Joint Relay Selection and Resource Allocation for Cooperative OFDMA Network

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Abstract

In this paper, the downlink resource allocation of OFDMA system with decode-and-forward (DF) relaying is investigated. A non-convex optimization problem maximizing system throughput with users' satisfaction constraints is formulated with joint relay selection, subcarrier assignment and power allocation. We first transform it to a standard convex problem and then solve it by dual decomposition. In particular, an Optimal resource allocation scheme With Time-sharing (OWT) is proposed with combination of relay selection, subcarrier allocation and power control. Due to its poor adaption to the fast-varying environment, an improved version with subcarrier Monopolization (OWM) is put forward, whose performance promotes about 20% compared with that of OWT in the fast-varying vehicular environment. In fact, OWM is the special case of OWT with binary time-sharing factor and OWT can be seen as the tight upper bound of the OWM. To the best of our knowledge, such algorithms and their relation have not been accurately investigated in cooperative OFDMA networks in the literature. Simulation results show that both the system throughput and the users' satisfaction of the proposed algorithms outperform the traditional ones.

Keywords: OFDM, cooperative communication, joint optimization, Decode-and-Forward relaying

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1. Introduction

Recently, cooperative communication has drawn much attention for its enhancement in cellular service coverage and network extensibility. Through deployment of the dedicated relay nodes, the edge users would achieve much better data rates and the central users can also benefit from spatial diversity. On the other hand, the technique of orthogonal frequency division multiplexing (OFDM) has been widely applied for its high spectrum efficiency and mitigation of inter-symbol interference (ISI) caused by multi-path fading. As a result, the incorporation of OFDM and relay has been accepted as the network infrastructure by the candidates of the emerging 4^{th} generation (4G) mobile communication system such as Long Term Evolution Advanced (LTE-A) and Worldwide Interoperability for Microwave Access (WiMAX).

The relay principles can be classified into two fundamental categories [1]: Amplify-and-Forward (AF) and Decode-and-Forward (DF). For the AF, the relay simply retransmits an amplified version of the original signal including noise without performing any decoding; while a DF relay decodes the message and transmits a re-encoded one. As the noise can be eliminated in DF, it generally has a better performance than AF. In this paper, the focus lies on DF.

The resource allocation for a cooperative OFDMA system with multiple mobile stations (MSs) and multiple relay nodes (RNs) involves many issues, such as relay selection, subcarrier assignment and power allocation. The resource allocation considered in [2] is to maximize system throughput with users' fairness constraints, whereas a fixed rather than dynamic power scheme is employed. In [3], the greedy-wise policy is used in both subchannel allocation and power allocation, which completely sacrifices the fairness of the cell-edge subscribers. A series of iterative water-filling algorithms are developed in [4] for power allocation, but the base station (BS) is assumed to make decisions separately from the RNs.

Some joint optimization schemes are proposed in [5][6][7][8] for cooperative OFDMA networks. A general optimization framework is proposed in [5], but the author has not taken the users' fairness into account. The scheme in [6] is based on an assumption of sum power constraint for BS and RNs, which is not realistic, and [7] entirely neglects the power constraint for BS. A joint allocation algorithm is discussed in [8], but the number of subcarriers that each RN could use is predetermined, which surely restricts the degree of freedom for transmission.

The authors in [9][10] discuss the power allocation problem in multi-cell environment. A two-stage resource allocation scheme is proposed in [9], while [10] is focused on the outage probability analysis in theory. The power allocation problem in [11,12] are studied with imperfect channel information. Some works like [13][14] combine the OFDM with multi-input and multi-output (MIMO) together, and enhance the sum rate by spatial diversity. The two-way relay channel is investigated in [15][16], which uses two phases to exchange information with higher spectral efficiency.

In this paper, a joint optimization concerning relay selection, subcarrier allocation and power control is investigated for downlink cooperative OFDMA networks. Since it is a non-convex problem, we first transform it to a convex one according to the perspective function. And to reduce the complexity further, a dual decomposition method is employed to decouple it into a master problem and several subproblems, where the power can be determined by a multi-level water-filling method. To work out the subcarrier allocation indicator precisely, we relax the indicator to real number and solve it via bisection method.

According to these decimal subcarrier indicators, an optimal allocation scheme with time-sharing (OWT) is proposed which allows the users to share a subcarrier in time. Owing to its poor adaption to fast-varying channel environments, an improved version with subcarrier monopolization (OWM) is put forward by confine the subcarrier indicators to binary digits. It works well in fast-varying channel, and besides has a simpler computational complexity. In fact, OWM is the special case of OWT with binary time-sharing indicator and OWT can be seen as the tight upper bound of the OWM. To the best of our knowledge, such algorithms and their relation have not been investigated accurately in cooperative OFDMA networks in the literature.

The rest of this paper is organized as follows. Section 2 describes the system model and gives the problem formulation. Section 3 transforms the problem to a convex optimization and solves it with OWT. In Section 4 an improved algorithm OWM is proposed and some comparisons are made with OWT in terms of complexity and environmental adaption. In section 5 the proposed algorithms are evaluated through simulations in different scenarios and the conclusion is drawn in section 6.

2. System Model and Problem Formulation

In this section, we first present the system model and some fundamental assumptions, and then formulate the problem with some constraints.

2.1 System Model

Consider a single-cell OFDMA system with half-duplex DF relaying containing one BS in the center, M RNs and K MSs. All the RNs are uniformly distributed around the BS on a ring [4]. The radius of the cell and the ring are r and $R\left(R>r\right)$ meters respectively. Since it is always the cell-edge users that are the bottleneck for system service, we focus the resource allocation on the users outside of the relaying ring, as shown in **Fig. 1**. All the K users are randomly distributed between the ring and the cell edge. The total B Hz bandwidth is divided into N subchannels.

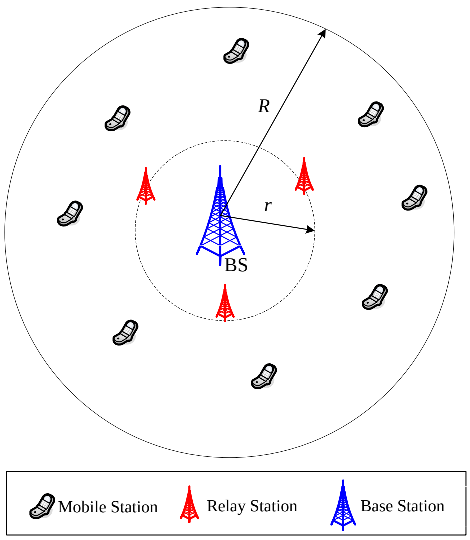

Fig. 1. System topology

The system works in a half time division duplex (half-TDD) way. In the first slot, the BS broadcasts information, and the RNs receive and decode the information they need. In the second slot, the RNs re-encode the signals and forward it to the corresponding users. Note that the received signals from the BS directly of cutting-edge users are too weak, so we can ignore it in analyzing results.

There are three resources in this model. The first one is the RN. Each user has to access one or more RNs to communicate with the BS, and the decision is made dynamically according to the simultaneous channel gain. The second resource is power. We assume that the maximum transmission power of the BS and RN is P_{B_T} and P_{R_T} individually. The BS assigns $p_{m,k}^{B}(n)$ power for the relay m on subcarrier n in the first hop, wherein the subscript k indicates the target user. Likewise, the m th RN allocates $p_{m,k}^{R}(n)$ power for user k on the n th subcarrier. Accordingly, a data link from the BS to the k th MS via RN m on subcarrier n is set up, denoted as $(B-m-k,n)$.

The last one is subcarrier. As is known, the spectrum is divided into a number of narrow subchannels which keep orthogonal one another in OFDM modulation. As long as each subcarrier is monopolized by one user or link, the intra-cell interference can be eliminated well. And that is exactly what the literature [1][2][3][4][5][6][7][8][9][10][11][12][13][14][15][16] has done. Obviously, the subcarrier assignment indicator will be formulated to a binary variable ρ, which leads the resource allocation to a mixed integer programming. It is NP-hard and can not be solved in polynomial time. To make it tractable, the subcarrier is often simply decided on principles like maximum weighted sum rate.

If the subcarrier indicator is relaxed to real number, the solution may be solved through some optimization methods. But the decimal assignment indicator of subcarriers seems to cause intra-cell interference.

However if try another perspective, like time, we may find that sharing subcarriers doesn't cause interference necessarily. In fact, the idea of time-sharing on subcarriers is first introduced into OFDMA systems in [17] but the author did not discuss any further. Considering a slow-varying wireless environment, users are sharing subcarriers by time in a relatively long term. At any given time unit, there is only one user occupying some subcarrier, and thus no intra-cell interference would be generated. Let's take a simple example. Two users share a subcarrier in $L=100$ unit time with $\rho_1=\rho_2=0.5$, so each user uses the subcarrier for 50 unit time. In this case ρ takes value from $[0,1]$ and also called as time-sharing factor [17].

2.2 Problem Description and Formulation

In this subsection, the optimization problem is formulated with combination of relay selection, subcarrier allocation and power control.

Considering a data link $(B-m-k,n)$, the capacity of the first hop is given by

$$C_{m,k}^{B}(n)=\frac{1}{2}\log_2\left(1+\frac{p_{m,k}^{B}(n)\cdot\left|H_{m}^{B}(n)\right|^2}{\sigma^2}\right) \tag{1}$$

where the multiplier $1/2$ is due to the fact that a link with relaying needs double time to transmit the same information than that of a non-relay link. $\left|H_m^B(n)\right|^2$ is the channel gain (with the pathloss included) from BS to the m th RN, which can be obtained through perfect channel estimation. σ^2 denotes the power of the additive white Gaussian noise (AWGN) on each subchannel at the receiver.

Similarly, the capacity on the second hop is expressed as

$$C_{m,k}^R(n) = \frac{1}{2}\log_2\left(1 + \frac{p_{m,k}^R(n)\cdot\left|H_m^k(n)\right|^2}{\sigma^2}\right) \tag{2}$$

where $\left|H_m^k(n)\right|^2$ is the channel gain from RN m to MS k. Clearly, the capacity of the DF link [18] is

$$C_m^k(n) = \min\left\{C_{m,k}^B(n), C_{m,k}^R(n)\right\} \tag{3}$$

Then, the system capacity in unit of bit/s/Hz over all RNs and subcarriers is given as

$$U(\mathbf{p}^B, \mathbf{p}^R, \boldsymbol{\rho}) = \frac{1}{N}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_m^k(n)C_m^k(n) \tag{4}$$

where $\mathbf{p}^B$ ($\mathbf{p}^R$) is the power vector of BS (RNs), and $\boldsymbol{\rho}$ is the subcarrier assignment indicator vector. $\mathbf{p}^B$ covers all the power variables on the BS allocated for the RNs, and all these variables need to be optimized:

$$\mathbf{p}^B = \left\{p_{m,k}^B(n) \middle| m = 1,2,\cdots,M,\ k = 1,2,\cdots,K,\ n = 1,2,\cdots,N\right\}$$

Similarly, $\mathbf{p}^R = \left\{p_{m,k}^R(n) \middle| m = 1,2,\cdots,M,\ k = 1,2,\cdots,K,\ n = 1,2,\cdots,N\right\}$

$\boldsymbol{\rho} = \left\{\rho_m^k(n) \middle| m = 1,2,\cdots,M,\ k = 1,2,\cdots,K,\ n = 1,2,\cdots,N\right\}$

Assuming that each user has a required minimum data rate R_k, which means when the rate available is less than R_k some bad cases like outage may occur. To maintain the fairness of the users, the following condition should be met:

$$\frac{1}{N}\sum_{m=1}^{M}\sum_{n=1}^{N}\rho_m^k(n)C_m^k(n) \geq R_k \quad \forall k \tag{5}$$

The objective of the problem in this paper is to

$$\text{maximize } U\left(\mathbf{p}^B, \mathbf{p}^R, \boldsymbol{\rho}\right) \tag{6}$$

subject to:

$$\begin{aligned}
&C1: \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} p_{m,k}^{B}(n)\rho_m^k(n) \le P_{B_T} \\
&C2: \sum_{k=1}^{K}\sum_{n=1}^{N} p_{m,k}^{R}(n)\rho_m^k(n) \le P_{R_T} \quad \forall m \\
&C3: \frac{1}{N}\sum_{m=1}^{M}\sum_{n=1}^{N} \rho_m^k(n)C_m^k(n) \ge R_k \quad \forall k \\
&C4: \sum_{m=1}^{M}\sum_{k=1}^{K} \rho_m^k(n) \le 1 \quad \forall n \\
&C5: \rho_m^k(n) \in [0,1] \quad \forall m,k,n \\
&C6: p_{m,k}^{B}(n) \ge 0, \quad p_{m,k}^{R}(n) \ge 0, \quad \forall m,k,n
\end{aligned} \tag{7}$$

The constraint C1 (C2) prevents the power actually used by BS (RN) from exceeding P_{B_T} (P_{R_T}). C3 ensures the fairness among users. C4,C5 set the time-sharing factor in $[0,1]$, and the total multiplexing in time is no more than 1. C6 confines the power allocated for the users or relays to a positive value.

3. Optimal Resource Allocation with Time-Sharing

In this section the problem (6)(7) is first transformed to a standard convex optimization, and then decomposed in the dual domain [19][20]. The water-filling and bisection method are used to search the solution of the subproblems. The last subsection gives the procedure of the optimal resource allocation with time-sharing (OWT).

3.1 Convex Transformation

Clearly, equation (3) takes its maximum only when $C_{m,k}^{B} = C_{m,k}^{R}$ holds. Setting (1) equal to (2) yields

$$p_{m,k}^{B}(n)\left|H_m^B(n)\right|^2 = p_{m,k}^{R}(n)\left|H_m^k(n)\right|^2 \tag{8}$$

If define $\eta_m^k(n) = \left|H_m^k(n)\right|^2 \Big/ \left|H_m^B(n)\right|^2$ as the channel gain's ratio of the two hops, we have the following relation:

$$p_{m,k}^{B}(n) = p_{m,k}^{R}(n)\cdot\eta_m^k(n) \tag{9}$$

Substituting $p_{m,k}^{B}(n)$ with (9), the problem (6)(7) is written as

$$\text{maximize } U(\mathbf{p}^R, \boldsymbol{\rho}) \tag{10}$$

subject to C2,C4,C5,C6 and

$$\begin{aligned}
&C1: \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} p_{m,k}^{R}(n)\eta_m^k(n)\rho_m^k(n) \le P_{B_T} \\
&C3: \frac{1}{N}\sum_{m=1}^{M}\sum_{n=1}^{N} \rho_m^k(n)C_{m,k}^{R}(n) \ge R_k \quad \forall k
\end{aligned} \tag{11}$$

The objective function $U(\mathbf{p}^R, \boldsymbol{\rho}) = \frac{1}{N}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_m^k(n)C_{m,k}^R(n)$ is not concave in (p,ρ) although $C_{m,k}^R(n)$ is a convex function of p. Moreover, the set of C1 (or C2) is not convex, either. As a result, the problem (10)(11) is not a convex programming.

Definition: For ease of process, a new power variable is defined as

$$\tilde{p}_{m,k}^R(n) = p_{m,k}^R(n)\cdot\rho_m^k(n) \tag{12}$$

In fact, compared with $p_{m,k}^R(n)$, the auxiliary power $\tilde{p}_{m,k}^R(n)$ has a more clear physical interpretation: the actual power allocated by RN m for user k on the n th subcarrier. Whereas $p_{m,k}^R(n)$ is somewhat virtual, because even it has a positive value, if there is no subcarrier assigned, the actual power should be void. If $\rho_m^k(n)\neq 0$, the second hop's capacity is re-written as

$$\tilde{C}_{m,k}^R(n) = \frac{1}{2}\log_2\left(1+\frac{\tilde{p}_{m,k}^R(n)\cdot\left|H_m^k(n)\right|^2}{\rho_m^k(n)\cdot\sigma^2}\right) \tag{13}$$

The problem (10)(11) is re-formulated as

$$\text{maximize}\sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}\rho_m^k(n)\tilde{C}_{m,k}^R(n) \tag{14}$$

subject to C4,C5,C6 and

$$C1:\ \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N}\tilde{p}_{m,k}^R(n)\eta_m^k(n)\le P_{B_T}$$

$$C2:\ \sum_{k=1}^{K}\sum_{n=1}^{N}\tilde{p}_{m,k}^R(n)\le P_{R_T}\quad \forall m$$

$$C3:\ \frac{1}{N}\sum_{m=1}^{M}\sum_{n=1}^{N}\rho_m^k(n)\tilde{C}_{m,k}^R(n)\ge R_k\quad \forall k$$

Note that the constant coefficient $1/N$ is neglected for simplicity. Evidently, the set of C1~C6 are convex. On the other hand, according to the perspective of a function [20], the objective function (14) is concave over the convex set $(\tilde{p},\rho)$. Hence, (14) is a convex maximization programming and thus the global optimum solution can be found in polynomial time. And the power variables $\tilde{p}_{m,k}^R(n)\quad \forall m,k,n$ and subcarrier indicators $\rho_m^k(n)\quad \forall m,k,n$ need to be optimized.

3.2 Dual Decomposition

It has been proved in [20] that the duality gap for convex optimization keeps zero. In this subsection the problem (14) is decomposed in the dual domain. We first give the Lagrange dual function as

$$\begin{aligned} & L\left(\mathbf{p}^R, \boldsymbol{\rho}, \alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}\right) \\ & = \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} \rho_m^k(n)\tilde{C}_{m,k}^R(n) + \alpha\left[P_{B_T} - \sum_{m=1}^{M}\sum_{k=1}^{K}\sum_{n=1}^{N} \eta_m^k(n)\tilde{p}_{m,k}^R(n)\right] \\ & \quad + \sum_{m=1}^{M}\beta_m\left[P_{R_T} - \sum_{k=1}^{K}\sum_{n=1}^{N}\tilde{p}_{m,k}^R(n)\right] + \sum_{k=1}^{K}\mu_k\left[\sum_{m=1}^{M}\sum_{n=1}^{N}\rho_m^k(n)\tilde{C}_{m,k}^R(n) - R_k\right] \\ & \quad + \sum_{n=1}^{N}\lambda_n\left[1 - \sum_{m=1}^{M}\sum_{k=1}^{K}\rho_m^k(n)\right] \end{aligned} \tag{15}$$

where $\mathbf{p}^R, \boldsymbol{\rho}$ are primal variables. The dual variable $\alpha, \boldsymbol{\beta}$ are the price per unit power of BS and RNs. $\boldsymbol{\mu}$ and $\boldsymbol{\lambda}$ correspond to users' required data rate and subcarrier occupation respectively. The dual objective is stated as

$$g(\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}) = \max_{\mathbf{p}^R, \boldsymbol{\rho}} L\left(\mathbf{p}^R, \boldsymbol{\rho}, \alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}\right) \tag{16}$$

And the dual problem of (14) is

$$\begin{aligned} & \min_{\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}} g(\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}) \\ & \text{s.t.} \quad \alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda} \geq \mathbf{0} \end{aligned} \tag{17}$$

We decompose the problem (17) into one master problem and $M \times K$ subproblems. The (m, k) subproblem is given as

$$\begin{aligned} l_{m,k}(\mathbf{p}^R, \boldsymbol{\rho}) & = \sum_{n=1}^{N} \rho_m^k(n)\tilde{C}_{m,k}^R(n) - \alpha\sum_{n=1}^{N}\eta_m^k(n)\tilde{p}_{m,k}^R(n) \\ & \quad - \beta_m\sum_{n=1}^{N}\tilde{p}_{m,k}^R(n) + \mu_k\sum_{n=1}^{N}\rho_m^k(n)\tilde{C}_{m,k}^R(n) - \sum_{n=1}^{N}\lambda_n\rho_m^k(n) \\ & = \left(1+\mu_k\right)\sum_{n=1}^{N}\rho_m^k(n)\tilde{C}_{m,k}^R(n) - \left(\alpha+\beta_m\right)\sum_{n=1}^{N}\left[1+\eta_m^k(n)\right]\tilde{p}_{m,k}^R(n) - \sum_{n=1}^{N}\lambda_n\rho_m^k(n) \end{aligned} \tag{18}$$

Using the Karush-Kuhn-Tucker (KKT) conditions, if the optimum solution exists, the partial derivatives must satisfy the following equation or inequations.

1) if $\rho_m^k(n)^* = 0$, then $\tilde{p}_{m,k}^R(n)^* = p_{m,k}^R(n)^* \cdot \rho_m^k(n)^* = 0$. The maximum is obtained at the boundary point $(0,0)$, and the differentiation there should be

$$dl_{m,k} = \frac{\partial l_{m,k}}{\partial \rho_m^k(n)} d\rho_m^k(n) + \frac{\partial l_{m,k}}{\partial \tilde{p}_{m,k}^R(n)} d\tilde{p}_{m,k}^R(n) \leq 0 \tag{19}$$

2) if $\rho_m^k(n)^* \neq 0$, we have

$$\begin{aligned} \frac{\partial l_{m,k}}{\partial \tilde{p}_{m,k}^R(n)^*} & = \frac{\left(1+\mu_k\right)}{2\ln 2} \cdot \frac{\rho_m^k(n)^* \cdot \gamma_m^k(n)}{\rho_m^k(n)^* + p_{m,k}^R(n)^*\gamma_m^k(n)} - \alpha\eta_m^k(n) - \beta_m \\ & \begin{cases} = 0 & \tilde{p}_{m,k}^R(n)^* > 0 \\ < 0 & otherwise \end{cases} \end{aligned} \tag{20}$$

$$\frac{\partial l_{m,k}}{\partial \rho_m^k(n)^*} = W_m^k(n) - \lambda_n \begin{cases} = 0 & \rho_m^k(n)^* \in (0,1) \\ > 0 & \rho_m^k(n)^* = 1 \end{cases} \tag{21}$$

where $W_m^k(n) = \frac{1+\mu_k}{2}\left\{\log_2\left[1+\frac{\tilde{p}_{m,k}^R(n)^*\gamma_m^k(n)}{\rho_m^k(n)^*}\right] - \frac{1}{\ln 2}\cdot\frac{\tilde{p}_{m,k}^R(n)^*\gamma_m^k(n)}{\rho_m^k(n)^* + \tilde{p}_{m,k}^R(n)^*\gamma_m^k(n)}\right\}$, and $\gamma_m^k(n)$ is defined as the channel noise ratio (CNR), $\gamma_m^k(n) = \left|H_m^k(n)\right|^2 / \sigma^2$.

3.3 Solution of the Subproblems

The subgradient method [19][20] is used to tackle the subproblems. By setting equation (20) to zero, we first get the actual power $\tilde{p}_{m,k}^R(n)^*$ allocated by RN m for the k th user on the n th subcarrier as

$$\tilde{p}_{m,k}^R(n)^* = \rho_m^k(n)^*\left[\frac{1}{2\ln 2}\cdot\frac{1+\mu_k}{\alpha\eta_m^k(n)+\beta_m} - \frac{1}{\gamma_m^k(n)}\right]_0^{P_{R_T}} \tag{22}$$

Compared with the definition of $\tilde{p}_{m,k}^R(n)$ in (12), we have

$$p_{m,k}^R(n)^* = \left[\frac{1}{2\ln 2}\cdot\frac{1+\mu_k}{\alpha\eta_m^k(n)+\beta_m} - \frac{1}{\gamma_m^k(n)}\right]_0^{P_{R_T}} \tag{23}$$

(22)(23) can be interpreted as the multi-level water-filling schemes. Different users adjust the water level through dual variables α, β_m, μ_k .However the solution for $\rho_m^k(n)^*$ is much more complicated.

Proposition**:** The solution of time-sharing factor $\rho_m^k(n)^*$ is sure to exist, and can be solved by the bisection method.

Proof: Two cases are considered separately:

a) If $\tilde{p}_{m,k}^R(n) = 0$, we have $\rho_m^k(n)^* = 0$. The reason is if no power is allocated to the data link $m-k$ on subcarrier n, no subcarrier shall be assigned on the link, either;

b) If $\tilde{p}_{m,k}^R(n) > 0$, equation (21) is a transcendental function and the solution cannot be put in a closed-form. We first give the analysis about the existence of the solution. We re-arrange (21) as

$$\rho_m^k(n) = f\left(\rho_m^k(n)\right) \tag{24}$$

where $f\left(\rho_m^k(n)\right) = \frac{\tilde{p}_{m,k}^R(n)\gamma_m^k(n)}{2^{X_m^k(n)}-1}$ and $X_m^k(n) = \frac{2\lambda_n}{1+\mu_k} + \frac{\tilde{p}_{m,k}^R(n)\gamma_m^k(n)}{\ln 2\cdot\left(\rho_m^k(n) + \tilde{p}_{m,k}^R(n)\gamma_m^k(n)\right)}$.

For ease of description, the subscript and superscript are omitted in the rest of the proof. Equation (24) can be simplified as two functions $y = \rho$ and $y = f(\rho)$. Examining the property of $y = f(\rho)$, it follows that,

i) Setting $\rho = 0$ yields $f(0) = \tilde{p}\gamma / \left(2^{2\lambda/(1+\mu)+1/\ln 2} - 1\right)$, which is the intersection on the y-axis;

ii) Setting $\rho \to +\infty$ yields $f(\infty) = \tilde{p}\gamma / \left(2^{2\lambda/(1+\mu)} - 1\right)$, which is the asymptote of $y = f(\rho)$.

It is clear from **Fig. 2** that the two curves are bound to intersect at a positive point ρ^{**}, which is the zero point of equation (21). Notice that since the intersection on the y-axis $f(0)$ is positive, ρ^{**} won't take an exact zero value as long as $\tilde{p}_{m,k}^{R}(n) > 0$. On the other hand, ρ^{*} is invalid beyond the bounds of the interval $[0,1]$. Finally, combined i) and ii), we have

$$\rho^{*} = \begin{cases} \rho^{**} & \rho^{**} \in [0,1) \\ 1 & \rho^{**} \in [1,+\infty) \end{cases} \tag{25}$$

hold for any $\tilde{p}_{m,k}^{R}(n) \geq 0$.

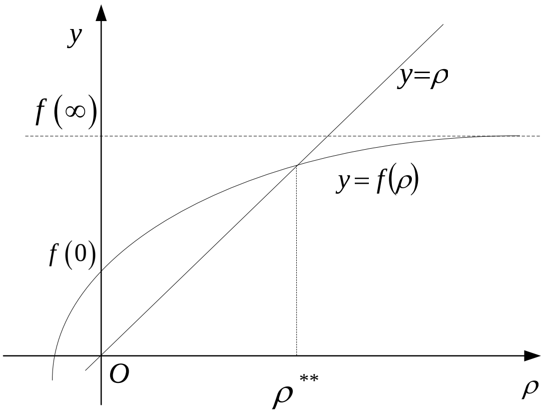

Fig. 2. the curves of the functions

3.4 Dual Updates

In this subsection the dual variables are updated by subgradient method. More specifically, the update may be performed as follows:

$$\begin{aligned} \alpha^{(i+1)} &= \left\{ \alpha^{(i)} - \varepsilon_{\alpha}^{(i)} \left[P_{B_T} - \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} \eta_m^k(n) \tilde{p}_{m,k}^{R}(n) \right] \right\}^{+} \\ \beta_m^{(i+1)} &= \left\{ \beta_m^{(i)} - \varepsilon_{\beta}^{(i)} \left[P_{R_T} - \sum_{k=1}^{K} \sum_{n=1}^{N} \tilde{p}_{m,k}^{R}(n) \right] \right\}^{+} \\ \mu_k^{(i+1)} &= \left\{ \mu_k^{(i)} - \varepsilon_{\mu}^{(i)} \left[\sum_{m=1}^{M} \sum_{n=1}^{N} \rho_m^k(n) \tilde{C}_{m,k}^{R}(n) - R_k \right] \right\}^{+} \\ \lambda_n^{(i+1)} &= \left\{ \lambda_n^{(i)} - \varepsilon_{\lambda}^{(i)} \left[1 - \sum_{m=1}^{M} \sum_{k=1}^{K} \rho_m^k(n) \right] \right\}^{+} \end{aligned} \tag{26}$$

where i is the iteration number, $\varepsilon^{(i)}$ is a sequence of scalar step sizes, and $\{\cdot\}^{+}$ is defined as $\{\cdot\}^{+} = \max(\cdot,0)$. To guarantee the convergence of the updates, the step size must meet some criterions [21]. In this paper we take it as

$$\varepsilon_z^{(i)} = c_z / i, \quad \text{for } z = \alpha, \beta, \lambda, \mu \tag{27}$$

where c_z is a constant to fine-tune the convergence speed.

The dual variables assist as follows:

1) α (or β_m) denotes the transmission cost per bit information of BS (RN). When the total power of the BS(RN) exceeds the maximum, the cost would be increased to restrain allocation by adjusting the water-level in equation (23). The cost of subcarrier occupation λ_n works in a similar way.
2) μ_k represents the system revenue per bit loaded to user k. When the data rate available of user k is below the demand, the revenue would be raised to promote the assignment of the subcarrier and power.

3.5 Resource Allocation with Time-Sharing

***Definition*:** To put the time-sharing scheme into practice, the first thing we need do is to transform these decimal time factors to integers by multiplying the sharing term L (which generally is a large number):

$$t_m^k(n) = \lfloor L \cdot \rho_m^k(n) \rfloor \tag{28}$$

The time $t_m^k(n)$ allocated for user k must be multiples of the resource granularity. If we take OFDM symbol (In this paper, each OFDM symbol contains N data subcarriers) as the resource granularity, the interpretation of (28) is that user k occupies the subcarrier n for $\lfloor L \cdot \rho_m^k(n) \rfloor$ symbols in every L OFDMA symbols. Finally, taking the lower bound is to guarantee the sum of the occupation time for all users won't exceed the total time L.

Notice that the truncation error occurs as a result of the round operation. Particularly, when $L \cdot \rho_m^k(n)^* < 1$, the data link $m-k$ cannot use the nth subcarrier at all. The procedure of OWT is described in **Table 1**.

Table 1. The procedure of OWT

Algorithm 1: OWT
1: set the total shared time L;
2: initialize $\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}, I_{\max}, \delta, \rho_m^k(n)$, set iteration number $i = 0$;
3: while $i < I_{\max}$ and $\max(\|\Delta(\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda})\|) > \delta$
4: compute $p_{m,k}^R(n)$ by (23);
5: compute $\tilde{p}_{m,k}^R(n)$ and search $\rho_m^k(n)$;
6: update $\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\lambda}$ by (26);
7: update iteration number as $i = i + 1$;
8: end while
9: calculate the share time $\lfloor L \cdot \rho_m^k(n)^* \rfloor$ for $m-k$;
10: perform the resource allocation according result in step 8 until L;

Remarks: OWT is performed every L symbols, and in this sense, the computational complexity is reduced to $1/L$ of the original one. The truncation error depends on L. Specifically, a large L would lead to a small error and vice versa. On the other hand, since the channel state is assumed time invariant, the channel variance is neglected during the sharing time. Consequently, OWT adapts to slow-varying channel well but degrades in fast-varying environment. This proposition would be validated in part 5.

4. Optimal Resource Allocation with Monopolization

The OWT algorithm proposed in section 3 fulfills the subcarrier demand of users precisely, but only works well in slow-varying environment. In this section, an improved version (OWM) with subcarrier monopolization is proposed, wherein the sharing is forbidden.

4.1 Problem Description and Formulation

With the constraint $C5: p_m^k(n) \in [0,1]$, the accurate solution for the time-sharing factor can be found by the bisection method. But obviously, the bisection search is very time-consuming. If we set the time-sharing factor binary, they can be decided easily without the exhaustive search. What is more, from the simulation results in section 5, we find most of the time-sharing factors are exact 1. This reveals that relaxing the subcarrier from decimal to binary should not cause much degradation.

Compared with OWT, the problem of OWM is almost the same except for the binary constraint. The constraint C5 is substituted as

$$C'5: \quad \rho_m^k(n) \in \{0,1\} \quad \forall m,k,n \tag{29}$$

The problem is formulated as

$$\text{maximize} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} \rho_m^k(n) \tilde{C}_{m,k}^R(n) \tag{30}$$

subject to

$$\text{C1,C2,C3,C4,C'5,C6,C7}$$

4.2 Solutions

After convex transformation and dual decomposition the formulas (19)(20)(21) are obtained. The power $p_{m,k}^R(n)$ is derived from (20). As for the subcarrier assignment indicator $\rho_m^k(n)$, from (19)(21) we have

$$\rho_m^k(n) = \begin{cases} 1 & W_m^k(n) > \lambda_n \\ 0 & W_m^k(n) < \lambda_n \end{cases} \tag{31}$$

Combining the constraint C4 and C'5, we conclude that for each subcarrier n at most one $\rho_m^k(n)$ equals 1 but it is not necessarily satisfied for (31). So we adjust it as

$$\rho_m^k(n)^* = \begin{cases} 1 & W_m^k(n) = \max_{x,y} W_x^y(n) \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

where $W_m^k(n)$ can be interpreted as the weighted data rate of link $m-k$ on subcarrier n. Equation (32) assigns subcarrier n to the link owning the max weighted data rate. Further, since the indicator ρ is decided by (32), there is no need to update the dual variable λ_n in master problem. The procedure of OWM is shown in **Table 2**.

Table 2. The procedure of OWM

Algorithm 2: OWM
1: initialize $\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}, I_{\max}, \delta, \rho_m^k(n)$, set iteration number $i=0$;
2: while $i < I_{\max}$ and $\max(\|\Delta(\alpha, \boldsymbol{\beta}, \boldsymbol{\mu})\|) > \delta$
3: compute $p_{m,k}^R(n)$ by (23);
4: compute $W_m^k(n)$ by (21) and decide $\rho_m^k(n)$ by (32);
5: update $\alpha, \boldsymbol{\beta}, \boldsymbol{\mu}$, by (26);
6: update iteration number as $i = i+1$;
7: end while

Remarks: There exist $M \times K \times N$ subcarrier assignment indicators which are solved through bisection method in OWT. To search them, the complexity is at least $O(M \times K \times N)$ while for OWM the complexity is $O(1)$. On the other hand, OWT performs every L time slices which equivalently reduces the complexity to $O(M \times K \times N/L)$. In general, L takes a large value and it follows that $O(L) = O(N)$. Therefore, the complexity of OWT is about $O(M \times K)$, a function of the number of RNs and MSs.

Note that OWT with $L=1$ is not strictly in accordance with OWM, because the ways to calculate the subcarrier assignment indicator are different. When $L=1$, OWT simply rounds ρ to an integer while OWM chooses the link with the max weighted data rate denoted by equation (32).

5. Simulation Results and Analysis

In this section, the performances of OWT and OWM are evaluated through simulation results, which mainly involve the system average throughput and users' satisfaction. We first show the system setups.

5.1 System Setup

A single-cell scenario with dedicated relays is considered, whose topology is shown in Fig.1. The inner radius is $r = 0.4$ km, and the outer radius is $R = 1$ km. There are $M = 3$ relay stations distributed on the inner circle uniformly and $K = 8$ mobile stations distributed between the circles randomly. In this paper we assume that each user has an identical required data rate. A bandwidth of $B = 2$ MHz spectrum is divided into $N = 128$ subcarriers with carrier center frequency $f = 2$ GHz. The AWGN variance over each subchannel is $\sigma^2 = -122$ dBm. The maximum transmission power of the base station and relay station are assumed to be identical: $P_{B_T} = P_{R_T} = P_T$.

The wireless links of both BS to RN and RN to MS are subject to independent identical distributed (i.i.d) Rayleigh fading. The ITU-R M.1225 channel profile in [22] is adopted, in which three test environments are defined: Indoor/Outdoor/Vehicular. To evaluate the adaption of the proposed schemes to time-varying scenarios, we employ three typical maximum Doppler shift 5Hz/70Hz/300Hz in [23] corresponding to low/medium/high value respectively. Shadowing effects are not considered here.

5.2 Simulation Results

The simulation results are averaged over 1000 independent trials and in each trial the locations of the MSs are also newly distributed.

5.2.1 Convergence

We first verify the convergence of the proposed schemes. **Fig. 3** and **Fig. 4** show the converging process of OWT and OWM in Indoor A profile with Doppler $f_d = 5$ Hz. **Fig. 3** is the dual variable α of BS power constraint and **Fig. 4** is the variable μ_1 of the first user's rate constraint. It reveals that the two schemes have almost the same convergence speed. After 50 iterations, the variables achieve about 90% to 95% of the optimal; after 100 iterations, the variables reach about 95% to 98%. Briefly, the schemes proposed converge well.

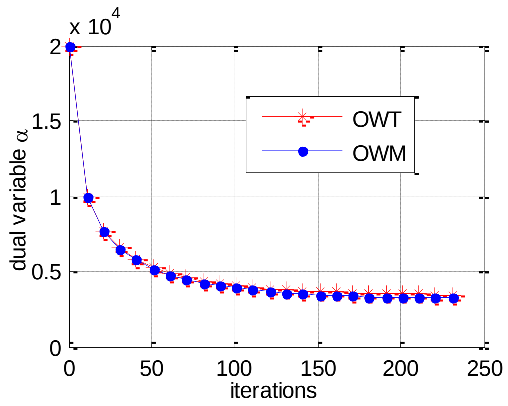

Fig. 3 Convergence of dual variable α

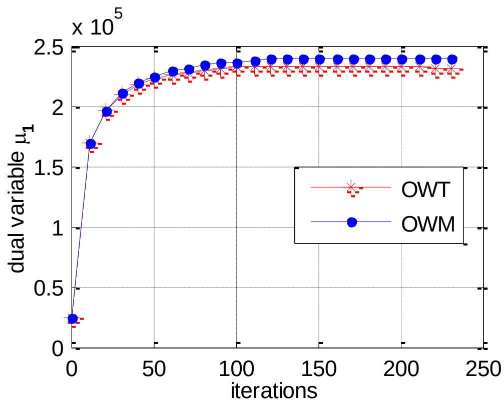

Fig. 4 Convergence of dual variable μ_1

5.2.2 Effects of L on OWT

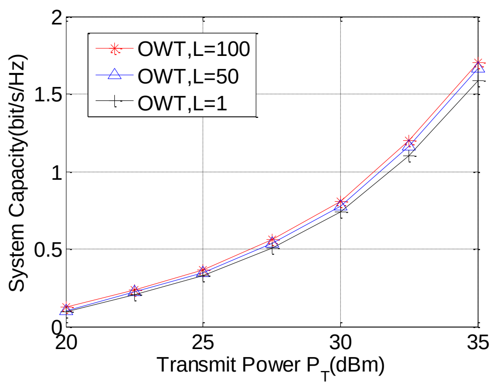

Fig.5 System capacity with sharing time

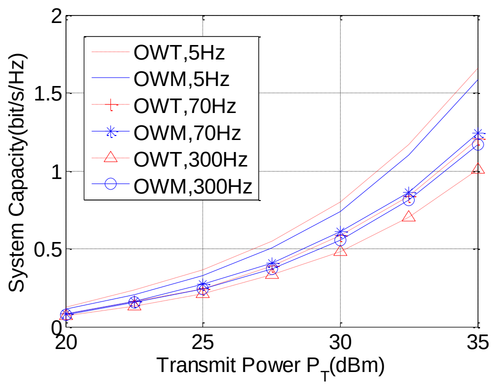

Fig.6 System capacities on various scenarios

Fig. 5 shows the effects of the total sharing time L on OWT in Indoor A profile with Doppler $f_d = 5$ Hz. As analyzed in section 3, rounding to integer causes truncation error and the system average capacity increases slowly as the total sharing time raise. Particularly, the error reaches the maximum when L takes 1. However, the performance increased is a bit slight with L, due to the fact found in simulations that most of the subcarriers are assigned to some user exclusively.

5.2.3 Adaption of OWT & OWM

Fig. 6 compares system average throughput of OWT ($L = 100$) and OWM in various channel environments as **Table 3**. We mainly investigate the adaptation to the time varying channels of the schemes.

Table 3. Channel model and Doppler shift

Channel model	Doppler shift
Indoor A	5Hz
Outdoor B	70Hz
Vehicular A	300Hz

From the **Fig. 6**, OWT performs well in the slow varying indoor scenarios, and OWM is very close to the upper bound, with no more than 10% degradation. In outdoor scenarios, the channel changes a bit sharp. The performance gap between OWT and OWM is counterbalanced by the time diversity and thus OWM outperforms OWT slightly. Furthermore, the channel experiences rapid variation in vehicular profile. In this case, OWT doesn't suit any more and has a large degradation up to 20 percent compared with OWM.

5.2.4 Comparison With Other Schemes

Fig. 7 and **Fig. 8** make a comparison of the algorithms proposed with some other schemes in Indoor A profile with $f_d = 5$ Hz. The schemes concerned are:

- [3] without SA (Subcarrier Allocation): The allocation proposed in [3] always assigns the subcarrier to the user has the best channel gains. This greedy-wise idea leaves the edge-users in starvation at all and completely deviates from our purpose. So we choose the scheme with random subcarrier assignment which looks after users' fairness, i.e. the scheme without Subcarrier Allocation in [3].
- OWM fixed RS: the same as OWM except for the relay selection (RS) policy, wherein the fixed selection in [9] is taken. The users always access the relay station with the best average channel gains.
-

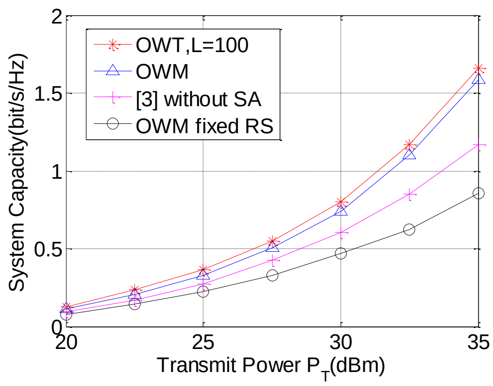

Fig. 7 Comparison of system capacity

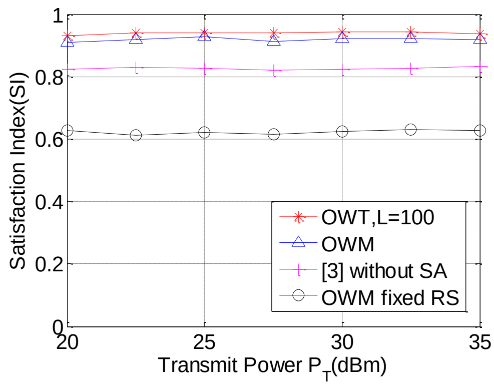

Fig. 8 Comparison of satisfaction index

The satisfaction index I defined in [7] is adopted to evaluate the fairness and service quality as follows:

$$I = \frac{1}{M}\sum_{m=1}^{M}\min\left(\frac{c_m}{\bar{c}_m},1\right) \tag{33}$$

which means all of the users meet their demands when satisfaction index I equals 1. Moreover, if we assume all users have an identical rate requirement, satisfaction index also reflects the fairness among users. Correspond to the transmit power in figures, the rate demand is 0.01 bit/s/Hz， 0.03 bit/s/Hz， 0.05 bit/s/Hz， 0.07 bit/s/Hz， 0.1 bit/s/Hz， 0.15 bit/s/Hz， 0.2bit/s/Hz in turn.

Fig. 7 shows the system average capacity of the four schemes while **Fig. 8** compares the satisfaction index in indoor scenario. Both the capacity and the satisfaction index of OWM are close to OWT, between which the gap is no more than 10%. This is mainly because a great part of the subcarriers are occupied by users exclusively and thus the performance degrades little in slow-varying channels. The algorithm in [3] without subcarrier allocation selects subcarrier randomly. In this case, the fairness between users can be achieved while it doesn't make full use of the multi-user diversity. OWM with fixed relay selection has a relatively lower performance due to some bad cases. Consider that when the burden of the relays is in great difference, many MSs share one "busy" relay's power while the extra power of the other relays cannot be exploited because of the stationary strategy. Finally, the system performance is bad.

6. Conclusion

In this paper, the downlink resource allocation problem in cooperative OFDMA networks is investigated. The issues of relay selection, subcarrier allocation and power control are jointly formulated to maximize the system average throughput with per-BS and per-RN power constraint. Owing to the extremely complexity of the original problem, we decouple it into subproblems in the dual domain after convex transformation and settle it with multi-level water-filling and bisection method. In particular, an optimal resource allocation scheme with subcarrier time-sharing OWT is proposed in slow-varying environments and an improved version OWM is proposed for general wireless channels, whose performance increases about 20% than OWT in fast-varying vehicular channels. The simulation results reveal that the proposed schemes converge rapidly and outperform the traditional algorithms both in system throughput and in users' fairness.

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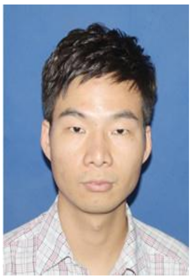

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