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Probabilities of Baccarat by Simulation

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Abstract

In Baccarat, the gambler can bet on either the Player or Banker. The only gambler's strategy is to consider the previous winning history on the round. The winning probabilities of Player or Banker are calculated by simulation using R. Conditional winning probabilities given that Player or Banker has won *i* consecutive times are also calculated by simulation. Conditional winning probability implies that the sequence of Baccarat results is an almost independent sequence of events. It has been shown that the total amount of returns in each round of games is almost identical to a random walk. Thus, one possible strategy is to catch the trend(the Player or the Banker) of the random walk and to bet on that side of the trend.

Keywords: Baccarat, winning probability, conditional probability, simulation, R.

1. Introduction to Baccarat

1.1. How to play Baccarat

Baccarat is a glamorous game with flexible limits, where the gambler decides to bet on Player(*Note, Player does not mean the gambler*) or Banker. The one which has the higher score, will be the winning hand. The minimum score is 0 and the maximum score is 9. There are various types of Baccarats, such as *chemin-de-fer, baccarat-banque* and *punto banco*. Among them *punto banco* is the most frequently played game. In this paper only *punto banco* is considered, and its playing method and rules are explained in the following subsection. More details about the game can be found in Baccarat (2000, 2010), and Tutorial-How to play Baccarat (2001).

1.2. Baccarat (*punto banco*) Rules

All cards from 2 to 9 are counted at face value and an Ace is counted as 1. Picture cards (King, Queen & Jack) and tens have a value of 0 or any combinations totaling 10 will likewise regard as 0.

The number of cards used for playing ranges from 1 up to 8 decks of which the choice is at the absolute discretion of the House. Both Player and Banker can only draw a maximum of 3 cards each.

At the beginning of the game, four cards will be dealt by the croupier: the first and third cards will go to the Player's hand and the second and fourth cards will go to the Banker's hand.

If either Player or Banker (or both) achieves a total of 8 or 9 on the initial deal (known as a 'natural'), no further cards are drawn and the outcome is immediately determined by comparing the two totals. However if neither has a natural, the play proceeds as follows:

- Player's rule

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If Player has an initial total of 0–5, a third card is drawn. If Player has an initial total of 6 or 7, it stands.

- Banker's rule
  - If Player stands(*i.e.*, has only two cards), Banker regards only his own hand and acts according to the same rule as the Player. That means a third card is drawn if Banker has 0–5 and it stands if Banker has 6 or 7.
  - If Player draws a third card, Banker acts according to the following more complex rules:
    * If Player draws a 2 or 3, Banker draws if it has 0–4, and stands if it has 5–7.
    * If Player draws a 4 or 5, Banker draws if it has 0–5, and stands if it has 6–7.
    * If Player draws a 6 or 7, Banker draws if it has 0–6, and stands if it has 7.
    * If Player draws an 8, Banker draws if it has 0–2, and stands if it has 3–7.
    * If Player draws an ace, 9, 10, or picture card, Banker draws if it has 0–3, and stands if it has 4–7.

The croupier will deal the cards according to the tableau and the croupier will announce the winning hand, either Player or Banker. Losing bets will be collected and the winning bets will be paid according to the rules of the house. Usually, winning bets on the Player are paid at even money and winning bets on Banker are paid 0.95 of the amount bet, a 5% commission is paid to the house (Commission Baccarat). For 'No Commission Baccarat' game, all winning bets on Banker are paid even money with one EXCEPTION when Banker wins with hand 6. In that case 50% commission is paid to the house. (For example: $100 winning bet on Banker will earn you $50).

However, if both Banker hand and Player hand have the same points at the end of the deal, it is a 'Tie' or 'Draw' game. All bets on Tie will be paid at 8 to 1 odds and all bets on Player and Banker remain in place and active for the next game (the gamer may or may not be able to retract these bets depending on casino rules).

So *Baccarat* is strictly a game of chance, with no skill or strategy involved; each player's moves are forced by the cards the player is dealt. In this paper we will examine this game from a probabilistic point of view through simulation.

Examples of studies of the Baccarat game in the literature are Kemeny and Snell (1957), Thorp and Walden (1966, 1973) and Dowonton and Lockwood (1975, 1976) where some Baccarats with different rules to *punto banco* are studied, such as *chemin-de-fer, baccarat-banque*. But in this paper we consider *punto banco* only.

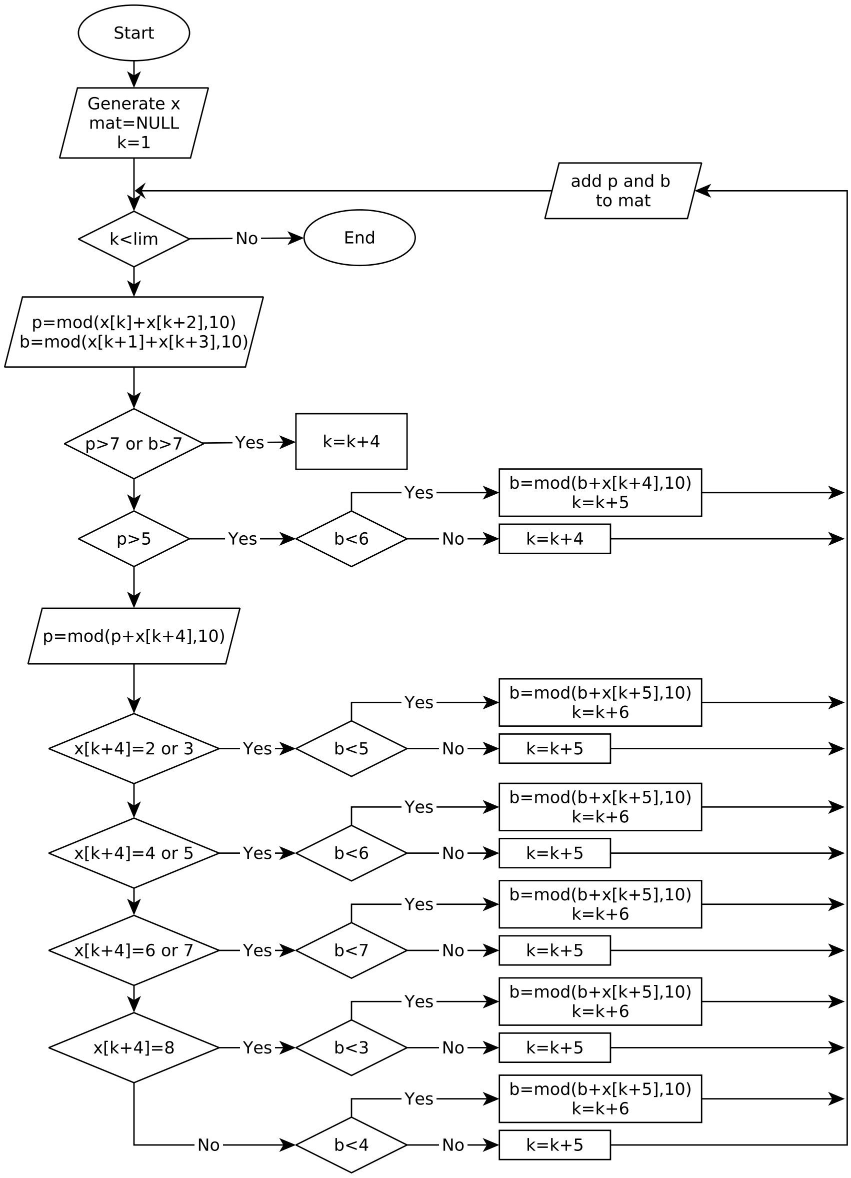
2. Simulation Using R

2.1. Outline

We use the open source and free software R to do the simulation of Baccarat. In this simulation, based on that the game starts with 8 decks of shuffled cards and ends when the number of the cards is less than 52, we call it a 'round'. Each game in a 'round' is called a 'play'.

We will stand in the perspective of Player: if we say 'win', it means Player wins and 'lose' means Player loses and the Banker wins.

The algorithm of generating all score pairs of Player and Banker is shown in Figure 1.



x: is a randomly ordered vector consisted with 32 number of 1's to 9's and 128 number of 0's.

lim: denotes the upper limit of the cards which can be used in one round. It is defined as $52 \times 8 -$ res, where res is at least how many cards should be left in a round.

mat: is a matrix with two columns used to store all the paired scores gotten in one round.

p: stands for the score of Player and b stands for the score of Banker.

Figure 1: *Flowchart of Baccarat simulation*

Table 1: Estimated p values and their standard deviation

Player	$\hat{p}$	s.e.
win	0.445391	0.000578
lose	0.459326	0.000580
tie	0.095283	0.000342

2.2. Probability of the outcomes for Player

First of all, we would like to calculate the probabilities of win, lose and tie for Player. Let us define several random variables.

N: total number of plays

X_w: total number of wins for Player

X_l: total number of loses for Player

X_t: total number of Ties

p_w: probability of win for Player

p_l: probability of lose for Player

p_t: probability of Tie

where $p_w, p_l, p_t \in (0, 1)$, $N = X_w + X_l + X_t$, and $p_w + p_l + p_t = 1$. So random variables X_w, X_l, X_t follow a multinomial distribution.

$$(X_w, X_l, X_t) \sim \text{Multi}(N, p_w, p_l, p_t).$$

The unbiased estimator of p_w is

$$\hat{p}_w = \frac{X_w}{N}.$$

Variance of $\hat{p}_w$ is

$$\text{Var}\,(\hat{p}_w) = \frac{\text{Var}(X_w)}{N^2} = \frac{p_w(1 - p_w)}{N}.$$

Because p_w is unknown, so we use $\hat{p}_w$ instead, $\text{Var}(\hat{p}_w) = \hat{p}_w(1 - \hat{p}_w)/N$.
Likewise, $\hat{p}_l = X_l/N$, $\text{Var}(\hat{p}_l) = \hat{p}_l(1 - \hat{p}_l)/N$, and $\hat{p}_t = X_t/N$, $\text{Var}(\hat{p}_t) = \hat{p}_t(1 - \hat{p}_t)/N$.

1000 rounds of game are simulated, it is not a difficult work to calculate the estimated p_w, p_l, p_t values and its standard errors. The simulation results are listed in Table 1.

The results mean that in one play the probability of win for Player is about 0.44539098, the probability of lose for Player is about 0.45932556, the probability of tie is about 0.09528346.

When betting on Player or Banker, the ‘Tie’ outcomes will not earn or lose you any money, so we can ignore the ‘Tie’ outcomes in this situation. The simulation results with ‘Tie’ omitted are shown in Table 2.

The simulation results show that if bet on Player, the average probability of win is about 0.4922989, the standard error is 0.0006114399, 95% CI is: (0.4911005, 0.4934973). The average probability of lose is about 0.5077011, the standard error is 0.0006114399, 95% CI is: (0.5065027, 0.5088995). The average probability of Tie is 0.09528346, the standard error is 0.0003415518, 95% CI is: (0.09461402, 0.0959529).

Table 2: Estimated p values and their standard deviations with 'Tie' plays omitted

Player	$\hat{p}$	s.e.
win	0.492299	0.000611
lose	0.507701	0.000611

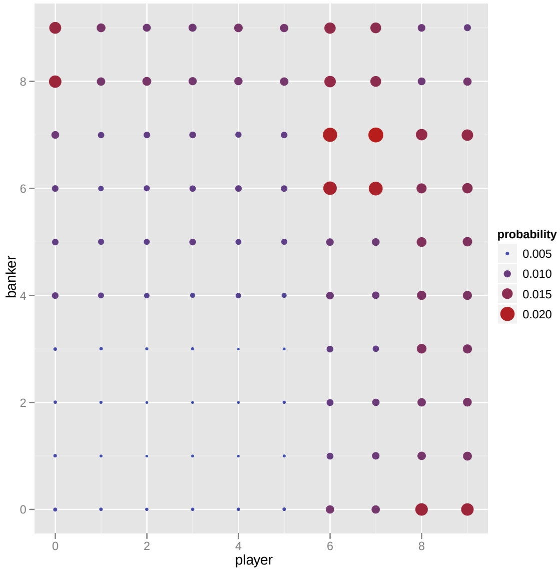


Figure 2: *Pairwise score plot of Player and Banker*

2.3. Expected return per play

Using the estimated probabilities in the previous section, we can calculate the expected return per Play(ERP) easily. Here we assume the bet on each play is 1.
If bet on Player, the ERP is calculated as

$$\text{ERP} = 0.4922989 - 0.5077011 = -0.0154022.$$

If bet on Banker

- for commission Baccarat the ERP is calculated as

$$\text{ERP} = 0.95 * 0.5077011 - 0.4922989 = -0.009982855.$$

- for no commission baccarat, the ERP is about -0.0144231(calculated through simulation).

If bet on Tie, the ERP is calculated as

$$\text{ERP} = 0.09528346 * 9 - 1 = -0.1424489.$$

We can note, all the expected incomes are negative which means the gambler is expected to lose money on the average. One interesting thing of note from a psychological aspect is that people would like to play no commission baccarat; however, in fact commission baccarat would have a higher ERP.

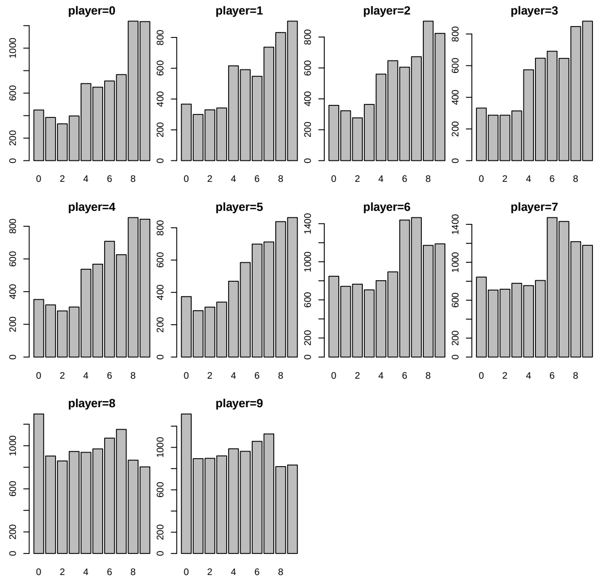


Figure 3: *Pairwise score plot of Player and Banker*

2.4. Comparison of the scores of Player and Banker

There are 100 possible pairs of scores for Player and Banker. We plot all the pairs in Figure 2 using the result of 1000 rounds of simulation. In the figure, horizontal and vertical axes represent the score of the Player and the score of the Banker, respectively in addition, the bigger points stand for the higher probability of the paired scores to appear. Notice that the size of the points are almost symmetric about the diagonal. Figure 3 uses a barplot to display the paired scores.

However we are usually not very interested in the score pairs but only in the final result of who wins.

2.5. Conditional probabilities

Do the previous outcomes effect the next play? We first calculate the probabilities of the outcomes under the condition of i consecutive wins for Player ($i \geq 1$).

We define another group of random variables.

N_{i^+}: number of plays under the condition of i consecutive wins for Player

$X_{w|i^+}$: number of wins under the condition of i consecutive wins for Player

$X_{l|i^+}$: number of loses under the condition of i consecutive wins for Player

$X_{t|i^+}$: number of ties under the condition of i consecutive wins for Player

$p_{w|i^+}$: probability of win under the condition of i consecutive wins for Player

$p_{l|i^+}$: probability of lose under the condition of i consecutive wins for Player

$p_{t|i^+}$: probability of tie under the condition of i consecutive wins for Player

Table 3: Probabilities and standard errors under the condition of i consecutive wins for Player

i^+	$\hat{p}_{w\|i^+}$	$s.e.$	$\hat{p}_{l\|i^+}$	$s.e.$	$\hat{p}_{t\|i^+}$	$s.e.$
1	0.444635	0.001164	0.460307	0.001168	0.095058	0.000687
2	0.447014	0.001759	0.457957	0.001763	0.095029	0.001038
3	0.445899	0.002652	0.459051	0.002659	0.095050	0.001565
4	0.448404	0.004018	0.454996	0.004023	0.096599	0.002387
5	0.438688	0.006046	0.463925	0.006076	0.097387	0.003612
6	0.435712	0.008992	0.461690	0.009040	0.102598	0.005502
7	0.449106	0.013865	0.466200	0.013905	0.084693	0.007761
8	0.443299	0.020592	0.470790	0.020690	0.085911	0.011616
9	0.401575	0.030759	0.488189	0.031364	0.110236	0.019651
10	0.441176	0.049164	0.480392	0.049469	0.078431	0.026620

Table 4: Probabilities and standard errors under the condition of i consecutive wins for Player('Tie' plays omitted)

i^+	$\hat{p}_{w\|i^+}$	$s.e.$	$\hat{p}_{l\|i^+}$	$s.e.$
1	0.492478	0.001223	0.507522	0.001223
2	0.494538	0.001758	0.505462	0.001758
3	0.494206	0.002523	0.505794	0.002523
4	0.494803	0.003641	0.505197	0.003641
5	0.487082	0.005197	0.512918	0.005197
6	0.492200	0.007411	0.507800	0.007411
7	0.481673	0.010832	0.518327	0.010832
8	0.466730	0.015381	0.533270	0.015381
9	0.482625	0.021955	0.517375	0.021955
10	0.495614	0.033112	0.504386	0.033112

where $p_{w|i^+}, p_{l|i^+}, p_{t|i^+} \in (0,1)$, and $p_{w|i^+} + p_{l|i^+} + p_{t|i^+} = 1$. So random variables $X_{w|i^+}, X_{l|i^+}, X_{t|i^+}$ follow a multiomial distribution.

$$(X_{w|i^+}, X_{l|i^+}, X_{t|i^+}) \sim \text{Multi}\left(N_{i^+}, p_{w|i^+}, p_{l|i^+}, p_{t|i^+}\right).$$

The unbiased estimator of $p_{w|i^+}$ is

$$\hat{p}_{w|i^+} = \frac{X_{w|i^+}}{N_{i^+}}.$$

Variance of $\hat{p}_{w|i^+}$ is

$$\text{Var}\left(\hat{p}_{w|i^+}\right) = \frac{\text{Var}(X_{w|i^+})}{N_{i^+}^2} = \frac{p_{w|i^+}(1 - p_{w|i^+})}{N_{i^+}}.$$

Because $p_{w|i^+}$ is unknown, so we use $\hat{p}_{w|i^+}$ instead, $\text{Var}(\hat{p}_{w|i^+}) = \hat{p}_{w|i^+}(1 - \hat{p}_{w|i^+})/N_{i^+}$.
Likewise, $\hat{p}_{l|i^+} = X_{l|i^+}/N_{i^+}$, $\text{Var}(\hat{p}_{l|i^+}) = \hat{p}_{l|i^+}(1 - \hat{p}_{l|i^+})/N_{i^+}$, and $\hat{p}_{t|i^+} = X_{t|i^+}/N_{i^+}$, $\text{Var}(\hat{p}_{t|i^+}) = \hat{p}_{t|i^+}(1 - \hat{p}_{t|i^+})/N_{i^+}$.

Based on the above formulas, the probabilities of the outcomes for Player under the condition of consecutive i wins are calculated. The results are shown in Table 3, the first column presents under the condition of i consecutive wins(We use symbol i^+ to present i consecutive wins, and i^- to present i consecutive loses.).

Usually, when betting on Player or Banker, 'Tie' plays can be omitted, Table 4 shows the simulation results with 'Tie' plays excluded.

Using the results above, we can get the 95% confidence intervals for each conditional probability. Figure 4 shows the CI plot(the probability of win and lose are calculated with 'Tie' plays excluded).

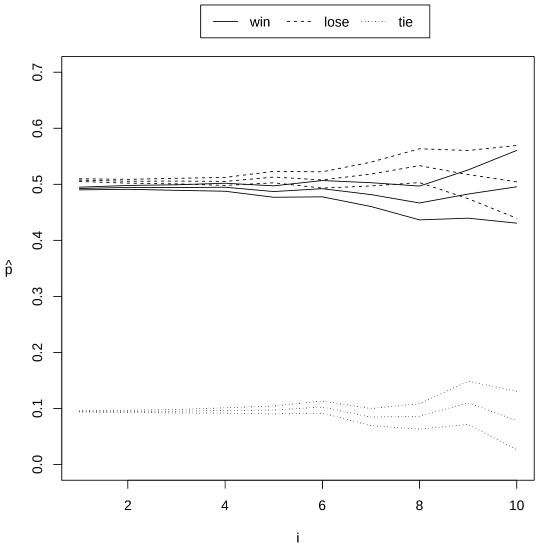


Figure 4: *95% CI for Player under the condition of consecutive i wins*

Table 5: Probabilities and standard errors under the condition of *i* consecutive losses for Player

i^-	$\hat{p}_{w\|i^-}$	*s.e.*	$\hat{p}_{l\|i^-}$	*s.e.*	$\hat{p}_{t\|i^-}$	*s.e.*
1	0.444550	0.001161	0.460074	0.001164	0.095376	0.000686
2	0.445301	0.001726	0.460035	0.001731	0.094664	0.001017
3	0.446477	0.002563	0.458465	0.002569	0.095058	0.001512
4	0.443371	0.003802	0.458304	0.003813	0.098325	0.002279
5	0.448370	0.005668	0.455903	0.005676	0.095727	0.003353
6	0.443931	0.008447	0.459827	0.008473	0.096243	0.005014
7	0.452806	0.012571	0.447066	0.012556	0.100128	0.007580
8	0.443284	0.019192	0.459701	0.019254	0.097015	0.011435
9	0.412371	0.028857	0.477663	0.029281	0.109966	0.018339
10	0.417219	0.040128	0.463576	0.040581	0.119205	0.026369

Table 6: Probabilities and standard errors under the condition of *i* consecutive losses for Player('Tie' plays omitted)

i^-	$\hat{p}_{w\|i^-}$	*s.e.*	$\hat{p}_{l\|i^-}$	*s.e.*
1	0.490697	0.001222	0.509303	0.001222
2	0.491793	0.001729	0.508207	0.001729
3	0.493701	0.002444	0.506299	0.002444
4	0.489602	0.003456	0.510398	0.003456
5	0.499903	0.004921	0.500097	0.004921
6	0.496957	0.006895	0.503043	0.006895
7	0.502278	0.009742	0.497722	0.009742
8	0.491857	0.014266	0.508143	0.014266
9	0.489185	0.020391	0.510815	0.020391
10	0.498452	0.027821	0.501548	0.027821

Similarly, the probabilities of the outcomes for Player under the condition of consecutive i loses are shown in Table 5 and Table 6.

Figure 5 shows the 95% confidence intervals for each conditional probability(the probability of win and lose are calculated with 'Tie' plays excluded).

Table 7: Outcomes of Plan 2

M	P_l	ERR
$M = 1$	0.018	−0.152
$M = 2$	0.034	−0.194
$M = 3$	0.055	−0.572
$M = 4$	0.073	−0.779
$M = 5$	0.085	−0.620
$M = 6$	0.095	−0.345
$M = 7$	0.111	−0.464
$M = 8$	0.121	−0.215
$M = 9$	0.138	−0.424
$M = 10$	0.150	−0.330

When comparing Table 3 and Table 5 with Table 1, we can find that the conditional probabilities are consistent with consecutive wins or loses. This implies almost independence of the game in probability. Therefore the previous outcomes have no effect to the next outcome. In theory, it is meaningless to decide which side to bet on according to the outcome sheet.

2.6. Some gamble methods

One of the most common gamble method is called the 'DoubleUp' Method: if you lose one bet you should bet twice the previous bet. For example, if you bet starting with \$1 and lose, you should bet \$2 in the next play, and if lose again, you should bet \$4 in the next play, and so on; if you win, you bet from the starting level \$1. So using this kind of method, no matter how much you had lost, as long as you win back one play, you will breakeven. It seems to be an always win method; however, it has a high risk because the bet increases so quickly and may exceed the maximum bet or you may not have enough money for the next bet.

Here are the simulations based on this method. We started with total \$100, and start with initial bet \$1.

Plan 1: We assume there is no maximum bet for betting, and we will not stop until the end of the round or until not enough money left to continue the game. We will always bet on Player in the simulation.

The results show that the probability of losing money in a round is about 0.4, but the expected return is \$$-1.786$. The reason is that even the probability of earning money is relative higher than losing money, but when we win we only win a little, when we lose we will lose a lot. So only using this kind of 'skill' will not help us make money in the long run.

Plan 2: All the assumptions are the same with Plan 1 except that we will stop when we have earned M amount of money. The simulation results are shown in Table 7, the second column is the probability of losing money in a round, the third column is the expected return per round(ERR). As we may note, the probability of losing money would increase as M increases, and no matter what M is, we are always expected to lose money. However, we have more than 98% probability to win money if we just stop when we win one dollar. Maybe it suggests us a good 'gambling skill'—if you win money, just stop and walk away.

Plan 3: All the assumptions are the same with Plan 2 except that we will bet on Player and Banker alternatively(if betting on the Player this play, then bet on Banker next play, let us call it the 'alternative' method). The simulation results are shown in Table 8.

Plan 4: All the assumptions are the same with Plan 2 except that we will bet on the one who wins in last play(if the Banker wins, then bet on the Banker in the next play, let us call it 'follow' method).

Table 8: Outcomes of Plan 3

M	P_l	ERR
$M = 1$	0.015	0.040
$M = 2$	0.034	−0.191
$M = 3$	0.052	−0.377
$M = 4$	0.067	−0.389
$M = 5$	0.080	−0.288
$M = 6$	0.093	−0.200
$M = 7$	0.112	−0.509
$M = 8$	0.128	−0.645
$M = 9$	0.145	−0.861
$M = 10$	0.163	−1.158

Table 9: Outcomes of Plan 4

M	P_l	Return
$M = 1$	0.015	0.040
$M = 2$	0.026	0.321
$M = 3$	0.044	0.143
$M = 4$	0.057	0.267
$M = 5$	0.077	−0.070
$M = 6$	0.091	−0.043
$M = 7$	0.111	−0.414
$M = 8$	0.123	−0.293
$M = 9$	0.141	−0.568
$M = 10$	0.152	−0.413

The simulation results are shown in Table 9. Compare Plan 4 with Plan 3, we note that the ‘follow’ method seems to be better than the ‘alternative’ method, because all the losing game probabilities are relatively smaller for the ‘follow’ method.

3. Stochastic Properties of the Return

Consider the results of the consecutive Baccarats. We illustrate the results by the marker as follows. A marker is placed at zero on the number line and a sequence of Baccarats is played. Suppose that the gamer bets B units to the Player. If the Player wins the game, the marker moves B units upward. If the Banker wins the game, the marker moves B units downward. If the game ties, the marker stays at the same level. Then the height of the marker at time t denotes the amount paid to the Player after t number of runs. The positive height implies the amount paid to the Player and the negative height to the Banker. In order to define this trajectory of the marker formally, let the sequence of random variables $\{Z_1, Z_2, \ldots\}$ denote the sequence of game results, and $\{B_1, B_2, \ldots\}$ be the sequence of the corresponding bets to the defined random variables. The random variable denoting the j^{th} game result is defined as

$$Z_j = \begin{cases} 1, & \text{if the Player wins,} \\ 0, & \text{if the Player ties,} \\ -1, & \text{if the Player loses.} \end{cases}$$

Also let

$$S_0 = 0 \quad \text{and} \quad S_t = \sum_{j=1}^{t} B_j Z_j.$$

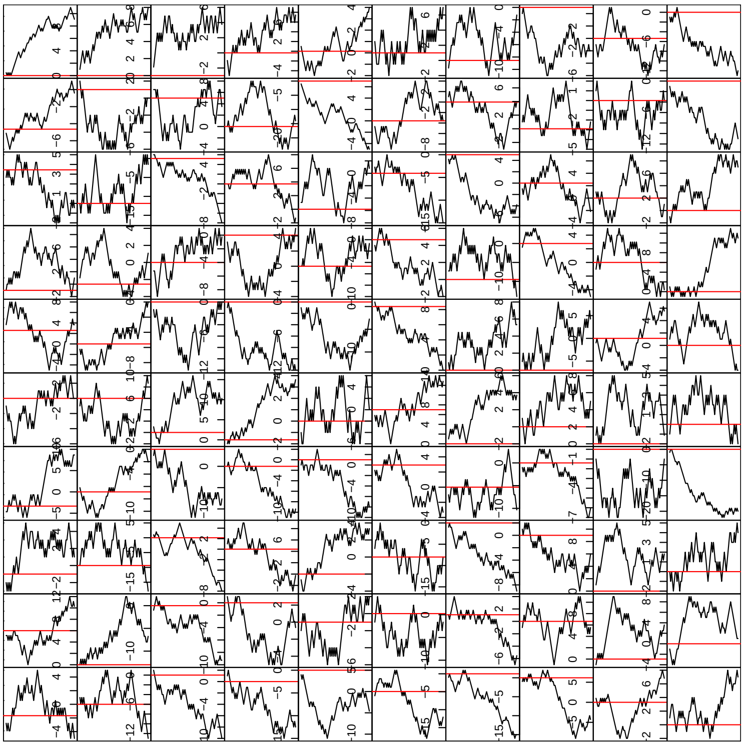


Figure 5: *Trajectory plots for 100 rounds game*

Then S_t denotes the total amount of returns for the Player after t number of runs.

In the previous section, we have shown that the game result is almost independent of the previous game results. Thus we may consider that random variables $\{Z_1, Z_2, \ldots\}$ are approximately *iid*. The amount of the bets $\{B_1, B_2, \ldots\}$ will be determined by the gamer. If the gamer bets b units to the Player at game t, then $B_t = b$. If the gamer bets b units to the Banker, then $B_t = -b$. The amount of the bet made by the gamer will be determined right before each game; however, it has been shown that there is no winning strategy in Baccarat in previous sections. Thus we assume

$$B_j = b.$$

That is, we bet a constant amount to the Player every time. Then the series $\{S_t\}$ is almost identical to a simple random walk.

Note that the random walk is a typical nonstationary stochastic process. Every random walk wanders away from the origin and is never guaranteed to return to the origin. The trajectory of S_t is illustrated in Figure 5 where the amount of the bet is a single unit, that is $b = 1$. In Figure 5, there are 100 panels, where each panel denotes the trajectory of a single round of Baccarat and the parallel line denotes the origin 0. As is seen in the figure, there is no round where the trajectory is always up and down about the origin. However, most of the panels the general shape of the trajectory shows two types typically. One type is to keep increasing (or decreasing) until the end of the round. The other is to keep increasing (or decreasing) for a certain period of time and then keep decreasing (or increasing) until the end of the round. This is a typical behavior of the simple random walk. Let the increasing trend in a certain period of times be the Player trend, and the decreasing trend be the Banker trend, for convenience.

Considering the similarity to the random walk of the game results, we may think that a possible good strategy for the Baccarat is to catch the trend of the trajectory at each round and bet (almost) continuously to the winning trend(the Player trend or the Banker trend). As long as the game is thought to be in the middle of the Player trend, bet on the Player side. As long as the game is thought to be in the middle of the Banker trend, bet on the Banker side. Otherwise, do not bet or reduce the amount of the bet until any trend(either the Player or the Banker) is caught. However catching the trend also corresponds to one of the game of chance as is the Baccarat itself. Surprisingly, it can be easily seen that the strategy has been used by most gamers in practice, whether or not they know this stochastic properties of the game results.

4. Conclusions

Baccarat is a popular but simple casino game. Gamblers hope to win the game and casinos want to have an advantage over the gamblers. In any casino game, the expected return for gambler is always negative due to the pre-determined odds. Despite the odds, players want to find whether there is any rule to win the game. We tried to find some winning rules by calculating the conditional probabilities; however, the simulation results show that no winning rule exists. The only possible winning strategy is to catch the trend(either the Player or the Banker) and to bet on that side. In every casino game, there should be no winning rule, otherwise, casinos will be bankrupt. Although the probabilistic results provide nothing different from our guess, it was meaningful to verify our guess as well as to calculate the expected return per play.

References

Baccarat (2000). Available from: http://baike.baidu.com/view/154935.htm

Baccarat (2011). *Wizard of Odds Consulting. 1 March 2011. Retrieved 24 June 2011*

Downton, F. and Lockwood, C. (1975). Computer studies of baccarat, I: Chemin-de-fer, *Journal of the Royal Statistical Society, Series A (General)*, **138**, 228–238.

Downton, F. and Lockwood, C. (1976). Computer Studies of Baccarat, II: Baccarat-Banque, *Journal of the Royal Statistical Society, Series A (General)*, **139**, 356–364.

Kemeny, J. G. and Snell, J. L. (1957). Game-Theoretic Solution of Baccarat, *American Mathematical Monthly*, **114**, 465–469.

Thorp, E. O. and Walden, W. E. (1966). A Favorable side bet in the Nevada Baccarat, *Journal of the American Statistical Association*, **6**, 313–328.

Thorp, E. O. and Walden, W. E. (1973). The fundamental theorem of card counting with applications to Trente-et-Quarante and baccarat, *International Journal of Game Theory*, **2**, 109–119.

Tutorial - How to play Baccarat (2001). *Gambling Info. Retrieved 24 June 2011*

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