAN FLUID WITH HEAT TRANSFER AND OHMIC HEATING

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ABSTRACT. The steady hydromagnetic flow of an electrically conducting non-Newtonian fluid due to the rotation of an infinite disk is studied with heat transfer with the inclusion of the ion slip as well as Ohmic heating. The governing nonlinear momentum equations and energy equations are solved using the finite difference method. The numerical results indicate the important effect of the ion slip and the non-Newtonian fluid characteristics on the velocity and temperature distributions.

1. Introduction

The flow due to an infinite rotating disk was studied by von Karman [1] who formulated the problem in the steady state and introduced similarity transformations to transform the governing nonlinear partial differential equations to nonlinear ordinary differential equations. Asymptotic solutions for the resulting nonlinear ordinary differential equations were given by Cochran [2]. Rotating disk flows of an electrically conducting fluids have interesting applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, viscometry and crystal growth processes. The effect of an externally applied uniform magnetic field on the flow due to a rotating disk was examined in [3,4]. In the above mentioned work, the Hall and ion slip effects were ignored in applying Ohm's law, as their effect can be ignored for small and moderate values of the magnetic field [5,6]. The influence of the Hall current and ion slip on the steady hydromagnetic flow with heat transfer due to an infinite rotating was studied [7,8]. The effect of uniform suction or injection on the flow of an electrically conducting fluid due to a rotating disk was studied taking the Hall current [9] and the ion slip [10] into consideration.

The problem of steady state heat transfer from a rotating disk at a constant temperature was examined by Millsaps and Pohlhausen [11] for a variety of Prandtl number. Sparrow and Gregg [12] studied the steady state heat transfer from a rotating disk maintained at a constant temperature to fluids at any Prandtl number. Attia [13] solved the unsteady state heat transfer in the presence of an

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applied uniform magnetic field. The steady flow of a non-Newtonian fluid above an infinite rotating disk with uniform suction was studied by Mithal [14]. The influence of a uniform magnetic field on the flow of the non-Newtonian fluid was studied by Attia [15].

In the present paper, the steady hydromagnetic flow of a viscous, incompressible, and electrically conducting non-Newtonian fluid due to the uniform rotation of an infinite disk in an axial uniform magnetic field is studied considering the ion slip and the Ohmic heating. The governing non-linear partial differential equations are solved numerically using the finite difference method. Some interesting effects for the magnetic field, the Hall current, the ion slip, and the non-Newtonian fluid characteristics on the velocity and temperature fields are discussed.

2. Basic Equations

The disk is assumed to be electrically insulating and rotating impulsively from rest in the z=0 plane about the z-axis with a uniform angular velocity ω. The fluid is assumed to be electrically conducting incompressible and has density ρ, kinematical viscosity ν, and electrical conductivity σ. An external axial uniform magnetic field is applied in the z-direction and has a constant flux density $\boldsymbol{B}_o$. The magnetic Reynolds number is assumed to be very small, and therefore the induced magnetic field is negligible. The electron-atom collision frequency is assumed to be high, then both the Hall effect and the ion slip effects are taken into consideration [5,6]. Due to the axial symmetry about the z-axis, the cylindrical coordinates (r,φ,z) are used. For the sake of definiteness, the disk is taken to be rotating in the positive φ direction. Due to the symmetry about the z=0 plane, it is sufficient to consider the problem in the upper half space only. If the Hall and ion slip terms are retained in generalized Ohm's law, then the equations of steady motion in cylindrical coordinates [5,6]

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}\right)+\frac{\sigma B_o^2}{(\alpha^2+Be^2)}(\alpha u-Bev)=\frac{\partial \tau_r^r}{\partial r}+\frac{\partial \tau_r^z}{\partial z}+\frac{\tau_r^r-\tau_\varphi^\varphi}{r} \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{uv}{r}\right)+\frac{\sigma B_o^2}{(\alpha^2+Be^2)}(\alpha v+Beu)=\frac{\partial \tau_\varphi^r}{\partial r}+\frac{\partial \tau_\varphi^z}{\partial z}+\frac{2\tau_\varphi^r}{r} \tag{3}$$

$$\rho\left(u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}\right)=\frac{\partial \tau_z^r}{\partial r}+\frac{\partial \tau_z^z}{\partial z}+\frac{\tau_z^r}{r} \tag{4}$$

where u, v, and w are the velocity components in the directions of increasing r, φ, and z, $Be=\sigma\beta B_o$ is the Hall parameter which can take positive or negative values, $\beta=1/nq$ is the Hall factor, n is the electron concentration per unit volume, $-q$ is the charge of the electron [5,6], $\alpha=1+BeBi$, and Bi is the ion slip parameter [5,6].

The boundary conditions are given as

$$z=0, u=0, v=r\omega, w=w_o, \tag{5a}$$

$$z\to\infty, u\to 0, v\to 0, p\to p_\infty. \tag{5b}$$

where p is denoting the pressure. Due to the uniform suction or injection, the vertical velocity component takes a constant non-zero value at z=0. Introducing von Karman transformations [1],

$$u = r\omega F, v = r\omega G, w = \sqrt{\omega\nu}H, z = \sqrt{\nu/\omega}\zeta, p - p_\infty = -\rho\nu\omega P$$

where ζ is a non-dimensional distance measured along the axis of rotation, F, G, H and P are non-dimensional functions of the modified vertical coordinate ζ, Eqs. (1)-(5) take the form

$$\frac{dH}{d\zeta} + 2F = 0 \tag{6}$$

$$\frac{d^2F}{d\zeta^2} - H\frac{dF}{d\zeta} - F^2 + G^2 - \frac{\gamma}{(\alpha^2 + Be^2)}(\alpha F - BeG) - \frac{1}{2}K((\frac{dF}{d\zeta})^2 + 3(\frac{dG}{d\zeta})^2 + 2F\frac{d^2F}{d\zeta^2}) = 0 \tag{7}$$

$$\frac{d^2G}{d\zeta^2} - H\frac{dG}{d\zeta} - 2FG - \frac{\gamma}{(\alpha^2 + Be^2)}(\alpha G + BeF) + K(\frac{dF}{d\zeta}\frac{dG}{d\zeta} - F\frac{d^2G}{d\zeta^2}) = 0 \tag{8}$$

$$\frac{d^2H}{d\zeta^2} - H\frac{dH}{d\zeta} - \frac{7}{2}K\frac{dH}{d\zeta}\frac{d^2H}{d\zeta^2} + \frac{dP}{d\zeta} = 0 \tag{9}$$

$$\zeta = 0, F = 0, G = 1, H = 0, \tag{10a}$$

$$\zeta \to \infty, F \to 0, G \to 0, P \to 0. \tag{10b}$$

where K is the parameter which describes the non-Newtonian behavior, $K=\mu_c/\mu\omega$, μ_c is the coefficient of cross viscosity, γ is the magnetic interaction number given by $\gamma=\sigma B_o^2/\rho\omega$ which represents the ratio between the magnetic force to the fluid inertia force.

Due to the difference in temperature between the wall and the ambient fluid heat transfer takes place. The energy equation, neglecting the dissipation term, takes the form [9,10],

$$\rho c_p(u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}) = k(\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}) + \frac{\sigma B_o^2}{\alpha^2 + Be^2}(u^2 + v^2) \tag{11}$$

where k and c_p are, respectively, the thermal conductivity and the specific heat at constant pressure of the fluid. The last term in Eq. (11) represents the Ohmic heating. The boundary conditions for the energy problem are that the temperature, by continuity considerations, equals T_w at the surface of the disk. Far from the disk, $T \to T_\infty$ where T_∞ is the temperature of the ambient fluid.

In terms of the non-dimensional variable $\overline{T} = (T - T_\infty)/(T_w - T_\infty)$ and using von Karman transformations, Eq. (11) takes the form (the bar will be dropped),

$$\frac{1}{\Pr}\frac{\partial^2 T}{\partial\zeta^2} - H\frac{\partial T}{\partial\zeta} + \frac{\gamma Ec}{\alpha^2 + Be^2}(F^2 + G^2) = 0 \tag{12}$$

where Pr is the Prandtl number given by, $Pr=c_p\mu/k$ and $Ec = \omega^2 r^2 / c_p(T_w - T_\infty)$ is the Eckert number. The boundary conditions, in terms of T and von Karman transformations, are expressed as

$$\zeta = 0 : T = 1, \zeta \to \infty, T \to 0, \tag{13}$$

The heat transfer from the disk surface to the fluid is computed by application of Fourier's law

$$q = -k\left(\frac{dT}{dz}\right)_w$$

Introducing the transformed variables, the expression for q becomes

$$q = -k(T_w - T_\infty)\sqrt{\frac{\omega}{\nu}}\frac{dT(0)}{d\zeta}$$

By rephrasing the heat transfer results in terms of a Nusselt number defined as, $N_u = q\sqrt{\nu/\omega}/k(T_w - T_\infty)$ the last equation becomes

$$N_u = -\frac{dT(0)}{d\zeta}$$

The system of non-linear ordinary differential equations (6)-(8) is solved under the conditions given by Eq. (10)to determine the velocity distribution, using the Crank-Nicolson method [16]. Then. Eq. (12) is solved under the boundary conditions (13) to find the temperature distribution. The resulting system of difference equations is solved in the infinite domain $0<\zeta<\infty$, by considering a finite domain in the ζ-direction instead with ζ chosen large enough such that the solutions are not affected by imposing the infinite asymptotic conditions at a finite distance. The independence of the results from the chosen length of the implemented finite domain and the size of mesh used was tested by numerical experiments. Computations are carried out for ζ_∞=10 which is adequate for the ranges of the parameters studied here.

3. Results and Discussion

Figures 1a-d present the profile of the radial velocity component F for various values of the Hall parameter Be, the ion slip parameter Bi, and the non-Newtonian parameter K and for γ=1 and K=0 and 1. Figure 1 indicates that for Be=-0.5 and Bi=0, the radial flow reverses its direction at a certain distance from the surface of the disk. Increasing Bi, for Be<0, reverses the direction of F for all ζ as a result of increasing the effective conductivity ($=\gamma/(\alpha^2+Be^2)$)) which increases the resistive force on F. Figure 1a shows also that for $BeBi$>0, increasing the magnitude of Bi increases F as a result of decreasing the effective conductivity which decreases the damping force on F. Large values of Bi decrease the effective conductivity more which is similar to the hydrodynamic case. Figure 1b depicts that when Be>0 and Bi>0, increasing Bi decreases F for some ζ. This is because the effect of Bi on the numerator in the magnetic force term in Eq. (7) is higher than its effect on the denominator which increases the damping force on F and consequently decreases F for some ζ. Also, for Bi<0, increasing the magnitude of Bi increases F for small ζ and then decreases it for larger ζ. This is corresponding to the crossover in the F-ζ chart with Bi. It is interesting to find that the changing of F with Bi depends on ζ. The effect of the parameter K is shown in Figs. 1c and 1d. The figures show that the effect of the parameter K is to reverse the direction of F for the case Bi=Be=0. For Be<0, as shown in Fig. 1c, the parameter K reverses the direction of F for all values of Bi. In Fig. 1d, for Be>0, K reduces the magnitude of F while keeping its direction where the value at which the crossover in the F-ζ charts occur increases with K.

Figures 2a-d present the profile of the azimuthal velocity component G for various values of the Hall parameter β_e, the ion slip parameter Bi and the non-Newtonian parameter K and for γ=1 and K=0 and 1. As shown in Fig. 2a, small negative values of Be increases G due to decreasing the magnetic damping force. Increasing Bi, with Be<0, reduces G, as a result of increasing the effective conductivity. Figure 2a indicated that for negative values of Bi, increasing the magnitude of Bi increases G as a result of decreasing the damping force on G. Figure 2b depicts the same findings. For $BeBi$>0, increasing Bi increases G, on the other hand, for $BeBi$<0, increasing the magnitude of Bi decreases G. Figures 2c and 2d indicate that increasing K increases G for all Be and Bi. The effect of Bi on G is more pronounced for K>0.

Figures 3a-d present the profile of the axial velocity component H for various values of the Hall parameter Be, the ion slip parameter Bi, and the non-Newtonian parameter K and for γ=1 and K=0 and 1. As shown in Fig. 3a, for Be=-0.5 and Bi=0, the axial velocity H reverses its direction at a certain ζ. Increasing Bi increases H, as a result of decreasing F, and reverses its direction for all ζ. Figure 3b indicates that for Be>0, increasing the magnitude of Bi, in general, decreases H as a result of increasing F. It is also shown in Fig. 3b that the axial flow is always towards the disk for all values of Bi. Figures 3c and 3d show the effect of the non-Newtonian parameter K in reversing the direction of H for the case Be=Bi=0. For Be<0, as shown in Fig. 3c, the parameter K reverses the direction of H for all values of Bi. In Fig. 3d, for Be>0, K reduces the magnitude of H while keeping its direction.

Figures 4a-d present the profile of the temperature T for various values of the Hall parameter Be, the ion slip parameter Bi, and the non-Newtonian parameter K and for γ=1, Pr=0.7 and K=0 and 1. It is shown in Figs. 4a and 4c that, for Bi>0, increasing Bi increases T as a result of increasing H but for Bi<0, increasing the magnitude of Bi decreases T due to decreasing H. On the other hand, Figs. 4b and 4d, shows that for Bi>0, increasing Bi decreases T as a result of decreasing H, however for Bi<0, increasing the magnitude of Bi increases T due to increasing H. It is shown in Fig. 4 that the effect of the ion slip parameter Bi on the temperature T becomes more apparent for higher values of K and for positive values of the Hall parameter than for negative values.

Tables 1 and 2 show the variation of the Nusselt number N_u with the Hall parameter Be, the ion slip parameter Bi, and the non-Newtonian parameter K for Pr=0.7 and 10, respectively. In these tables γ=1 and Ec=0.2. It is shown that, for Be<0 and all values of S, increasing Bi decreases N_u due to the decrease in the incoming axial flow at near-ambient temperature to the disk which decreases the heat transfer from the surface of the disk. On the other hand, for Be>0 and for all values of S, increasing Bi increases the incoming flow and then increases N_u. The influence of Bi on N_u is more clear for Be>0 than for Be<0. Increasing the parameter K reduces N_u for all Be, Bi and Pr. This is expected since the effect of the non-Newtonian behavior is to restrain the axial flow towards the disk and then prevent bringing the fluid at near-ambient temperature to the disk which decreases N_u.

Table 3 shows the variation of the pressure at infinity p_∞ with the Hall parameter Be, the ion slip parameter Bi, and the non-Newtonian parameter K for Pr=0.7. In these tables γ=1 and Ec=0.2. It is shown that, for Be<0, increasing Bi increases the magnitude of p_∞ for all values of K. For Be<0, increasing the non-Newtonian parameter K increases p_∞, whereas for Be>0, increasing the non-Newtonian parameter K decreases p_∞. The influence of Bi on p_∞ is more clear for Be<0 than for Be>0.

4. Conclusions

The steady hydromagnetic flow of an electrically conducting non-Newtonian fluid due to the rotation of an infinite disk in the presence of an externally applied uniform magnetic field is studied with heat transfer considering the Hall effect, the ion slip and the Ohmic heating. The inclusion of the Hall effect, the ion slip and the non-Newtonian fluid characteristics reveals some interesting phenomena and it is found that both the magnitudes and signs of the Hall and ion slip parameters are effective. It is interesting to find, in the Newtonian case, that the radial and axial flows reverse direction with the distance ζ for $Be<0$ and all values of Bi. The effect of the non-Newtonian parameter K is to reverse the axial and radial flow for $Be=Bi=0$. Also, in the non-Newtonian case, both the axial and radial flows reverse direction for all ζ for $Be<0$ and all values of Bi. For $Be>0$, the dependence of the velocity components in the ion slip changes with ζ. The heat transfer at the surface of the disk is found to vary with the magnitude and sign of the Hall and ion slip parameters. The effect of Bi on the temperature as well as the heat transfer at the surface of the disk is more apparent for higher values of K and for $Be<0$ than $Be>0$. On the other hand, the effect of non-Newtonian parameter K on the heat transfer is more clear for $Be>0$ than $Be<0$.

Table 1. Variation of the steady state value of N_u with Be, Bi and K for $Pr=0.7$

N_u	$K=0$	$K=0.5$	$K=1$
$Be=0$, $Bi=0$	0.1646	0.1164	0.0729
$Be=-0.5$, $Bi=0$	0.1136	0.0601	0.0158
$Be=-0.5$, $Bi=0.5$	0.0474	-0.0083	-0.0543
$Be=-0.5$, $Bi=1$	-0.0556	-0.1135	-0.1636
$Be=-0.5$, $Bi=-0.5$	0.1573	0.1061	0.0635
$Be=-0.5$, $Bi=-1$	0.1875	0.1383	0.0973
$Be=0.5$, $Bi=0$	0.2321	0.1877	0.1474
$Be=0.5$, $Bi=0.5$	0.2425	0.1982	0.1587
$Be=0.5$, $Bi=1$	0.2509	0.2069	0.1682
$Be=0.5$, $Bi=-0.5$	0.2184	0.1743	0.1333
$Be=0.5$, $Bi=-1$	0.1996	0.1564	0.1147

Table 2. Variation of the steady state value of N_u with *Be*, *Bi* and *K* for *Pr*=10

N_u	K=0	K=0.5	K=1
Be=0, *Bi*=0	0.2346	-0.0119	-0.2276
Be=-0.5, *Bi*=0	0.1115	-0.1204	-0.2947
Be=-0.5, *Bi*=0.5	-0.3415	-0.5891	-0.7725
Be=-0.5, *Bi*=1	-1.1095	-1.3574	-1.5532
Be=-0.5, *Bi*=-0.5	0.3841	0.1707	0.0066
Be=-0.5, *Bi*=-1	0.5574	0.3601	0.2059
Be=0.5, *Bi*=0	0.5659	0.3485	0.1410
Be=0.5, *Bi*=0.5	0.6872	0.4839	0.2967
Be=0.5, *Bi*=1	0.7702	0.5787	0.4069
Be=0.5, *Bi*=-0.5	0.3791	0.1457	0.0888
Be=0.5, *Bi*=-1	0.0827	-0.1649	-0.4338

Table 3. Variation of the steady state value of p_∞ with *Be*, *Bi* and *K* for *Pr*=0.7

N_u	K=0	K=1
Be=0, *Bi*=0	-0.0321	-0.00767
Be=-0.5, *Bi*=0	-0.0042	0.2979
Be=-0.5, *Bi*=0.5	-0.0549	-0.6618
Be=-0.5, *Bi*=1	-0.2305	-1.3811
Be=-0.5, *Bi*=-0.5	-0.0028	-0.1290
Be=-0.5, *Bi*=-1	-0.0191	-0.0512
Be=0.5, *Bi*=0	-0.1299	0.02496
Be=0.5, *Bi*=0.5	-0.1330	-0.0240
Be=0.5, *Bi*=1	-0.1396	-0.0256
Be=0.5, *Bi*=-0.5	-0.1336	-0.0304
Be=0.5, *Bi*=-1	-0.1506	0.0451

Fig. 1 Steady state profile of F for various values of Be, Bi and K. (a) K=0, Be≤0; (b) K=0, Be≥0; (c) K=1, Be≤0; and (d) K=1, Be≥0.

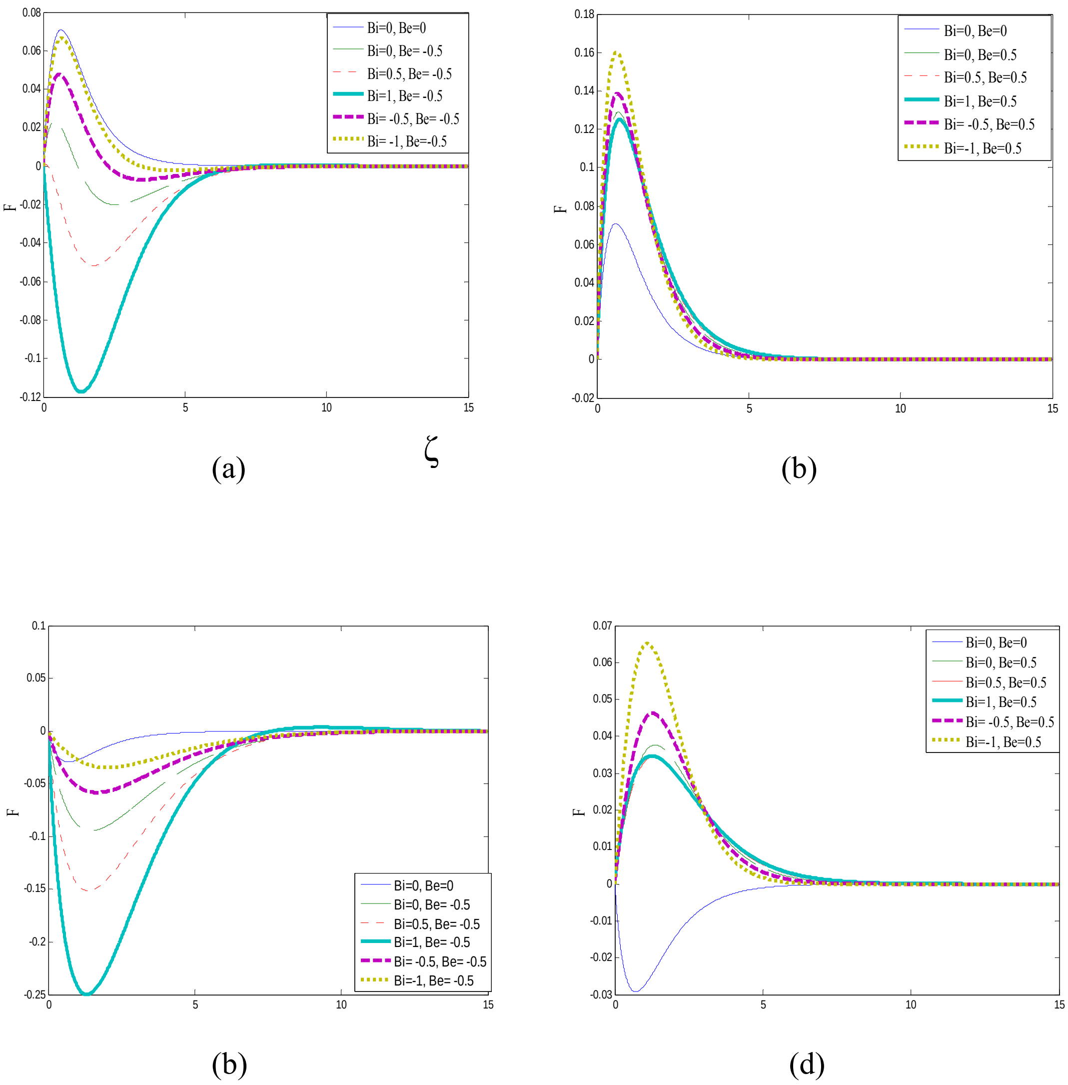


Fig. 2 Steady state profile of G for various values of Be, Bi and K. (a) K=0, Be≤0; (b) K=0, Be≥0; (c) K=1, Be≤0; and (d) K=1, Be≥0.

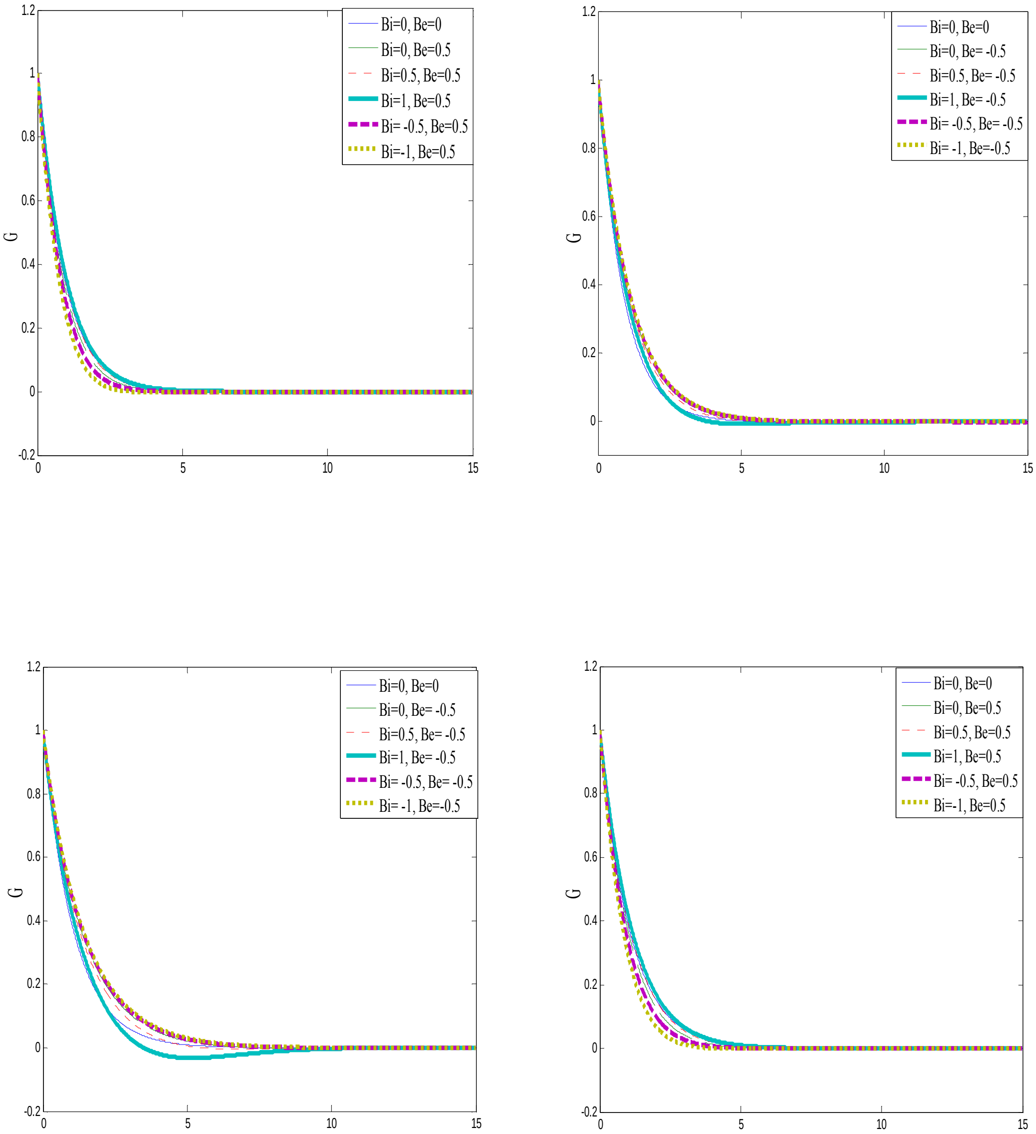


Fig. 3 Steady state profile of H for various values of Be, Bi and K. (a) K=0, Be≤0; (b) K=0, Be≥0; (c) K=1, Be≤0; and (d) K=1, Be≥0.

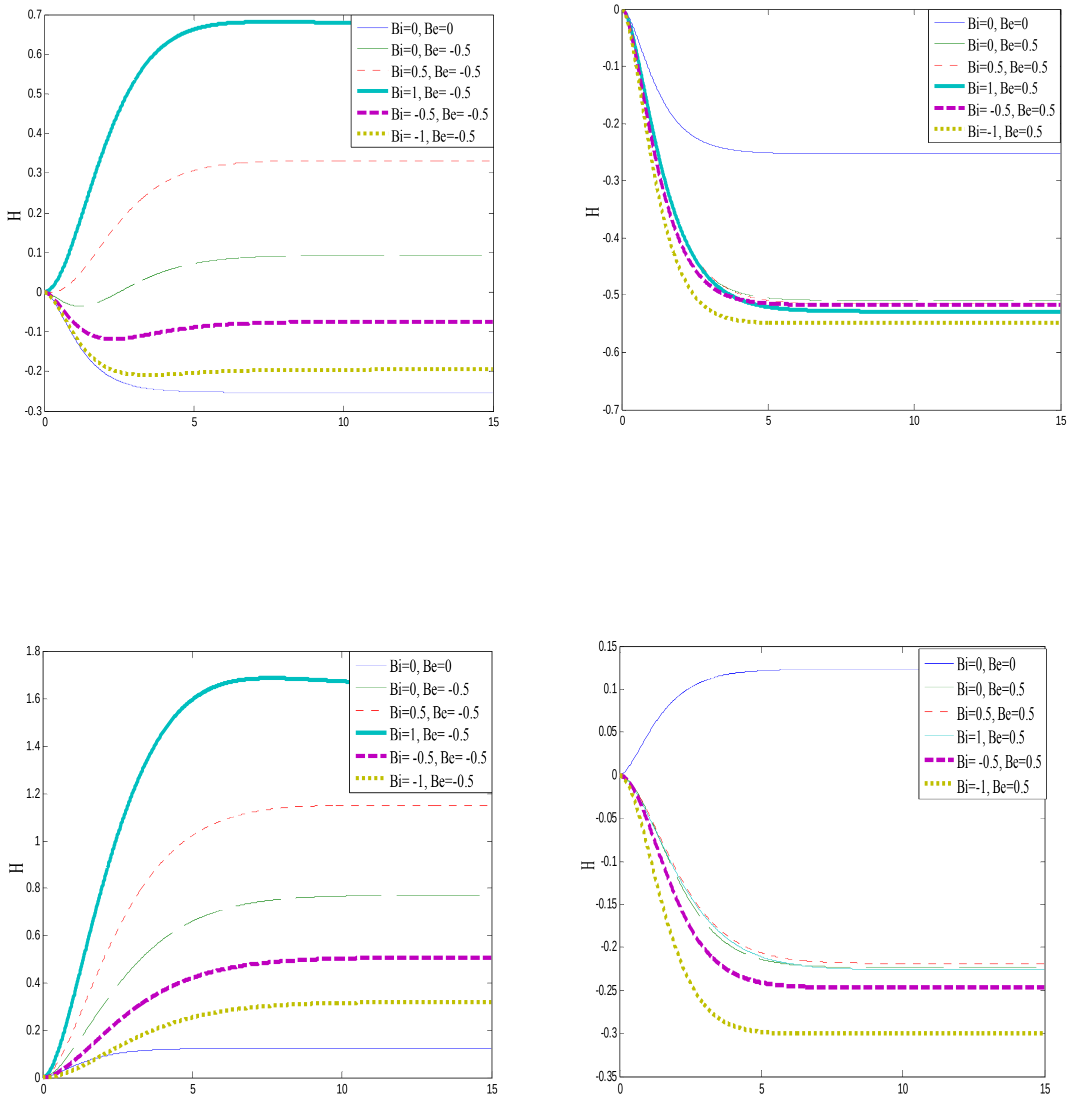
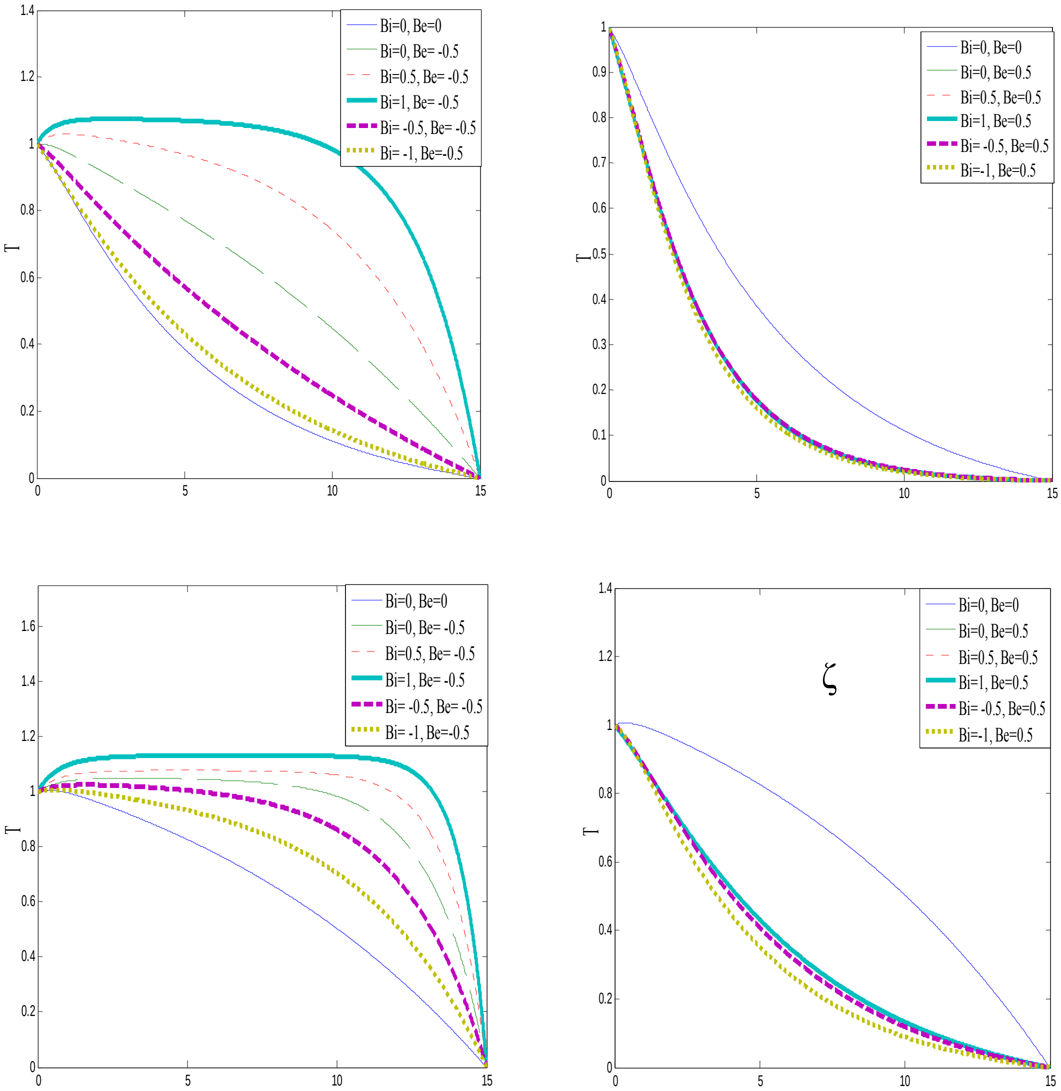


Fig. 4 Steady state profile of T for various values of Be, Bi and K. (a) K=0, Be≤0; (b) K=0, Be≥0; (c) K=1, Be≤0; and (d) K=1, Be≥0.



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