Journal of the Korea Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 16, No. 4, December 2012, 233–244

Creativity Development in Probability through Debate[1]

OH, Taek-Keun*
Gyeonggi Science High School for the Gifted, 135 Suil-ro, Jangan-gu, Suwon,
Gyeonggi-do 440-210, Korea; E-mail: tech0523@dreamwiz.com

LEE, Kyeong Hwa
Department of Mathematics Education, Seoul National University,
Seoul 151-748, Korea; Email: khmath@snu.ac.kr

(Received October 20, 2012; Revised December 5, 2012; Accepted December 20, 2012)

The purpose of this study is to investigate the relationship between creativity development and debate in solving a probability task. We developed the probability task with instructional strategies facilitating debating among students. 33 students in grade 11 who were identified as gifted participated in this study. The findings indicated that debating leads students to critical and reflective thinking on prior learning regarding probability concepts, which nurtured creative ideas on sample space.

Keywords: probability, creativity, debating, gifted education
MESC Classification: B20, B70, D30
MSC2010 Classification: 97C40, 97D10

INTRODUCTION

The classical definition of probability is based on the premise that all outcomes are equally likely. However, equally likeliness of outcomes cannot be satisfied in various contexts with complicated data. Alternative viewpoints such as frequentist and subjectivist viewpoint have been developed to handle this complicated data (Dubucs, 1993; Konold, 1991; Von Mises, 1957). Since there are several definitions of probability, the probability concept itself is ambiguous in nature (Lee, 1996). Previous studies show that tasks with ambiguity help students feel the need for mathematical justification and flexi-

[1] A draft version of the article was presented at KSME 2012 Fall Conference on Mathematics Education at Korea National University of Education, Cheongju, Chungbuk 363-791, Korea; November 2–3, 2012 (*cf.* Oh & Lee, 2012).

* Corresponding author

ble change of perspectives (*e.g.* Lee & Lee, 2010). Therefore, we decided to integrate the ambiguity of probability concepts into mathematics lessons so that students could have a chance to change their perspectives and think critically. This study attempted to investigate what aspects of creativity emerge when students, who were identified as gifted, are engaged in probability task solving and debating induced from ambiguous features of probability.

THEORETICAL BACKGROUND

It has been proposed that the definition of creativity in mathematics education for the gifted is different among researchers. Sriraman (2005) states that creativity is a procedure which leads to novel and insightful solutions for a given problem, raising new questions, and looking at well-known problems in a new way using the imagination. Shriki (2010) divided creativity into two groups: process and product. In process, creativity is not only conceptual thinking that contains fluency, originality, and flexibility, but also cognitive ability such as decision-making skills. In product, creativity is an ability that helps a person make unexpected, original, and useful output. Krutetskii (1976) emphasized the flexibility of the thinking process, originality of ideas, and elaboration in refining ideas measuring creativity. On the basis of this research, this study focused on flexibility, originality and elaboration of mathematical creativity.

If various approaches for a given situation are allowed, students may bring simultaneously a variety of interpretations. At this point, if the collision of different interpretations occurs, they will need to justify their claims in front of their opponents or the audience, and this naturally leads to debate (Lee, 2005). The goal of the debate is to persuade a claim, when there are people with different opinions about an issue. In debates, students will experience cognitive conflict while they are faced with a different argument or opposing evidence. Cognitive conflict is a cognitive non-equilibrium situation that the learner experiences while acquiring new information which is unexplained by existing cognitive structures (Piaget, 1977). Stylianides & Stylianides (2008) suggested that the goal of the conflict teaching approach in mathematics teaching is to help students reflect on their current mathematical understanding and recognize the importance of modifying these understandings to resolve the contradiction. Peterson & Eeds (1990) stated that debate is the ideal mechanism for generating meaning in collaboration, promoting reflective thinking, and modifying meaning. Additionally, Gilles (1993) emphasized the cyclical process of debate. In other words, as students return to a topic or subtopic that interests them, through debate, they create new ideas, respond to different ideas, and modify their own ideas.

The probability concept is ambiguous in nature because it contains a process that replaces subjective belief with numbers as objective knowledge (Lee, 1996). Freudenthal (1973) verified that a lot of controversies occurred at every moment of consideration of probability as a mathematical concept. Borovcnik, Bentz & Kapadia (1991) argued that paradoxes must be treated in probability education.

Mathematics is considered a paradigm of infallible secure knowledge. Recently, it has become widely accepted that mathematics knowledge develops through a process of refinement through conjectures and refutations (Lakatos, 1976). According to Cobb & Yackel (1996) and Cobb & Bowers (1999), constructivism and the sociological perspective of learning mathematics are complementary theories to explain students' learning process. Seo (2005) presented a model of mathematics in the classroom for gifted students by applying social constructivism in the following way: forming subjective knowledge, objectifying, forming objective knowledge, and individual re-forming.

Based on this research, this study designed the following model of a mathematics classroom for gifted students: forming subjective knowledge, forming objective knowledge, and re-forming the subjective knowledge.

METHODOLOGY

Task

Equally likeliness of outcomes is based on symmetry of events. However, symmetry can be interpreted differently. In particular, for events occurring in succession, symmetry can occur for a variety of interpretations. We focus on this point in this study. The probability task was developed as shown in Figure 1.

Here, Gab looked at the situation *as a whole*, so he considered the set of six shortest paths as a sample space. On the other hand, Eul was handling the situation *locally*. She considered the set of two selections which is one of left and right road as sample spaces. In other words, she thought that each path was made up of a chain selection.

Instructional Design

Based on social constructivism, this study designed the following model of a mathematics classroom for gifted students.

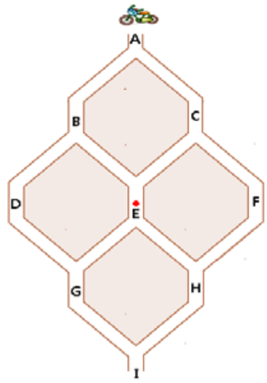
Stage 1: Forming subjective knowledge

In this stage, all students worked individually on the worksheet. This was done to develop an experience in which students take two different perspectives on the solutions.

Although they disagreed with them, its main purpose was to allow students to objectify themselves.

Read the following and answer the questions given below. ((1) ~ (3))

Four congruent hexagons are shown as follows. The length and width are all same.



Find the probability of the shortest distance going from point A to point I, which includes point E in the process of travelling.

Two students *Gab* and *Eul* described their different solutions to the problem are as follows;

Solution of Gab (G):

A number of cases that we can go from A to I the shortest distance is 6. Among them, the cases that go through E are 4. Therefore the probability is

$$\frac{4}{6}=\frac{2}{3}$$

Solution of Eul (E):

The road divides into two different roads. At an intersection, choosing either way will result in a same outcome. At this point, the probability through E is two cases that A→B→E or A→C→E, so the probability is

$$\left(\frac{1}{2}\times\frac{1}{2}\right)+\left(\frac{1}{2}\times\frac{1}{2}\right)=\frac{1}{2}$$

(1) Criticize the solution of Eul in Gab's viewpoint.
(2) Criticize the solution of Gab in Eul's viewpoint.
(3) Which is more valid? Explain your answer.

Figure 1. Task and Worksheet for students

Stage 2: Forming objective knowledge

All students participated in this debate class. They presented their subjective judgment on the task, justified their views, and refuted other perspectives. This made students share various opinions. In this stage, public criticism of students actively took place, and the teacher did not speak assess, or judge the students' presentation. This was to prevent students from revising their judgments due to external authority.

Stage 3: Re-forming the subjective knowledge

After debating, all students were asked to modify or complement their prior judgment. This made students reflect on what they had accomplished. This is the process of internalization of knowledge generated through public criticism. Also, this stage was designed to identify changes in an individual students' perspective.

Participants

We selected a science high school for the gifted as the theoretical sampling (Merriam, 1997). Participants were 33, grade 11, students, and they were divided into two classes; P and Q. All of them are regarded as gifted students, because they got through the barriers of the entrance examination for gifted. The researcher participated in the class as a participant observer.

RESULTS AND DISCUSSION

Flexibility

Before the class debate, students' typical reaction was to select one of two solutions. Even if students thought that both claims made sense, they tried to select one of two. The following are examples of such episodes.

Q8: What's the right answer, either?

Teacher: Well, what do you think?

Q8: I think Gab makes more sense. But I think 'Eul' is probably true also.

Teacher: So both are right, you think?

Q8: Then there exists two kinds of answers. It does not make any sense!

Teacher: Really? Then I want you to think about it. And write down what you think.

Q8: Is there a correct answer?

During the debate, there were a lot of students who learned to accept two arguments to be valid at the same time. Before the debate, the number of students that took position G (Gab)-B (Between or Beyond)-E (Eul) was 4-3-8 in P and 3-3-12 in Q. After the debate, the number of students changed to 3-6-6 in P and 2-11-5 in Q. In detail, each student changed their subjective judgments as shown in Figure 2 and 3.

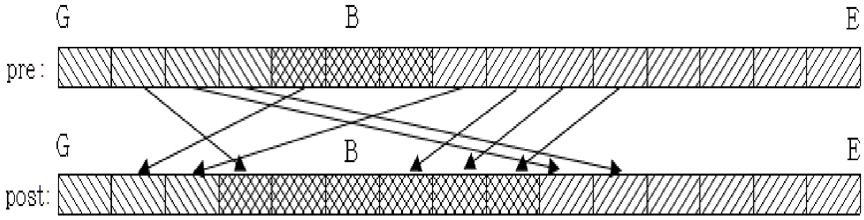


Figure 2. The shift on the position of 15 students in P

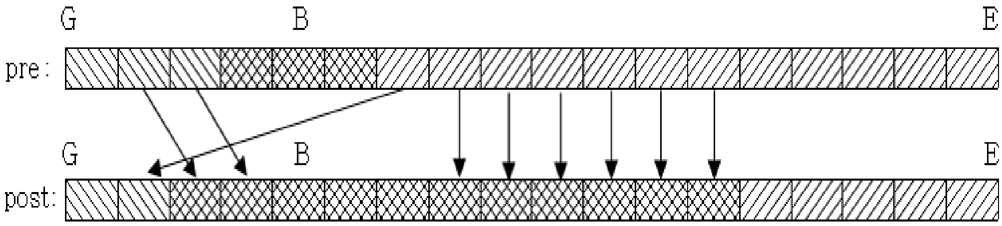


Figure 3. The shift on the position of 18 students in Q

The number of students that chose B's point of view rose from 3 to 17. They have recognized both the overall randomness and local symmetry, and they have the flexibility to accept both of the two perspectives. We can see, through changes in student responses before and after the debate, that students have acquired flexibility by looking at the following evidence.

P7 (Before): According to Gab, when you select any road, if the number of paths followed is different, then chance to choose the way is different. It does not make sense.

P7 (After): The difference between the two perspectives lies in the timing of the selection. Gab is starting, and Eul is at the fork. If you know the road ahead, Gab is right and reasonable. If you do not know the way, Eul is right. Thus, according to the level of subjects' information, it's different.

Q3 (Before): When someone rides a bike, he already knows that there are six kinds of paths. Only then can he move the shortest distance. We do not know which path he would prefer, so we can say that all six kinds of paths have the same probability to each other. In other words, this guy is not moving at random from each fork.

Q3 (After): It depends on the prerequisites for the person's information.

At first, even though P7 belonged to B, he thought that did not make sense that there would be different probabilities. However, through debating, he accepted Gab's claim based on the premise that the subject knew the way. On the other hand, Q3 thought, at first, that he can move the shortest distance only if he had already known the way. But, after debate, he accepted the opponent's claims. Furthermore, he was open to new ideas. We will explain this again after talking about originality.

Elaboration

- Gab's camp: Contextualization

Before the class debate, students who choose G had justified their claims by the definition of probability which is the ratio of the number of particular cases over the number of the entire set. The following responses are examples of students who supported G in stage 1:

P1 (Before): The length of all paths is equal, so the chances are all the same. Because there are four of six passes for E, the probability is 2/3.

P2 (Before): There are six full paths, because four kinds of paths pass through E.

Q2 (Before): I think Gab's words are reasonable, because he considered the number of E of the total number of cases

However, they were not fully aware of the equipossibility of events which is the basic premise of the definition. During debate, they encountered arguments from students who belong to E such that the six paths could not have the same probability. In this way, they justified their claims by presenting the context in which the case has same probability:

P1: (While drawing on the board bike and paths) this is my explanation to support Gab's position. Once you go on a bike suddenly change the way, you can get hurt. Gab thinks that cyclists are familiar with the road. So he thinks that… (Along a path down on the board) 'Today, I will go this way.' In other words, he will consider the road ahead in his mind.

PP: Aha!

P1: If he is riding the road in the usual way, he might know that there are six paths. Among them, there are only 4 paths passing E, as Gab said. Thus, 4/6 (i.e. 2/3).

PP: Oh! (Students clap and cheered.)

This *contextualization* led many students who belonged to E to begin to accept G's claim. P1 argued about the choice of the road ahead before departure. To justify such a situation, he emphasized the fact that an unexpected change of direction would cause an accident.

- Eul's camp

Students belonging to E noted that, rather than the number of paths, each path is determined by the judge at the crossroads. That is, because the probability of selecting any one of two ways at the crossroads is the same, they showed that the probability of each path may be different from each other. This can be seen in the following dialogue (in stage 1):

P8 (Before): Gab's failure is that he assumes that six kinds of odds are equal to each other. In fact, the probabilities of paths are different such as 1/4, 1/8.

During debate, students belonging to E were critical of G's claims on the basis described above. However, by the same logic, a point was raised from the G camp (P4) and B camp (P2) that the probabilities of choosing one of two at the crossroads may differ:

P4: A number of cases that we can go from E to I the shortest distance is 2. On the other hand, we can go from D or F to I by only one way. Therefore, we cannot say that the probability from B (C) to D or E (E or F) is 1/2.

P6: Strictly speaking, the probability of choosing a path at the fork also does not fit all. If a person riding a bike has a tendency of going to the right, the probability may vary.

Since the symmetry of the paths is not guaranteed, P4 thought that chances at a crossroads were different from each other. On the other hand, P6 argued that it is necessary to consider the *statistical tendency*. On the other hand, 9 students who belonged to E justified their claims using the *abstraction* or *generalization* strategy that realistic context should be removed because they should not assume the condition which is not given in mathematical problems. However, two students containing P8 changed their position from E to G, and 9 students accepted those claims. For the latter, P6, by himself, gave the values of the probability such that the left road is a, and the right road is b as shown in Figure 4. Furthermore, since the tendency may vary at each moment, shown in Figure 5, P9 developed a more extended representation:

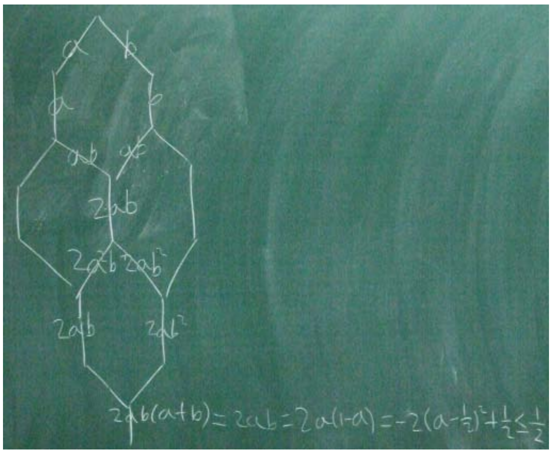


Figure 4. Tendency of judgment at an intersection (by P6)

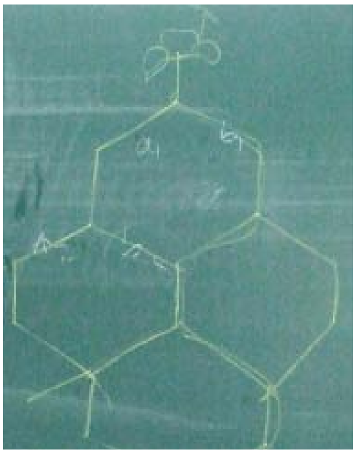


Figure 5. Generalization of the tendency (by P9)

Originality

Originality is associated with novelty which differs from the conventional. In this study, we have included in the category of originality a new perspective that integrates perspectives which have been discussed in debate and a new interpretation that is different from the intention of the task.

- The integration of two perspectives:

There is a student, P11, who had creatively integrated two perspectives. The following example is his claim, during the debate and in stage 3:

P11 (After): If you take a bike ride, you already know the way to go. Otherwise, for every crossroad, the road may be chosen. In the former case, you can think that the 6 paths have the same possibility, so the probability is 2/3 like in Gab's claim. In the latter case, as in Eul's claim, the probability is 1/2. Therefore, if the probability of selection before departure is p, then the probability passing through E is as Eq. (1).

$$p\times\frac{2}{3}+(1-p)\times\frac{1}{2}=\frac{1}{6}p+\frac{1}{2} \tag{1}$$

At first, I thought people like a ball. But as in Gab's claim, because you can choose the path ahead, I thought that it is right to consider Gab's words.

P11 did not only accept the two perspectives, but also integrated from a higher perspective. In other words, he was thinking of a new sample space which depends on a time of path selection. This was a unique attempt. The conversion of these ideas is seems to have been acquired by a reflection on conventional thinking through debate.

- New interpretation of the situation

In the course of debate, Q3, who supported G in the first stage, did invent the conditional probability as shown in Figure 6.

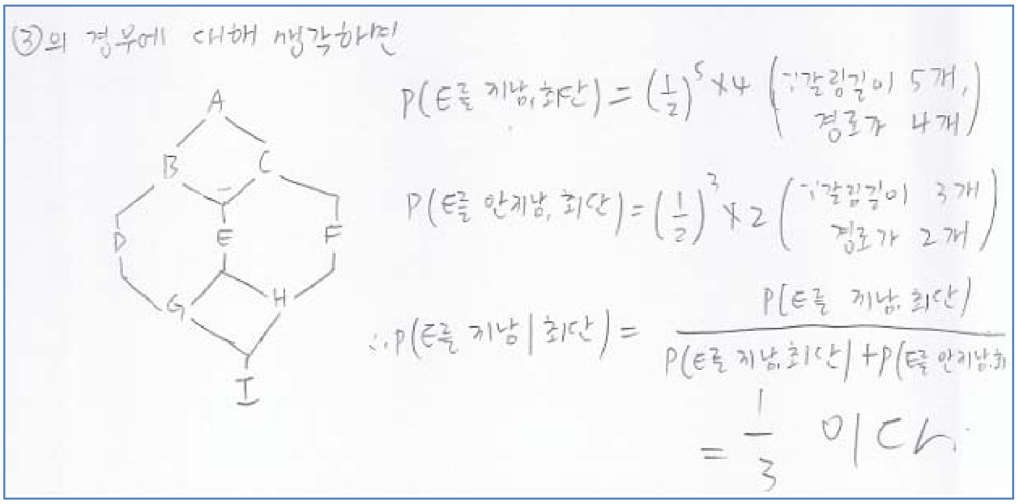


Figure 6. Conditional probability by new interpretation (written by Q3)

This solution has arisen by actively considering E's claim that the bike rider does not know anything. Q3 has changed positions from G to B by accepting E's claim. Furthermore, he noted the fact that if the person does not know any information then he does not know the direction of the shortest path. Thus, the entire path was not limited to the shortest path. In other words, the sample space has expanded. Q3's value can be justified by Bayes' theorem as depicted in Eq. (2).

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)} = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A^C)P(A^C)} \tag{2}$$

Q7 presented in this perspective during class debate, and 4 students in class Q agreed to his new claim.

CONCLUSION

The findings indicate that debating stimulated by cognitive conflict promoted critical thinking, and the creativity of gifted students allowed for more flexibility and elaboration. Moreover, during the debate, some students presented original interpretations, which are beyond these two views, and they are as follows: the integration of two views and a new interpretation of the situation. These led to new values of the probability.

These results lead us to the following conclusions.

First, debating was facilitated by a task raising cognitive conflict. The task enabled students to produce different results using different interpretations of the situations aroused their curiosity. The students were categorized into three groups, and this enabled them to develop critical thinking by participating in the debate actively.

Second, debating promoted students reflection on prior probability concepts. Many students began to consider through the debate that equally likeliness of outcomes is valid in a given situation. This process provided practice with re-thinking the concept of probability

Lastly, a creative perspective on sample space for the given task emerged among students by integrating or coordinating the shared ideas.

REFERENCES

Borovcnik, M. Bentz, H. J., & Kapadia, R. (1991). A probabilistic perspective. In: R. Kapadia & M. Borovcnik (Eds.), *Chance Encounters: Probability in Education* (pp. 27–71). Dordrecht, Netherlands: Kluwer Academic Publishers.

Cobb, P. & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher* **28(2)**, 4–15. ERIC EJ587003

Cobb, P. & Yackel, E. (1996). Constructive, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist* **31(3/4)**, 175–190. [Reprinted in: T. P. Carpenter, J. Dossey, & J. Koehler (Eds.)(2004). *Classics in mathematics education research* (pp. 208–226). Reston, VA: National Council of Teachers of Mathematics.] ERIC ED389535 Available from: http://www.eric.ed.gov/PDFS/ED389535.pdf

Dubucs, J. P. (1993) *Philosophy of Probability*. Dordrecht: Kluwer Academic Publishers.

Freudenthal, H. (1973). *Mathematics as an educational task.* Dordrecht, Netherlands: Kluwer Academic Publishers.

Gilles, C. (1993). We make an idea: Cycles of Meaning in Literature Discussion Groups. Portsmouth, NH, USA: Heinenmann.

Konold, C. (1991). Understanding Students' Beliefs about Probability. In: E. V. Glasersfeld (Ed.) *Radical Constructivism in Mathematical Education* (pp. 139–156). Dordrecht, Netherlands: Kluwer Academic Publishers.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children*. Chicago, IL, USA: University of Chicago Press.

Lakatos, I. (1991). *Proofs and refutations: the logic of mathematical discovery* (in Korean:

증명과 반박: 수학적 발견의 논리). Seoul: Muneumsa. Translated by J. H. Woo from English: Proofs and refutations - the logic of mathematical discovery. Cambridge, UK / New York, NY, USA: Cambridge University Press. (Original work published 1976). ME **1980x.**02032 ME **1984l.**00985

Lee, D. H. & Lee, K. H. (2010). How the mathematically gifted cope with ambiguity? *Journal of Korea Society of Educational Studies in Mathematics: School Mathematics* **12(1)**, 79–95.

Lee, D. W. (2005). *Debate: Persuasion of Position and Views.* Seoul: Communication Books.

Lee, K. H. (1996). *A study on the didactic transposition of the concept of probability* (Doctoral dissertation). Retrieved from http://library.snu.ac.kr/index.ax

Merriam, S. B. (1998). *Qualitative Research and Case Study Applications in Education.* San Francisco: Jossey-Bass Publishers. ERIC ED415771

Oh, T. & Lee, K. (2012). Creativity Development in Probability through Debate. In: J. Cho, S. Lee & Y. Choe (Eds.), *Proceedings of KSME 2012 Fall Conference on Mathematics Education at Korea National Universityof Education, Cheongju, Chungbuk 363-791, Korea; November 2–3, 2012* (pp. 377–388).

Peterson, R. & Eeds, M. (1990). *Grand Conversation: Literature Group in Action.* Ontario: Scholastic Canada Ltd. ERIC ED317967

Piaget, J. (1977). Problem in equilibration. In: M. H. Apple & L. S. Goldberg (Eds.), *Topic in Cognitive Development. Vol. 1 Equilibration: Theory, Research, and Application* (pp. 3–13). New York: Plenum. [Originally published 1972. In: C. Nodine, J. Gallagher & R. Humphrey (Eds.), *Piaget and Inhelder on Equilibration.* Philadelphia: Jean Piaget Society.]

Seo, D. Y. (2005). A model of mathematics classroom for gifted students applying social constructivism. *Journal of Korea Society of Educational Studies in Mathematics: School Mathematics* **7(3)**, 237–252

Shriki, A. (2010). Working like real mathematicians: developing prospective teachers' awareness of mathematical creativity through generating new concepts. *Educ. Stud. Math* **73(2)**, 159–179. ME **2010c.**00149

Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Journal of Secondary Gifted Education* **17(1)**, 20–36.

Stylianides, A. J., & Stylianides, G. J. (2008). 'Cognitive conflict' as a mechanism for supporting developmental progressions in students' knowledge about proof. In: *Proceedings of the 11th International Congress on Mathematical Education, Monterrey, Mexico; 6–13 July 2008.* Available from: http://tsg.icme11.org/document/get/283

Von Mises, R. (1957). *Probability, Statistics, and Truth.* London, UK: George Allen and Unwin Ltd.