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JORDAN $*$-HOMOMORPHISMS BETWEEN UNITAL C^*-ALGEBRAS

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Abstract. In this paper, we prove the superstability and the generalized Hyers-Ulam stability of Jordan $*$-homomorphisms between unital C^*-algebras associated with the following functional equation

$$f\left(\frac{-x+y}{3}\right)+f\left(\frac{x-3z}{3}\right)+f\left(\frac{3x-y+3z}{3}\right)=f(x).$$

Moreover, we investigate Jordan $*$-homomorphisms between unital C^*-algebras associated with the following functional inequality

$$\left\|f\left(\frac{-x+y}{3}\right)+f\left(\frac{x-3z}{3}\right)+f\left(\frac{3x-y+3z}{3}\right)\right\|\leq\|f(x)\|.$$

1. Introduction

The stability of functional equations was first introduced by Ulam [33] in 1940. More precisely, he proposed the following problem:

Given a group G_1, a metric group (G_2,d) and a positive number ϵ, does there exist a $\delta>0$ such that if a function $f:G_1\longrightarrow G_2$ satisfies the inequality $d(f(xy),f(x)f(y))<\delta$ for all $x,y\in G1$, then there exists a homomorphism $T:G_1\to G_2$ such that $d(f(x),T(x))<\epsilon$ for all $x\in G_1$?

As mentioned above, when this problem has a solution, we say that the homomorphisms from G_1 to G_2 are stable. In 1941, Hyers [7] gave a partial solution of Ulam's problem for the case of approximate additive mappings under the assumption that G_1 and G_2 are Banach spaces. In 1978, Th. M. Rassias [27] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy differences. This phenomenon of stability that was introduced by Th. M. Rassias [27] is called *generalized Hyers-Ulam stability* or *Hyers-Ulam-Rassias stability*.

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Theorem 1.1. *Let $f : E \longrightarrow E'$ be a mapping from a norm vector space E into a Banach space E' subject to the inequality*

$$\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p) \tag{1.1}$$

for all $x, y \in E$, where ϵ and p are constants with $\epsilon > 0$ and $p < 1$. Then there exists a unique additive mapping $T : E \longrightarrow E'$ such that

$$\|f(x) - T(x)\| \leq \frac{2\epsilon}{2 - 2^p}\|x\|^p \tag{1.2}$$

for all $x \in E$. If $p < 0$, then the inequality (1.1) *holds for all $x, y \neq 0$, and* (1.2) *for $x \neq 0$. Also, if the function $t \mapsto f(tx)$ from $\mathbb{R}$ into E' is continuous for each fixed $x \in E$, then T is $\mathbb{R}$-linear.*

Recently, C. Park and W. Park [26] applied the Jun and Lee's result to the Jensen's equation in Banach modules over a $\mathbb{C}^*$-algebra. B. E. Johnson [15, Theorem 7.2] also investigated almost algebra $*$-homomorphisms between Banach $*$-algebras: Suppose that $\mathcal{U}$ and B are Banach $*$-algebras which satisfy the conditions of [15, Theorem 3.1]. Then for each positive ϵ and K there is a positive δ such that if $T \in L(\mathcal{U}, B)$ with $\|T\| < K$, $\|T^\vee\| < \delta$ and $\|T(x^*)^* - T(x)\| \leq \delta\|x\|$, then there is a $*$-homomorphism $T' : \mathcal{U} \to B$ with $\|T' - T\| < \epsilon$. Here $L(\mathcal{U}, B)$ is the space of bounded linear maps from $\mathcal{U}$ into B, and $T^\vee(x, y) = T(xy) - T(x)T(y)$. See [15] for details.

Throughout this paper, let A be a unital $\mathbb{C}^*$ -algebra with norm $\|\cdot\|$ and unit e, and B a unital $\mathbb{C}^*$-algebra with norm $\|\cdot\|$. Let $\mathcal{U}(A)$ be the set of unitary elements in A, $A_{sa} = \{x \in A | x = x^*\}$, and $I_1(A_{sa}) = \{v \in A_{sa} | \; \|v\| = 1, v \text{ is invertible}\}$. During the last decades several stability problems of functional equations have been investigated by many mathematicians. A large list of references concerning the stability of functional equations can be found in [1]–[14], [18, 21, 30, 31, 32, 34].

Definition 1.2. Let A, B be two C^*-algebras. A $\mathbb{C}$-linear mapping $f : A \to B$ is called a Jordan $*$-homomorphism if

$$\begin{cases} f(a^2) = f(a)^2 \\ f(a^*) = f(a)^* \end{cases}$$

for all $a \in A$.

C. Park [24] introduced and investigated Jordan $*$-derivations between unital C^*-algebras associated with the following functional inequality

$$\|f(a) + f(b) + kf(c)\| \leq \left\| kf\left(\frac{a+b}{k} + c\right) \right\|$$

for some integer k greater than 1 and proved the generalized Hyers-Ulam stability of Jordan $*$-derivations between unital C^*-algebras associated with the following functional equation

$$f\left(\frac{a+b}{k} + c\right) = \frac{f(a) + f(b)}{k} + f(c)$$

for some integer k greater than 1 (see also [23, 19, 17, 20, 25]).

In this paper, we investigate Jordan $*$-homomorphisms between unital C^*-algebras associated with the following functional inequality

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\| \le \|f(a)\|.$$

We moreover prove the generalized Hyers-Ulam stability of Jordan $*$-homomorphisms between unital C^*-algebras associated with the following functional equation

$$f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) = f(a).$$

2. Jordan $*$-homomorphisms

In this section, we investigate Jordan $*$-homomorphisms between unital C^*-algebras.

Lemma 2.1. *Let $f : A \to B$ be a mapping such that*

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3c}{3}\right) + f\left(\frac{3a+3c-b}{3}\right) \right\|_B \le \|f(a)\|_B \tag{2.1}$$

for all $a, b, c \in A$. Then f is additive.

Proof. Letting $a = b = c = 0$ in (2.1), we get

$$\|3f(0)\|_B \le \|f(0)\|_B.$$

So $f(0) = 0$. Letting $a = b = 0$ in (2.1), we get

$$\|f(-c) + f(c)\|_B \le \|f(0)\|_B = 0$$

for all $c \in A$. Hence $f(-c) = -f(c)$ for all $c \in A$. Letting $a = 0$ and $b = 6c$ in (2.1), we get

$$\|f(2c) - 2f(c)\|_B \le \|f(0)\|_B = 0$$

for all $c \in A$. Hence

$$f(2c) = 2f(c)$$

for all $c \in A$. Letting $a = 0$ and $b = 9c$ in (2.1), we get

$$\|f(3c) - f(c) - 2f(c)\|_B \le \|f(0)\|_B = 0$$

for all $c \in A$. Hence

$$f(3c) = 3f(c)$$

for all $c \in A$. Letting $a = 0$ in (2.1), we get

$$\|f(\frac{b}{3}) + f(-c) + f(c - \frac{b}{3})\|_B \le \|f(0)\|_B = 0$$

for all $a, b, c \in A$. So

$$f(\frac{b}{3}) + f(-c) + f(c - \frac{b}{3}) = 0$$

for all $a, b, c \in A$. Let $t_1 = c - \frac{b}{3}$ and $t_2 = \frac{b}{3}$ in the last equation, we get

$$f(t_2) - f(t_1 + t_2) + f(t_1) = 0$$

for all $t_1, t_2 \in A$. This means that f is additive. $\square$

Now we prove the superstability problem for Jordan $*$-homomorphisms as follows.

Theorem 2.2. *Let $p < 1$ and θ be nonnegative real numbers, and let $f : A \to B$ be a mapping satisfying $f(0) = 0$, $f(3^n ux) = f(3^n u)f(x)$ for all $u \in \mathcal{U}(A)$ and all $x \in A$ and*

$$\left\| f\left(\frac{b-a}{3}\right) + f\left(\frac{a-3\mu c}{3}\right) + \mu f\left(\frac{3a+3c-b}{3}\right) \right\|_B \leq \|f(a)\|_B, \tag{2.2}$$

$$\|f(3^n u^*) - f(3^n u)^*\|_B \leq 2\theta 3^{np}, \tag{2.3}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} \ ; |\lambda| = 1\}$, all $u \in \mathcal{U}(A)$, $n = 0, 1, 2, \ldots$ and all $a, b, c \in A$. Then the mapping $f : A \to B$ is a Jordan $*$-homomorphism.

Proof. Let $\mu = 1$ in (2.2). By Lemma 2.1, the mapping $f : A \to B$ is additive. Letting $a = b = 0$ in (2.2), we get

$$\|f(-\mu c) + \mu f(c)\|_B \leq \|f(0)\|_B = 0$$

for all $c \in A$ and all $\mu \in \mathbb{T}^1$. So

$$-f(\mu c) + \mu f(c) = f(-\mu c) + \mu f(c) = 0$$

for all $c \in A$ and all $\mu \in \mathbb{T}^1$. Hence $f(\mu c) = \mu f(c)$ for all $c \in A$ and all $\mu \in \mathbb{T}^1$. By Theorem 2.1 of [22], the mapping $f : A \to B$ is $\mathbb{C}$-linear. By (2.3), we get

$$f(u^*) = \lim_{n\to\infty} \frac{1}{3^n} f(3^n u^*) = \lim_{n\to\infty} \frac{1}{3^n} f(3^n u)^* = \left(\lim_{n\to\infty} \frac{1}{3^n} f(3^n u)\right)^* = f(u)^*$$

for all $u \in \mathcal{U}(A)$. Since f is $\mathbb{C}$-linear and each $x \in A$ is a finite linear combination of unitary elements (see [16, Theorem 4.1.7], i.e., $x = \sum_{i=1}^m \lambda_i u_i$ ($\lambda_i \in \mathbb{C}, u_i \in \mathcal{U}(A)$),

$$\begin{aligned} f(x^*) &= f\left(\sum_{i=1}^m \bar{\lambda}_i {u_i}^*\right) = \sum_{i=1}^m \bar{\lambda}_i f({u_i}^*) = \sum_{i=1}^m \bar{\lambda}_i f(u_i)^* \\ &= \sum_{i=1}^m \lambda_i f(u_i)^* = f\left(\sum_{i=1}^m \lambda_i u_i\right)^* = f(x)^* \end{aligned}$$

for all $x \in A$. Since $f(3^n ux) = f(3^n u)f(x)$ for all $u \in \mathcal{U}(A)$, $x \in A$ and all $n = 0, 1, 2, \ldots,$

$$f(ux) = \lim_{n\to\infty} \frac{1}{3^n} f(3^n ux) = \lim_{n\to\infty} \frac{1}{3^n} f(3^n u)f(x) = f(u)f(x)$$

for all $u \in \mathcal{U}(A)$, $x \in A$. Since f is $\mathbb{C}$-linear and each $x \in A$ is a finite linear combination of unitary elements, i.e., $x = \sum_{i=1}^m \lambda_i u_i$ ($\lambda_i \in \mathbb{C}, u_i \in \mathcal{U}(A)$),

$$\begin{aligned} f(xy) &= f\left(\sum_{i=1}^m \lambda_i u_i y\right) = \sum_{i=1}^m \lambda_i f(u_i y) = \sum_{i=1}^m \lambda_i f(u_i) f(y) \\ &= f\left(\sum_{i=1}^m \lambda_i u_i\right) f(y) = f(x)f(y) \end{aligned} \tag{2.4}$$

for all $x, y \in A$. Replacing y by x in (2.4), we get $f(x^2) = f(x)^2$ for all $x \in A$. Therefore, the mapping $f : A \to B$ is a Jordan $*$-homomorphism, as desired. □

Theorem 2.3. *Let $p > 1$ and θ be a nonnegative real number, and let $f : A \to B$ be a mapping satisfying* (2.2) *and* (2.3). *Then the mapping $f : A \to B$ is a Jordan $*$-homomorphism.*

Proof. The proof is similar to the proof of Theorem 2.2. □

We prove the generalized Hyers-Ulam stability of Jordan $*$-homomorphisms between unital C^*-algebras.

Theorem 2.4. *Suppose that $f : A \to B$ is a mapping for which there exists a function $\varphi : A \times A \times A \to \mathbb{R}^+$ such that*

$$\sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{b}{3^i}, \frac{c}{3^i}\right) < \infty, \tag{2.5}$$

$$\lim_{n\to\infty} 3^{2n} \varphi\left(\frac{a}{3^n}, \frac{b}{3^n}, \frac{c}{3^n}\right) = 0, \tag{2.6}$$

$$\|f(3^n u^*) - f(3^n u)^*\|_B \le \varphi(3^n u, 3^n u, 3^n u), \tag{2.7}$$

$$\left\| f\left(\frac{\mu b - a}{3}\right) + f\left(\frac{a - 3c}{3}\right) + \mu f\left(\frac{3a - b}{3} + c\right) - f(a) + f(c^2) - f(c)^2 \right\|_B \le \varphi(a, b, c) \tag{2.8}$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}^1$. Then there exists a unique Jordan $*$-homomorphism $h : A \to B$ such that

$$\|h(a) - f(a)\|_B \le \sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \tag{2.9}$$

for all $a \in A$.

Proof. Letting $\mu = 1$, $b = 2a$ and $c = 0$ in (2.8), we get

$$\left\| 3f\left(\frac{a}{3}\right) - f(a) \right\|_B \le \varphi(a, 2a, 0)$$

for all $a \in A$. Using the induction method, we have

$$\left\|3^n f\left(\frac{a}{3^n}\right) - f(a)\right\| \leq \sum_{i=0}^{n-1} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \tag{2.10}$$

for all $a \in A$. In order to show the functions $h_n(a) = 3^n f(\frac{a}{3^n})$ form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replace a by $\frac{a}{3^m}$ and multiply by 3^m in (2.10), where m is an arbitrary positive integer. We find that

$$\left\|3^{m+n} f\left(\frac{a}{3^{m+n}}\right) - 3^m f\left(\frac{a}{3^m}\right)\right\| \leq \sum_{i=m}^{m+n-1} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \tag{2.11}$$

for all positive integers. Hence by the Cauchy criterion the limit $h(a) = \lim_{n\to\infty} h_n(a)$ exists for each $a \in A$. By taking the limit as $n \to \infty$ in (2.10) we see that

$$\|h(a) - f(a)\| \leq \sum_{i=0}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right)$$

and (2.9) holds for all $a \in A$. Let $\mu = 1$ and $c = 0$ in (2.8), we get

$$\left\|f\left(\frac{b-a}{3}\right) + f\left(\frac{a}{3}\right) + f\left(\frac{3a-b}{3}\right) - f(a)\right\|_B \leq \varphi(a, b, 0) \tag{2.12}$$

for all $a, b, c \in A$. Multiplying both sides (2.12) by 3^n and Replacing a, b by $\frac{a}{3^n}, \frac{b}{3^n}$, respectively, we get

$$\begin{aligned} &\left\|3^n f\left(\frac{b-a}{3^{n+1}}\right) + 3^n f\left(\frac{a}{3^{n+1}}\right) + 3^n f\left(\frac{3a-b}{3^{n+1}}\right) - 3^n f\left(\frac{a}{3^n}\right)\right\|_B \\ &\leq 3^n \varphi\left(\frac{a}{3^n}, \frac{b}{3^n}, 0\right) \end{aligned} \tag{2.13}$$

for all $a, b, c \in A$. Taking the limit as $n \to \infty$, we obtain

$$h\left(\frac{b-a}{3}\right) + h\left(\frac{a}{3}\right) + h\left(\frac{3a-b}{3}\right) - h(a) = 0 \tag{2.14}$$

for all $a, b, c \in A$. Putting $b = 2a$ in (2.14), we get $3h(\frac{a}{3}) = h(a)$ for all $a \in A$. Replacing a by $2a$ in (2.14), we get

$$h(b - 2a) + h(6a - b) = 2h(2a) \tag{2.15}$$

for all $a, b \in A$. Letting $b = 2a$ in (2.15), we get $h(4a) = 2h(2a)$ for all $a \in A$. So $h(2a) = 2h(a)$ for all $a \in A$. Letting $3a - b = s$ and $b - a = t$ in (2.14), we get

$$h\left(\frac{t}{3}\right) + h\left(\frac{s+t}{6}\right) + h\left(\frac{t}{3}\right) = h\left(\frac{s+t}{2}\right)$$

for all $s, t \in A$. Hence $h(s) + h(t) = h(s + t)$ for all $s, t \in A$. So, h is additive. Letting $a = c = 0$ in (2.12) and using the above method, we have $h(\mu b) = \mu h(b)$

for all $b \in A$ and all $\mu \in \mathbb{T}$. Hence by Theorem 2.1 of [22], the mapping $f : A \to B$ is $\mathbb{C}$-linear.

Now, let $h^{'} : A \to B$ be another $\mathbb{C}$-linear mapping satisfying (2.9). Then we have

$$\begin{aligned}
\|h(a) - h^{'}(a)\|_B &= 3^n \left\| h\left(\frac{a}{3^n}\right) - h^{'}\left(\frac{a}{3^n}\right) \right\|_B \\
&\le 3^n \left[\left\| h\left(\frac{a}{3^n}\right) - f\left(\frac{a}{3^n}\right) \right\|_B + \left\| h^{'}\left(\frac{a}{3^n}\right) - f\left(\frac{a}{3^n}\right) \right\|_B \right] \\
&\le 2 \sum_{i=n}^{\infty} 3^i \varphi\left(\frac{a}{3^i}, \frac{2a}{3^i}, 0\right) \\
&= 0
\end{aligned}$$

for all $a \in A$. By (2.6), (2.7), (2.8) and similar to the proof of Theorem 2.2, the mapping $h : A \to B$ is a Jordan $*$-homomorphism. $\square$

Corollary 2.5. *Suppose that $f : A \to B$ is a mapping with $f(0) = 0$ for which there exist constant $\theta \ge 0$ and $p_1, p_2, p_3 > 1$ such that*

$$\left\| f\left(\frac{\mu b - a}{3}\right) + f\left(\frac{a - 3c}{3}\right) + \mu f\left(\frac{3a - b}{3} + c\right) - f(a) + f(c^2) - f(c)^2 \right\|_B$$
$$\le \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}),$$

$$\|f(3^n u^*) - f(3^n u)^*\|_B \le \theta(3^{np_1} + 3^{np_2} + 3^{np_3})$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}$. Then there exists a unique Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\|_B \le \frac{\theta\|a\|^{p_1}}{1 - 3^{(1-p_1)}} + \frac{\theta 2^{p_2}\|a\|^{p_2}}{1 - 3^{(1-p_2)}}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) := \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3})$ in Theorem 2.4, we obtain the result. $\square$

Theorem 2.6. *Suppose that $f : A \to B$ is a mapping with $f(0) = 0$ for which there exists a function $\varphi : A \times A \times A \to B$ satisfying* (2.7), (2.8) *and* (2.8) *such that*

$$\sum_{i=1}^{\infty} 3^{-i} \varphi(3^i a, 3^i b, 3^i c) < \infty, \tag{2.16}$$

$$\lim_{n\to\infty} 3^{-2n} \varphi(3^i a, 3^i b, 3^i c) = 0 \tag{2.17}$$

for all $a, b, c \in A$. Then there exists a unique Jordan $*$-homomorphism $h : A \to B$ such that

$$\|h(a) - f(a)\|_B \le \sum_{i=1}^{\infty} 3^{-i} \varphi(3^i a, 3^i 2a, 0) \tag{2.18}$$

for all $a \in A$.

Proof. Letting $\mu = 1$, $b = 2a$ and $c = 0$ in (2.8), we get

$$\left\| 3f\left(\frac{a}{3}\right) - f(a) \right\|_B \leq \varphi(a, 2a, 0) \tag{2.19}$$

for all $a \in A$. Replacing a by $3a$ in (2.19), we get

$$\|3^{-1}f(3a) - f(a)\|_B \leq 3^{-1}\varphi(3a, 2(3a), 0)$$

for all $a \in A$. On can apply the induction method to prove that

$$\|3^{-n}f(3^n a) - f(a)\|_B \leq \sum_{i=1}^{n} 3^{-i}\varphi(3^i a, 2(3^i a), 0) \tag{2.20}$$

for all $a \in A$. In order to show the functions $h_n(a) = 3^{-n}f(3^n a)$ form a convergent sequence, we use the Cauchy convergence criterion. Indeed, replace a by $3^m a$ and multiply by 3^{-m} in (2.20), where m is an arbitrary positive integer. We find that

$$\|3^{-(m+n)}f(3^{m+n}a) - 3^{-m}f(3^m a)\| \leq \sum_{i=m+1}^{m+n} 3^{-i}\varphi(3^i a, 2(3^i a), 0) \tag{2.21}$$

for all positive integers. Hence by the Cauchy criterion the limit $h(a) = \lim_{n\to\infty} h_n(a)$ exists for each $a \in A$. By taking the limit as $n \to \infty$ in (2.20) we see that

$$\|h(a) - f(a)\| \leq \sum_{i=1}^{\infty} 3^{-i}\varphi(3^i a, 2(3^i a), 0)$$

and (2.18) holds for all $a \in A$.

The rest of the proof is similar to the proof of Theorem 2.4. □

Corollary 2.7. *Suppose that $f : A \to B$ is a mapping with $f(0) = 0$ for which there exist constant $\theta \geq 0$ and $p_1, p_2, p_3 < 1$ such that*

$$\left\| f\left(\frac{\mu b - a}{3}\right) + f\left(\frac{a - 3c}{3}\right) + \mu f\left(\frac{3a - b}{3} + c\right) - f(a) + f(c^2) - f(c)^2 \right\|_B$$
$$\leq \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3}),$$

$$\|f(3^n u^*) - f(3^n u)^*\|_B \leq \theta(3^{np_1} + 3^{np_2} + 3^{np_3})$$

for all $a, b, c \in A$ and all $\mu \in \mathbb{T}$. Then there exists a unique Jordan $*$-homomorphism $h : A \to B$ such that

$$\|f(a) - h(a)\|_B \leq \frac{\theta\|a\|^{p_1}}{3^{(1-p_1)} - 1} + \frac{\theta 2^{p_2}\|a\|^{p_2}}{3^{(1-p_2)} - 1}$$

for all $a \in A$.

Proof. Letting $\varphi(a, b, c) := \theta(\|a\|^{p_1} + \|b\|^{p_2} + \|c\|^{p_3})$ in Theorem 2.7, we obtain the result. □

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