Journal of the Korean Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 16, No. 2, June 2012, 125–135 ⓒ 2012 Korean Society of Mathematical Education

Using Mathematician's Creativity Methods in Mathematics Education[1]

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(Received May 14, 2011; Revised June 10, 2012; Accepted June 20, 2012)

Students not only learn mathematics knowledge, but also have the capability of mathematical creativity. The latter has been thought an important task in mathematics education by more and more mathematicians and mathematics educators. In this paper, mathematicians' methods of creating mathematics are presented. Then, the paper elaborates on how these methods can be utilized to enhance mathematical creativity in the schools.

Keywords: mathematician, mathematics creativity methods, student, mathematics education
MESC Classification: D40
MSC2010 Classification: 97C70, 97D40

1. INTRODUCTION

Today, almost every country's mathematics curricula in the world stress not only the mathematics knowledge which meets the need of the people's daily lives and society, but also the mathematical creativity which means creating some new things to students in their mathematics learning. The reason is very clear, because students' creativity today means the country's creativity in the future. In the traditional mathematics education, learning mathematics is almost learning mathematics knowledge. It is obvious that learning mathematics knowledge is different to forming the ability of the mathematical creativity, although both have some connections. How to make students to form the ability of the mathematical creativity is becoming the important research subject in mathematics education community in the world. The author believes that it is best to culture students'

[1] A draft version of the article was presented at the 16th Korean International Seminar of Mathematics Education on Creativity Development and Gifted Students at Chungnam Nat'l Univ., Yuseong-gu, Daejeon 305-764, Korea; August 12, 2011 (*cf.* Zhang, 2011)

creativity in the mathematics classrooms if the mathematics teacher can "imitate" mathematicians' creativity behaviors to a certain degree. Of course, it is different between students' learning mathematics and mathematicians' works, so, it is impossible to make students' behaviors the same as mathematicians' researches, but mathematicians' works surely can be illuminable to mathematics education.

2. MATHEMATICIANS' CREATIVITY METHODS

What is the mathematicians' creativity? Although the researches to the mathematicians' creativity can be traced to the beginning of last century, the mathematicians' creativity still is a relatively unexplored field, the related researches are insufficient. The existing researches in the mathematicians' creativity mainly manifests in three aspects which are not linear fully.

In the first aspect, the researchers stressed the behaviors of mathematicians' creativity processes. In 1902, a famous periodical published an extensive questionnaire about mathematical creativity; then, Henri Poincare, a famous French mathematician, made a lecture about mathematician's creativity (cf. Srirman, 2009). Based on the above questionnaire and lecture, Hadamard (1945) investigated the psychology of mathematicians' creativity, and posed the process of mathematical creativity *preparation-incubation-illumination-verification* which obviously was influenced by the Gestalt psychology of his time.

After Hadamard, some researchers continued to explore the mathematicians' creativity, for example, Ervynck (1991) brought forward three stages of mathematical creativity, but his theory's influence is small. Hadamard's theory of four-stage mathematical creativity still has considerable influence even today, and some researches think that it is effective to contemporary mathematicians' works (Srirman, 2009).

In the second aspect, the researchers want to describe the exact meanings of mathematicians' creativity, in another words, they try to define the mathematicians' creativity. Many scientists and psychologists define the creativity, but according to Haylock (1987) there seems to be no commonly accepted definition. Psychologists often define the creativity as "useful", "produce an unexpected work" and "adaptive", obviously, mathematicians, mathematics educators, and mathematics philosophers don't agree these definitions. The definitions from mathematicians, mathematics educators, and mathematics philosophers stress the process of mathematical creativity "novel", "original", "unusual" and "insightful". For example, Sriraman (2009) defines the mathematical creativity as "the process that results in unusual and insightful solutions to a given problem, irrespective of the level".

In the third aspect, the researchers want to find the factors in mathematicians' creativity, that is, what leads mathematicians to creativity. Someone attribute the mathematical creativity to aesthetics, someone to creativity thinking, someone to social and culture, and someone to divine, etc. (Sriraman, 2009). The author of this paper agrees the definition of the mathematics creativity given by Srirman, but adds a condition which is new. To mathematics science, the new means the first emergence in the mathematics history; and to mathematics education, the new means the first presence in students' knowledge.

On one hand, it is very important to research the mathematicians' creativity, because of the results of researches will foster development of mathematics science and mathematics education; on the other hand, up to now, this kind of research is insufficient. The author invites more researchers, including mathematicians, mathematics educators and mathematics philosophers join in this research community.

Which methods do mathematicians use in their creativity? The answer of this question is difficult, because every mathematician has his/her characteristics. Every mathematician has his/her mathematics belief, research expertise, and favor, so, mathematical creativity often is personal. Handbook of Creativity which contains a comprehensive review of all research available in the field suggests that most the approaches used in the study of creativity can be subsumed under six categories: mystical, pragmatic, psychodynamic, psychometric, social-personality, and cognitive (Sternberg, 2000). Complexity of studying creativity approaches reflexes complexity of creativity methods from one side.

Whether do study mathematical creativity methods have no sense because of methods' so many? It is not necessary to be so pessimistic. There are a few methods which are more important and more common, because most mathematicians will use them when they make creativity (Barbeau, 1985). By reading existing literature and interviewing with some mathematicians, the author suggests that following methods are more important and more common, these methods are posing new mathematics problems, individual exploring or cooperation research, non-logical thinking, persisting and putting aside. The following are explanations in detail.

Posing new mathematics problems

Of course, mathematician can solve existing problems. At any time, there are many unsolved problems; these are problems the members of the mathematics community face commonly. For example, before Andrew Wiles solved the Fermat's last theorem, it was an existing problem. Except solving the existing problems, mathematician must pose new mathematics problems and try to solve them.

In fact, the problems most mathematicians solve in their academic lives are the problems posed by them. Because the problems are the beginning of mathematical creativity, posing new problems become an important step, without it, no creativity. So, the methods of posing new mathematics problems should belong to the methods of mathematical creativity.

Individual exploring or cooperation research

In the history of mathematical creativity, some mathematicians like individual exploring, and others like cooperation research.

Aforementioned the model of Hadamard's mathematical creativity, in fact, points to individual mathematician's creativity. So, in Hadamard's view, the mathematical creativity should be a kind of mathematician individual behavior. Many famous mathematicians in mathematics history were lonely individual explorer, and Wiles' proving the Fermat's last theorem was a recent example of individual exploring. Individual exploring mathematics, of course, relates to mathematician's individual character; but it also relates to mathematics' traits. Mathematics is a thinking science; thinking is mathematician's main way to get result. Thinking is related to lone, quiet, and undisturbed. So, individual exploring is understandable.

Although mathematical creativity is thought as a lonely work by many people; it is undeniable that mathematicians' cooperation has been existed. Of course, the scope of mathematicians' cooperation is smaller than some nature science, for example, medicine. Some mathematicians like cooperating with other mathematicians because of getting advantage from the cooperation. Different mathematician has his/her characteristics, some are good at whole, and some are expert in details, the others can pose a kind of thoughts, etc. When these mathematicians cooperate, different ideas and thoughts will impact each other to result in problem-solving effectively. Obviously, cooperation research is also related to mathematicians' traits.

Mathematics papers are main forms of the results of mathematicians' creativity, it is very clear to find the coexisting of individual exploring and cooperation research in mathematical creativity from the writer number of the papers published in mathematics periodicals. For example, there are 84 papers in last five issues of *Annals of Mathematics,* a famous mathematics periodical which found in 1884. Among them, 35 papers (about 42%) are the results of individual exploring, and 49 papers (about 58%) are the results of cooperation research.[2]

Non-logical thinking

Most people think that mathematicians' thinking is logical because of the logical feature of mathematics knowledge. This is not correct. Mathematics knowledge is logical, and no logic no mathematics; but mathematicians use more non-logical thinking in math-

[2] http://annals.math.princeton.edu/

ematical creativity. Using logical thinking only, mathematicians will create nothing.

The process of the mathematical creativity includes posing a mathematics problem, understanding the problem, solving the problem and finishing and publishing the result (Zhang, 2010).

In the first stage or the stage of posing a problem, a mathematician needs pose a new and valuable problem. The mathematician needs using non-logical thinking in this stage. Aesthetics or intuition or some other non-logical thinking will make the mathematician believe that one problem is more fruitful than other problem.

In the second stage or the stage of understanding the problem, the mathematician needs understand deeply the related concepts and other mathematicians' related works. Obviously, logical thinking is important in this stage.

In the third stage or the stage of solving the problem, the mathematician uses kinds of strategies to solve this problem. In this stage, the non-logical thinking such as intuition, imagery, analogy, and induction plays an important role. For example, Felix Klein often used the modeling thinking to solve the problems in his creativity (Glas, 2002). Charles Sanders Peirce thought that visual thinking is very important in this process (Compos, 2009). In the fourth stage or the stage of finishing and publishing, the mathematician finishes the problem by writing it formally logically and publishes it. The mathematician mainly uses logical thinking in this stage.

In short, mathematicians use logical and non-logical thinking for mathematical creativity, but non-logical seems more important in the creativity process. Recent investigation also showed that non-logical thinking, such as imaginary, plays the important role in mathematicians' creativity (Sriraman, 2009).

Persisting and putting aside

A real mathematics problem is not easy to solve. The failure cannot stop a mathematician's continuing exploring, because he/she knows that the valuable problem cannot be solved easily. In fact, the mathematician has prepared psychologically to hard work for solving this problem when he/she puts forward or choices the problem. To solve this problem, the mathematician often uses kinds of methods and strategies. After a period of time, the mathematician maybe feels that continuing work on this problem will not solve it. In this case, the mathematician will not give up this problem; he/she puts it aside and begins another problem.

A mathematician often has a few problems simultaneously, the problem he/she is doing presently can be called the *explicit problem*, and the problems which are put aside by the mathematician can be called *implicit problems*. Both the explicit problem or implicit problems occupy the mathematician' brain. At one time, just one problem is the explicit problem which the mathematician does it consciously; and the others are the implicit problems which be thought by brain unconsciously. The mathematician' brain thinking the implicit problems like the computer's background processing. The neuropsychology has proved that human's brain is very complex and high developed, its power is beyond people's imagination.

Even the power of the most advanced computer up to now cannot compare with it. Whether be conscious or unconscious, the mathematician's brain thinks these problems simultaneously. At a certain moment after the mathematician's struggle consciously and unconsciously, the flash of inspiration appears suddenly even without any omen, and the one of the problems occupied in the mathematician's brain is solved accompanied by the mathematician's "Aha!" So, mathematician's persisting and putting aside don't conflict, and putting aside doesn't mean to abandon the problem and is a form of persisting.

3. USING MATHEMATICIAN'S CREATIVITY METHODS IN MATHEMATICS EDUCATION

In mathematics education, is there also mathematical creativity? The author agrees the views of many mathematicians and mathematics educators to this question. "Between the work of a student who tries to solve a difficult problem in mathematics and a work of invention (creation)…there is only a difference of degree" (Polya, 1954). Creativity as a feature of mathematical thinking is not a patent of the mathematician! (Krutetskii, 1976).

According to the aforementioned understanding of mathematical creativity, student's proving a theorem through exploring (this theorem although existing in mathematics, but it is new to the student), posing a new method (this method maybe is common to a mathematician, but it is designed by the student independently), and solving a problem (this problem is not problem to mathematicians, but is a real problem to this student) are all creativity. Undeniably, in the traditional mathematics education, learning mathematics knowledge is overemphasized, so, the space of student's mathematical creativity is very small. In some countries, the space even does not exist.

Now, the traditional education which overemphasized mathematics knowledge learning and ignored mathematics creativity must be changed; mathematical creativity should possess its position in mathematics education. It is obvious that culturing the ability of students' mathematical creativity is different to making them mastering mathematics knowledge. How to culturing the ability of students' mathematical creativity? Mathematicians' creativity methods have some inspirations to mathematics education which gives mathematical creativity a right space.

Posing problem by themselves

Posing new and valuable problems is the beginning of mathematician's creativity. In the traditional mathematics education, students are restrained in the space of solving regular mathematics problems passively, and have little opportunities to putting problems actively themselves. In order to culture the ability of students' mathematical creativity, the mathematics teachers should provide their students opportunities to posing problems and guide them posing the problems. In the beginning, the teachers can model posing mathematical problems for the students, gradually, the students get the opportunities to posing problems independently; In the beginning, the students are guided to posing simply problems, gradually, they should posing more difficult problems; in the beginning, the students can posing low-level problems, gradually, they should be guided to posing high-level problems which involved with advanced mathematics thinking. Obviously, culturing students' posing mathematical problems needs the mathematics teachers having the ability of posing mathematics problems.

The question is whether mathematics teachers have such ability. If mathematics teachers have not the ability to pose mathematics problems themselves, it is impossible to want them to culture the students' posing problems. Some researches revealed that mathematics teachers' ability of posing problems is unsatisfied. For example, some researchers found that the majority of the questions and problems used by teachers focus on memorization and procedural understanding rather than on mathematical reasoning and conceptual understanding (Stein, Smith, Henningsen, & Silver, 2000; Stevenson & Stigler, 1992); and some found that when asked to generate extensions to a given mathematics problem, both teachers and prospective teachers generate problems that are predictable, undemanding, ill-formulated, and unsolvable (Crespo, 2003; Nicol, 1999; Silver, Downs & Leung, 1996). The mathematics teachers' low-level posing problem is rooted in having not got related training from their teacher education. So, in current mathematics teacher education, it is important to pay attention to posing problems.

Using non-logical thinking

Non-logical thinking plays an important role in mathematician's creativity, but it is not paid attention in mathematics education because of ignoring the creativity. Since the mathematical creativity has been stressed in mathematics education now, still just paying attention to logical thinking without non-logical thinking is not reasonable. Burton (1999) wrote a paper titled:

> *Why is intuition so important to mathematicians but missing from mathematics education?*

Today a more general question should be asked:

> *Why is non-logical thinking so important to mathematicians but missing from mathematics education?*

Mathematics teachers should possess the belief that logic and non-logical thinking are equal important to students' mathematics learning in today's mathematics education, and they should give the students more time and opportunities to use non-logical thinking in mathematics classrooms. In order to give students more opportunities to use non-logical thinking, the teachers should not overemphasize proofs logically, students need to be encouraged to make more guesses; the teachers should, of course, want the students to analyze logically mathematical propositions and phenomena, meanwhile, the students need to have opportunities to imagine and compare between mathematical propositions and phenomena; the students certainly need to make many symbol operations in mathematics learning, but they should also have many opportunities to operate the figures, teaching aids, and true objects.

The theory of functional asymmetry in the human brain can illuminate the training of the non-logical thinking in mathematics education. This theory suggests that the left hemisphere is usually connected with logical thinking, whereas the right hemisphere acts mainly with the help of non-logical thinking. Several investigations show that the two cerebral hemispheres process stimuli in different ways. The left hemisphere processes stimuli sequentially, whereas the right hemisphere is specialized in parallel processing (Pehkonen, 1997). Obviously, the left hemisphere is better suited for arithmetic, solving procedural mathematics exercises, and analytic deduction in mathematics education, whereas the right hemisphere is better in intuitions, imaginary, and solving non-procedural mathematics problems. The left hemisphere is overused will prevent the development of the right hemisphere, that is, if rules and algorithms are overused in mathematics education, then, students' mathematical creativity will be prevent at some extent. Investigations, nonverbal expression, laboratory work and multisense learning will foster the development of the right hemisphere; they also foster the development of the students' mathematical creativity.

Facing real problems

What is a real problem? A real problem is a problem which the solver has no existing methods to solve. A real mathematics problem may come from either the mathematics field or other field, generally, it is complex or (and) difficult. The problems mathematicians face are all real. In order to solve these real problems, the mathematicians must create some new ways or use old ways creatively to solve them. In the traditional mathematics education, there are just a lot of "mathematics exercises", but not real problems. To the "mathematics exercises", if the students can use existing methods correctly, they will get correct answers. No real problem in mathematics education makes the students losing the important opportunities to experience creativity.

If the real mathematics problems are introduced to mathematics education, mathematics education will change significantly. The mathematics problems the students face should include both mathematics exercises and real problems, and the students are both exercisers and problem-solvers. Pehkonen (1997) lists four reasons of problems solving in mathematics education: firstly, problem solving develops general cognitive skills; secondly, problem solving fosters creativity; thirdly, problem solving is a part of the mathematical application process; fourthly, problem solving motivates pupils to learn mathematics. In briefly, problem solving in mathematics education not only fosters students' mathematical creativity but also promotes their mathematics knowledge learning.

Mathematics teachers should grasp the core thought of problem solving which is students should create themselves methods to solve the problems. In order to create new methods, the students maybe need to spend many time and energy, maybe they need cooperate with their peers. There is a wrong way to deal with problem solving in current mathematics education, which is that some teachers teach the methods of problem solving firstly, then the students use these methods to solve problems. This way will turn the creativity of problem solving back to using of general mathematics methods, turn the real mathematics problems to mathematics exercises. In the process of problem solving, teachers maybe provide the students some helps, but they cannot substitute the students' creativity, the students are hosts of mathematical creativity.

Creating suitable teaching and learning atmosphere for mathematical creativity

Mathematicians need a suitable atmosphere for their mathematical creativity, similarly, students' mathematical creativity also need a suitable teaching and learning atmosphere. How do mathematics teachers create a suitable teaching and learning atmosphere which will foster the students' mathematical creativity?

Firstly, the activities of the mathematical creativity should meet the students' characteristics. Some students like thinking quietly and alone, the others favor cooperation with peers. So, when the students make mathematics research, the teachers should allow some students explore independently, also agree the others cooperate with their peers. Every student has his/her mathematics thinking characteristic. Some students are good at algebra thinking, and the others are expert in geometric thinking. The teachers should allow the students use their favorite thinking style in mathematics learning and creativity.

Secondly, teachers should encourage the students to mathematical creativity. Like as mathematician's creativity, student's mathematical creativity also is a risk. The failure of student' mathematical creativity is possible, but it cannot become barrier of student' mathematical creativity. The teacher should make the student recognize that the failure is the normal phenomena of mathematical creativity, no failure no success. The students should make their mathematical creativity unanxiously, they should not worry being accused by their teachers because of the failure of mathematical creativity, they will get praise and encouragement whether success or failure.

The thirdly, the new evaluation system should be built. The traditional mathematics learning evaluation system just is for the students' mathematics knowledge learning but not for mathematical creativity. The new system should include the evaluation of both mathematical knowledge learning and mathematical creativity. Up to now, there is no such system, so, many problems in the new evaluation system should be researched. It is sure that the process evaluation should be a main mode of the evaluation of mathematical creativity. Through the process evaluation, the students' emotion and attitude to mathematical creativity, their creativity ability, and their progresses in mathematical creativity can be assessed effectively.

Although school mathematics is different with mathematics science, students' mathematical work is also different with mathematicians' research; the author believes that mathematicians' methods of mathematics creativity have some illumination to mathematics education which begins to stress the creativity. Because the students' mathematical creativity is not independent fully, the mathematics teachers play an important role in guiding and helping the students' mathematical creativity. So, the mathematics teachers should have related knowledge and ability. Unfortunately, many mathematics teachers have not such knowledge and ability, because they have not the experience of the creativity in their mathematics lives and have not been trained how to culture students' creativity in teacher education. Today, experience and guiding methods of the mathematical creativity must an important part in mathematics teacher's continuing education and in preservice mathematics education.

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